Game Theoretic Stochastic Routing for Fault Tolerance and Security in Computer Networks

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Abstract

We introduce the Game-Theoretic Stochastic Routing (GTSR) framework, a proactive alternative to today's reactive approaches to route repair. GTSR minimizes the impact of link and router failure by (1) computing multiple paths between source and destination and (2) selecting among these paths randomly to forward packets. Besides improving fault-tolerance, the fact that GTSR makes packets take random paths from source to destination also improves security. In particular, it makes connection eavesdropping attacks maximally difficult as the attacker would have to listen on all possible routes. The approaches developed are suitable for network layer routing as well as for application layer overlay routing and multi-path transport protocols such as SCTP.

Through simulations, we validate our theoretical results and show how the resulting routing algorithms perform in terms of the security/fault-tolerance/delay/throughput trade-off. We also show that a beneficial side-effect of these algorithms is an increase in throughput, as they make use of multiple paths.

Keywords: Multi-path Routing, Stochastic Routing, Game Theory, Network Security, Fault Tolerance.

I. Introduction

"Traditional" routing protocols forward packets over a single path from source to destination. This means that, even if redundant resources are available, a single failure (accidental or due to malicious activities) temporarily interrupts all connections that use the compromised route. Eventually, the underlying routing protocol will react to the failure and correct it, but this can take a non-negligible amount of time, depending on how often routers ping/poll one another and how frequently routing updates are distributed. Another undesirable aspect of single-path routing is that packet interception or eavesdropping can be achieved with a minimum amount of resources, because the path over which packets travel is predictable. For example, by tapping into one of the links along a path, an attacker can reconstruct an unencrypted file transfer

from eavesdropped packets, and by breaking into one router, a man-in-the-middle attack can be launched on encrypted transfers that utilize key exchange [1].

In this paper, we describe *game-theoretic stochastic routing*, which can be viewed as a proactive alternative to today's reactive approaches to route repair. GTSR explores the existence of multiple paths between network nodes and routes packets to minimize predictability. It discovers all paths between a source-destination pair and determines next-hop probabilities, i.e., the probability with which a packet takes a particular next-hop along one of the possible paths. Unlike security and fault tolerance mechanisms that are based on detection and response, GTSR takes a *proactive* approach to making connections less vulnerable to failures or attacks.

Throughout this paper we use *nodes* to refer to the network elements that forward packets. Such nodes can be network layer routers; end systems (or hosts) that participate in application layer routing (e.g., in an overlay network such as RON [2]); or act as transport layer end-points (e.g., in SCTP that supports splitting of data streams over multiple paths [3]). From this perspective, GTSR can operate at any layer of the protocol stack. In the remainder of this section, we put GTSR in perspective by reviewing some existing approaches to failure prevention and recovery.

Failures, Interception, and Eavesdropping

The Internet is particularly vulnerable to failures and attacks because data packets travel along a physical infrastructure that is easily probed and is composed of thousands of elements each one of them vulnerable to attacks and failures. Dynamic routing protocols are designed to try to circumvent failures in network nodes and links. However, these protocols are purely reactive and wait for failures to be detected before taking corrective measures. While some failures can be easily discovered (e.g., if there is no carrier signal, the physical layer interface will report an error), intermittent failures are especially difficult to detect and routing often takes considerable time to recover from them. In addition, while some link-state algorithms can find alternate paths relatively quickly, distance-vector algorithms may require considerably more time in large, poorly connected networks. In either case, during the detection of the cut/failure and the search for a new route, the connection remains broken.

Selective faults/attacks are especially challenging. Suppose, for example, that a node of the network is compromised and that it selectively intercepts packets from specific end-to-end connections. If routing hello packets are not intercepted, other nodes do not compute new routes

to exclude the compromised node. This means that application or transport-layer retransmissions will keep following the same route and will continue to be subject to interception. A more "active" form of attack exploiting compromised nodes is to have them generate spurious routing updates, each reporting very low costs to all or some destinations. As a result, other nodes will funnel packets to these compromised nodes.

Privacy and integrity techniques (e.g., end-to-end encryption, VPNs, and secure tunnels) are effective in protecting against eavesdropping and some forms of interception (e.g., selective interception). However, if an encryption key is stolen or when defending against attackers with inside information, these techniques lose much of their effectiveness. Furthermore, man-in-the-middle attacks may still be possible. To protect a data transmission network against a wide range of attacks, one needs a suite of mechanisms spanning several layers of the protocol stack. In particular, end-to-end security mechanisms (e.g., end-to-end encryption for privacy and message authentication for integrity/authenticity) should be complemented by security mechanisms at other layers (e.g., link-layer encryption). GTSR is one such mechanism and complements security measures taken at other layers.

Exploring Multiple Paths

(Deterministic) flooding — i.e., sending each packet along every possible path — is the simplest form of multi-path routing and provides a routing mechanism that is extremely robust with respect to link/node failures. However, this type of routing is not practical because it greatly increases overhead . Flooding also significantly simplifies packet eavesdroppingWe propose to explore multiple paths through *statistical flooding*. In statistical flooding, packets are sent through all available paths but generally no packet is sent more than once (except for the possibility that error control at the transport layer may require a packet to be re-sent). Flooding thus occurs "on-average" and is randomized to minimize predictability.

Stochastic routing is the mechanism used to achieve statistical flooding. In this type of routing, nodes select the next hop where to forward a packet in a random fashion. The next-hop probabilities — i.e., the probabilities of selecting particular next-hop destinations — are design parameters and determining these probabilities is the subject of this paper. Our main challenge is determining of next-hop probabilities that ensure:

1) *Delivery*, i.e., all packets reach their desired destination with probability one.

2) *Timeliness*, i.e., the delay is acceptable.
3) *Statistical flooding* is achieved, i.e., all paths are explored by the packet ensemble.

With respect to 3, some network elements may be more susceptible to failures/attacks than others and therefore it may be desirable to have non-uniform statistical flooding. For example, as a fault/attack detector starts to suspect that a particular element is compromised, it can adjust the next-hop probabilities to start avoiding that element. With GTSR this can be done progressively, which allows for a measured response and provides additional flexibility in addressing the trade-off between the false alarm rate and detection speed (cf., [4] for a discussion on this trade-off).

Scope

One question to ask is "where can the GTSR methodology be effectively applied in today's networks?" Clearly, GTSR will only be effective if there are multiple paths to explore. At the network layer, we see at least two domains in which this technique may prove useful:

1) Inside Internet service providers (ISPs) where there are often multiple independent paths to the exit point for that ISP. GTSR would be part of the suite of tools that the ISP utilizes to provide secure networking to its clients.
2) Inside an organization that builds multi-path redundancy in its network to improve security (and also to augment throughput). An organization may utilize multiple connections to a single ISP, or even use multiple ISPs to connect to the outside network and employ GTSR to spread data across independent paths provided by distinct ISPs.

Although GTSR can also be used for inter-domain routing, it has been noted that Autonomous Systems (ASs) typically have limited information about the network topology and may have different (and often conflicting) path-selection goals [5]. This can limit the viability of GTSR for inter-domain routing.

At application layer overlay networks, the applicability of GTSR depends on the degree to which links in the overlay network are truly independent (i.e., do not share any physical links/nodes). While the presence of dependent paths does not affect the applicability of GTSR, if there are no independent paths at the network layer, then GTSR may not significantly increase security/reliability. On the other hand, if attacks target application layer routers instead of packet transmission (i.e., if the risk is that end-hosts taking part in the forwarding may be compromised),

then GTSR can increase reliability and security as long as the overlay network has multiple distinct paths between the source and destination.

At the transport layer, protocols such as SCTP allow data streams to be split over several paths. Again, the utility of splitting the flow depends on whether the different physical paths are distinct.

Outline

The remainder of this paper is organized as follows. In the next section, we review related work. Section III develops techniques for designing "optimal" stochastic routing policies. Simulations of these stochastic routing policies are presented in Section IV. Finally, Section V provides concluding remarks and points towards future research. Detailed derivations of the results in Section IV are included in the Appendix. A subset of the results in this paper was presented at the 11th IEEE Int. Conf. on Comput. Communications and Networks [6].

II. Related Work

A. Network robustness and security

Virtual Private Networks (VPNs) [7] have been used as a way to securely interconnect a (typically small) number of sites. While private networks use dedicated lines, VPNs try to implement private networks atop a publicly-accessible communication infrastructure like the Internet. VPNs typically employ some combination of encryption, authentication, and access control techniques to allow participating sites to communicate securely.

The emergence of IPsec [8] as an IETF standardized protocol has prompted VPN solutions to use IPsec as the underlying network-layer protocol. As in any encryption-based mechanism, the key challenge in IPsec is that secret keys must remain secret. As previously discussed, if an attacker is able to infiltrate a node or has "inside" information, shared and/or private keys may be compromised and the corresponding communication channels become insecure.

Onion routing [9] is another approach to security that focuses on hiding the identities of the communicators. It uses several layers of encryption, where each layer is used to encrypt the transmission between routers on each end of a link. Because of the many layers of encryption, routers are unable to decrypt the data or even the source and destination addresses. All that a router can decipher is the next-hop information. While onion routing is very effective for

anonymity, it is computationally heavy: each connection must be built and torn down, routers must encode and decode packets, and memory-intensive source routing is used. Secure BGP (S-BGP) [10] makes use of public key and an authorization infrastructure, as well as IPsec to verify the authenticity and authorization of control traffic generated by the Border Gateway Protocol.

Another related effort is the Resilient Overlay Networks (RON) project [2] whose goal is to improve the performance and robustness of network-layer routing. RON nodes monitor current routing paths and decide whether to choose other routes (by selecting alternate application-layer paths through other RON nodes) in order to meet application-specific performance requirements. GTSR could be implemented in such networks to improve security and fault tolerance.

The approach developed here is, in some ways, similar to that presented in [11], where members of a group cooperate to maintain their anonymity to the server. Specifically, users send their request not directly to the server but to random users in the group. Each user can then either forward the request to the server or to another member of the group. In GTSR, randomization is applied at the network layer with the goal to protect the data packet not from the destination, but from intermediate attackers.

Equal-Cost Multi-Path (ECMP) [12] and OSPF Optimized Multi-Path (OSPF-OMP) [13] are, in a way, multi-path routing algorithms. While these methods insist that the alternative paths be of equal cost, MPLS allows different paths, besides shortest paths, to be utilized. However, none of these methods is stochastic; rather, routers employ techniques to ensure that a connection will utilize a single path[1]. Hence a link failure will result in cut connections and eavesdropping at a single link will expose connections going through that link. Furthermore, these algorithms were developed to increase throughput but not to make routing robust to attacks or failures. Hence, there is no mechanism in place to avoid links that may have been compromised or have been subject to high failure rates. In short, these, as well as other load balancing techniques, seek to optimize performance metrics such as bit-rate and delay, while in GTSR we optimize fault tolerance and security.

[1]One reason that a connection is restricted to a single path is that many of today's implementations of TCP do not work well under persistent packet reordering. However, as is discussed in Section IV, advancements in TCP have relieved this limitation.

B. Game theoretical approaches to network routing

Game theoretical approaches have been proposed for many resource allocation problems in computer networks, but none very closely related to GTSR. We review the most relevant results in routing and refer the reader to the survey in [14] for applications of game theory to other networking problems, such as bandwidth allocation, congestion control, power control in wireless networks, and medium access control.

The authors of [15] consider the selection of one among L parallel queuing systems to transport (or process) data. Two classes of packets need to be processed: the α-class packets can be queued whereas the β-class packets are always blocked when the server is busy. The problem is formulated as a game played between two players P_α and P_β, each one controlling the fraction of packets of a particular class that go through each of the L queues. This problem has a unique Nash equilibrium (NE) when P_α minimizes the queuing delay and P_β the blocking probability.

The paper [16] considers a network with arbitrary topology in which every link has two queues with different priorities. Both queues are FIFO, but transmission of packets from the low priority queue only takes place when the high priority queue becomes empty. An independent nonzero-sum game is played at each node of the network, where one player (router) decides which outgoing link to use and the other player (network) decides whether the packet should be placed in the high or the low-priority queue. The authors show by simulation that the resulting routing algorithm decreases the overall number of packets that are dropped by timeout.

In [17], a group of nodes in a sensor network wants to construct a routing tree rooted at a destination node. This problem is formulated as a nonzero-sum game, in which every node is viewed as a player that independently decides its next hop to maximize an estimate of end-to-end path reliability, from which it subtracts a next-hop communication cost. For each candidate next-hop node, the path-reliability estimate depends on the reliability of the next-hop node and the probability that the link between the two nodes will fail. It turns out that computing the NE for this game is NP-hard.

The authors of [18] study the uniqueness of NE for the "selfish routing" problem, where a number of players share a network. Each player generates data at a fixed rate and needs to decide how to split its transmission among a set of alternative end-2-end paths. The players are selfish in the sense that each one attempts to minimize its own objective function, which

depends on the overall distribution of flows through the shared network. Under a similar problem setup, [19, 20] investigate how far the noncooperative NE is from the cooperative social optimum. Under appropriate simplifying assumptions on the network structure and the cost-functions, these papers establish worst-case bounds for the ratio between the noncooperative and the cooperative solutions. This line of research was further extended in [21] to the framework of repeated games, where the routing game is played repeatedly with a discounting factor. While the above mentioned papers are focused on theoretical studies, [22] presents an extensive simulation study showing that selfish routing can achieve close to optimal average latency in Internet-like environments.

The paper [23] considers incentive-based routing, in which inter-domain lowest-cost paths are computed based on per-packet costs reported by a network of ASs. The game aspect derives from the fact that the ASs may not report true costs in an attempt to engineer the flows so as to maximize their own profits. It is shown that a Vickrey-Clarke-Groves (VCG) mechanism produces "strategyproof" prices to be paid to each AS for forwarding packets in transit. Strategyproof essentially means that the ASs have no incentive for lying about their costs and should therefore report true values. However, [24] later showed that the VCG mechanism is not strategyproof in the context of repeated games, which they argue have more practical significance. The authors of [25] also consider the issue of avoiding selfish behavior in routing. They propose a reputation-based scheme to prevent some nodes from refusing to carry other node's traffic (the "free-loader's" problem).

III. Routing Games

Within each deterministic router, there exists a routing table that associates each possible destination IP address with the address and physical interface of a next-hop router. The goal is to, at every hop, get "one-step closer" to the destination or, in case the final destination is in the local subnet, the address and physical interface of the host. Stochastic routing utilizes a distinct concept called *next-hop probabilities*, which map each possible destination IP with a probability distribution of the next-hop. Packets are forwarded by selecting the next-hop at random, but according to the next-hop probability distribution. From an end-to-end perspective, this results in data packets following random paths. The main challenges in stochastic routing is the determination of next-hop probabilities that ensure: *delivery*, *timeliness*, and *statistical flooding*. The key technical insight explored here is to formalize the stochastic routing problem as

an abstract game between two players: the designer of the routing algorithm, which is represented by *routers*, and an *attacker* that attempts to intercept packets. In practice, minimizing the impact of an attack is equivalent to minimizing the impact of a worst-case failure.

We consider zero-sum games in which routers target at minimizing the time it takes for a packet to be safely transmitted, while the attacker's goal is to maximize this time. It is well known from the game theory literature that the solution to such games requires the use of mixed (randomized) policies (cf., e.g., [26]). In practice, the mixed solution of the minimax problem provides the probability distributions needed for the next-hop probabilities. By formalizing the problem as an optimization with a time cost, we achieve both delivery and timeliness. Statistical flooding is a consequence of the saddle solution.

There are several alternatives to formalize a game that results in adequate routing policies. In this paper, we consider two alternatives: *off-line routing games* and *on-line games*.

In *off-line games*, the attacker starts by selecting one link or one physical interface at a particular node that she will scan for packets. This choice is made before routing starts but is not conveyed to the router, whose task is to design the next-hop probabilities to minimize the overall probability that the packet will be intercepted, assuming that the attacker made an intelligent choice. This setup can be generalized to attacks at nodes and even mixed attacks at both nodes and links. We shall see that the computation of routing policies for on-line games amounts to solving a linear program, requiring information about the overall network topology as in link-state routing.

In *on-line games*, the attacker is not forced to select a single link/node before routing starts. Instead, the attacker is allowed to scan one physical interface at every node. However, she will not be able to catch all packets that travel through the interface selected, only a fraction of these. For highly secure nodes this fraction would be zero, whereas it would have high values for less secure nodes. The computation of the routing policies that arise from this game can be done using dynamic programming, amenable to distributed computation as in distance-vector routing.

It is important to emphasize that the abstract games described above *are not intended to represent realistic attack scenarios*, they mostly intend to capture the facts that (1) simultaneous failures in multiple links/nodes are unlikely and therefore one should optimize routing for robustness against a finite number of worst-case faults and that (2) an attacker will (hopefully!) have a finite set of resources available. Moreover, we will show that the attack models described

above provide computational tools to obtain stochastic routing polices with adequate properties. Similarly, although the number of hops is generally a poor measure of routing-path quality, minimum-hop optimization is a useful tool to compute deterministic routing tables because it generally provides adequate routing trees.

As with deterministic routing, topology changes require the re-computation of the next-hop probabilities, regardless of the type of game considered. Failure to do so will generally result in lost packets. However, with stochastic routing some degree of functionality is preserved even before the next-hop probabilities are updated. This is because some of the packets will still make it through, as long as at least one of the paths from source to destination remains viable.

A. Stochastic routing policies

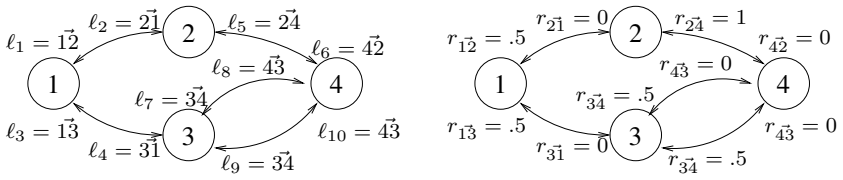
We consider a data transmission network with nodes $\mathcal{N} := \{1, 2, \ldots, n\}$ connected by unidirectional links. We denote by $\mathcal{L}$ the set of all links and use the notation $\vec{ji}$ to represent a link from node j to node i. We assume that all the nodes in the network are connected in the sense that it is possible to reach any node from any other node through a finite sequence of links. The source and destinations nodes are denoted by n_{src} and n_{end}, respectively.

In a stochastic routing framework, each *stochastic routing policy* is characterized by a list of probabilities $R := \{r_\ell : \ell \in \mathcal{L}\}$, such that

$$\sum_{\ell \in \mathcal{L}[k]} r_\ell = 1, \quad \forall k \in \mathcal{N}, \tag{1}$$

where the summation is taken over the set of links $\mathcal{L}[k] \subset \mathcal{L}$ that exit from node k. Under this policy, when a packet arrives at a node $k \in \mathcal{N}$, it will be routed with probability r_ℓ through the link $\ell := \overrightarrow{k\,k'} \in \mathcal{L}$ to the next-hop node k'. The distribution $\{r_\ell : \ell \in \mathcal{L}[k]\}$ determines the entries of the next-hop probabilities at node k, associated with the path from node n_{src} to node n_{end}. In this case, the routing table will actually be a matrix, as the next-hop may depend not only on the final destination node n_{end} but also on the source node n_{src}.

In the sequel, we denote by $\mathcal{R}_{\text{sto}}$ the set of lists that satisfy (1) and therefore $\mathcal{R}_{\text{sto}}$ represents the set of all stochastic routing policies. In general, stochastic routing policies can exhibit cycles. Since cycles introduce delivery delays and can only increase the probability of packet interception, we restrict our attention to *cycle-free* routing policies, i.e., policies for which a



(a) (b)

Fig. 1. (a) Example of a network with 4 nodes $\mathcal{N} := \{1, 2, 3, 4\}$ and 10 links $\mathcal{L} := \{\ell_1, \ell_2, \ldots, \ell_{10}\}$. In this network all connections between the nodes are assumed bidirectional, which corresponds to two unidirectional links between every two nodes that are connected. (b) Stochastic routing policy from the source node $n_{\text{src}} = 1$ to the destination node $n_{\text{end}} = 4$. Under this policy, at each node the packets are routed with equal probability among the possible options. This policy is cycle-free.

packet will never pass through the same node twice. Formally, $R \in \mathcal{R}_{\text{sto}}$ is cycle-free when there is no sequence of links

$$\mathcal{S} := \{\overrightarrow{k_1 k_2}, \overrightarrow{k_2 k_3}, \ldots, \overrightarrow{k_{k-1} k_k}, \overrightarrow{k_k k_1}\} \subset \mathcal{L}, \tag{2}$$

with positive probabilities $r_\ell > 0$ for all $\ell \in \mathcal{S}$ starting and ending at the same node k_1. We denote by $\mathcal{R}_{\text{no-cycle}}$ the subset of $\mathcal{R}_{\text{sto}}$ consisting of cycle-free policies. Figure 1 shows an example network and a cycle-free stochastic routing policy.

B. On-line games

In on-line games, the attacker has the capability of scanning a certain percentage of packets that go through every node. She then has to decide on which physical interfaces she should concentrate her efforts. For each node $k \in \mathcal{N}$, we denote by p_k the total percentage of packets that the attacker can scan on node k. In the context of on-line games, a *stochastic attack policy* is a list of probabilities $A := \{a_\ell : \ell \in \mathcal{L}\} \in \mathcal{R}_{\text{sto}}$ that specifies which percentage of packets will be scanned on each output interface of each node. The probability of a packet being scanned in the node k interface connected to the link $\ell := \overrightarrow{k\,k'} \in \mathcal{L}$ is therefore given by $p_k\, a_\ell$. Since $A \in \mathcal{R}_{\text{sto}}$, all probabilities a_ℓ that exit node k must add up to one, and therefore the total percentage of packets scanned in node k is indeed p_k. The selection of links by the router is done according to a stochastic routing policy $R := \{r_\ell : \ell \in \mathcal{L}\} \in \mathcal{R}_{\text{sto}}$ as described before.

For each link $\ell \in \mathcal{L}$, we denote by $\tau_\ell > 0$ the time it takes for a packet to traverse it assuming that it was not intercepted. When there is interception we assume that this time increases by T_ℓ.

Denoting by $\mathbf{T}^*$ the amount of time that takes a packet to get from the source node n_{src} to the destination node n_{end}, the router selects a stochastic routing policy $R \in \mathcal{R}_{\text{sto}}$ so as to minimize the expected value of $\mathbf{T}^*$, whereas the attacker selects a stochastic attack policy $A \in \mathcal{R}_{\text{sto}}$ to maximize it. The problem just formulated is a zero-sum game for which we will attempt to find an optimal saddle pair of policies $R^* \in \mathcal{R}_{\text{sto}}$, $A^* \in \mathcal{R}_{\text{sto}}$ for which

$$\mathrm{E}_{R^*,A^*}[\mathbf{T}^*] = \min_{R\in\mathcal{R}_{\text{sto}}} \max_{A\in\mathcal{R}_{\text{sto}}} \mathrm{E}_{R,A}[\mathbf{T}^*] = \max_{A\in\mathcal{R}_{\text{sto}}} \min_{R\in\mathcal{R}_{\text{sto}}} \mathrm{E}_{R,A}[\mathbf{T}^*], \tag{3}$$

In the above equation, the subscripts on the expected value E emphasize the fact that the expectation depends on the routing/attack policies. The choice of a routing policy R^* for which (3) holds guarantees the *best possible timeliness (smallest $\mathbf{T}^*$), assuming an intelligent attacker that does her best to prevent this*. This is achieved by statistical flooding, as not fully exploring the available paths would be used by the attacker to her advantage.

The computation of the optimal routing and attack policies can be done using dynamic programming. To this effect, for each node $k \in \mathcal{N}$ we denote by V_k^* the *optimal cost-to-go from node k*, which is the average time it takes to send a packet from node k to node n_{end} using optimal policies for each player. Once the optimal cost-to-go have been computed for all nodes, the optimal routing policy can be computed as follows: for each node k, the next-hop routing distribution $\{r_\ell^* : \ell \in \mathcal{L}[k]\}$ over the set of links $\mathcal{L}[k]$ that exit node k is the solution to the following minimization:

$$\underset{\substack{r_\ell:\ell\in\mathcal{L}[k]\\ \sum_\ell r_\ell=1}}{\arg\min} \max_{\substack{a_\ell:\ell\in\mathcal{L}[k]\\ \sum_\ell a_\ell=1}} \sum_{\vec{ki},\vec{kj}\in\mathcal{L}[k]} r_{\vec{ki}} a_{\vec{kj}} m^*_{ijk}, \quad m^*_{ijk} := \begin{cases} V_i^* + \tau_{\vec{ki}} & i \neq j,\ k \neq n_{\text{end}} \\ V_i^* + \tau_{\vec{ki}} + p_k T_{\vec{ki}} & i = j,\ k \neq n_{\text{end}} \\ 0 & k = n_{\text{end}} \end{cases} \tag{4}$$

The optimal cost-to-go V_k^* can be computed with the help of the following function T that transforms a vector of cost-to-go $V := [V_1\ V_2\ \cdots\ V_N]$ into another vector of cost-to-go $V' := [V_1'\ V_2'\ \cdots\ V_N'] = T\big([V_1\ V_2\ \cdots\ V_N]\big)$, where the kth element of V' is given by

$$V_k' = \min_{\substack{r_\ell:\ell\in\mathcal{L}[k]\\ \sum_\ell r_\ell=1}} \max_{\substack{a_\ell:\ell\in\mathcal{L}[k]\\ \sum_\ell a_\ell=1}} \sum_{\vec{ki},\vec{kj}\in\mathcal{L}[k]} r_{\vec{ki}} a_{\vec{kj}} m_{ijk}, \ \ m_{ijk} := \begin{cases} V_i + \tau_{\vec{ki}} & i \neq j,\ k \neq n_{\text{end}} \\ V_i + \tau_{\vec{ki}} + p_k T_{\vec{ki}} & i = j,\ k \neq n_{\text{end}} \\ 0 & k = n_{\text{end}} \end{cases} \tag{5}$$

We defer to Section III-B.2 how these optimizations can be performed and proceed to state several important properties of the policies just defined (refer to the Appendix for their proofs).

Theorem 1: Assume that all the $\tau_\ell > 0$, $\ell \in \mathcal{L}$. Then

1) The vector $V^* := [V_1^* \; V_2^* \; \cdots \; V_N^*]$ of optimal cost-to-go is the unique fixed point of the function T defined by (5).
2) For any (not necessarily optimal) vector of cost-to-go $V(0)$, the optimal cost-to-go can be obtained by $V^* = \lim_{i\to\infty} \underbrace{T(T(\cdots(T}_{i \text{ times}}(V(0)))\cdots)$.
3) The stochastic routing policy $R^* \in \mathcal{R}_{\text{sto}}$ defined by (4) and the stochastic attacker policy $A^* \in \mathcal{R}_{\text{sto}}$ defined by a similar expression with the min and max interchanged form a saddle point-pair for the game, i.e., (3) holds. □

1) Heterogeneous attacks: By choosing distinct percentages p_k for different nodes, we can take into account the fact that some nodes may be more secure than others. In practice, by choosing high values for p_k, we are implicitly assuming that the node k is less secure. One can also encode in the p_k external information about where an attack is more likely to occur and/or succeed. This information could be obtained, e.g., from intrusion detection sensors that provide indications where an attack may be occurring. In this case, the routing algorithm could actually adapt to the changing perception, as the intrusion detection sensors gain more confidence about which hosts have been compromised. The percentages p_k can also be viewed as design parameters that allow a traffic engineer to shape the amount of data sent that goes through different sections of the network. For example, by increasing the values of the p_k one can obtain routing policies that avoid links that are especially costly or that are somehow undesirable. To some extent, the percentages p_k can play the role of link-costs in shortest-path routing.

2) Computational issues: As mentioned above, the optimal routing policies can be obtained from (4), as soon as the vector of optimal cost-to-go $V^* := [V_1^* \; V_2^* \; \cdots \; V_N^*]$ is available. According to Statement 2 of Theorem 1, one can obtain V^* using an iteration of the form

$$V(t+1) = T\big(V(t)\big), \tag{6}$$

where T is defined by (5) and the initial vector $V(0)$ can be anything (e.g., equal to zeros for every node). In general, V^* is only obtained as $t \to \infty$ but one can usually stop the iteration after a finite number of steps, at the expense of getting a slightly sub-optimal policy. Typically, the iteration is stopped when there is a small change in $V(t)$. The number of steps needed is usually on the order of the maximum diameter of the graph.

To compute each iteration of (6), we need to solve the minimax optimization that appears in

the definition (5) of the operator T. This can be done using the following linear program:

$$\frac{1}{V_k'} = \max \sum_{\vec{ki}\in\mathcal{L}[k]} \tilde{r}_{\vec{ki}} \quad \text{subject to} \quad \tilde{r}_{\vec{ki}} \geq 0, \sum_{\vec{ki}\in\mathcal{L}[k]} \tilde{r}_{\vec{ki}} m_{ijk} \leq 1, \quad \forall \vec{kj} \in \mathcal{L}[k], \tag{7}$$

from which one obtains V_k', which is the kth element of $T(V)$ [26, Section 2.3]. The constants m_{ijk} that appear in (7) are defined in (5). Once the optimal cost-to-go V^* has been obtained, the optimal next-hop routing distribution $\{r_\ell^* : \ell \in \mathcal{L}[k]\}$ over the set of links that exit node k can be obtained from $r_{\vec{ki}}^* = \frac{\tilde{r}_{\vec{ki}}}{\sum_{\vec{kj}\in\mathcal{L}[k]} \tilde{r}_{\vec{kj}}}$,where the $\tilde{r}_{\vec{ki}}$ are the solutions to a linear program like (7), but with the m_{ijk} replaced by the m_{ijk}^* that appear in (4).

The computation of V_k' in (7) only requires knowledge of the entries V_i corresponding to the neighbors i of k. In practice, this means that the computation of the kth element of the vector of cost-to-go $V(t+1)$ can be performed at the node k, provided that this node knows the elements of the vector $V(t)$ that correspond to their neighbors. Hence, the computation of the optimal cost-to-go V^* can be distributed in the same way that RIP is. In RIP or similar distance-vector routing computations, at each stage the router performs an operation with complexity $O(d)$, where d is the out-degree of the router. In the case of on-line games, the router must solve a linear programming problem with worst-case complexity $O(d^3)$, but for which there are methods that typically require much more modest complexity. Figure 2 shows an example of routing policies obtained for on-line games.

C. Off-Line Games

In off-line games the attacker is only allowed to select a single link for scanning and this choice must be done ahead of time. As in on-line games, we assume that the attacker may not possess sufficient resources to scan every packet in the link selected. To this effect, we take as given percentages of the packets p_ℓ, $\ell \in \mathcal{L}$ that the attacker would be able to scan if she were to select link ℓ. By choosing different percentages for different links we can take into account the fact that some links may be more secure or more desirable than others (cf. discussion in Section III-B.1). Shortly, this formulation will be extended to node-attacks as opposed to link-attacks.

Assuming that there is more than one independent path from source to destination, the optimal action for the attacker is to select links according to some distribution. In the context of off-line games a *stochastic attack policy* $A = \{a_\ell : \sum_\ell a_\ell = 1, \ell \in \mathcal{L}\}$ consists of a distribution over the

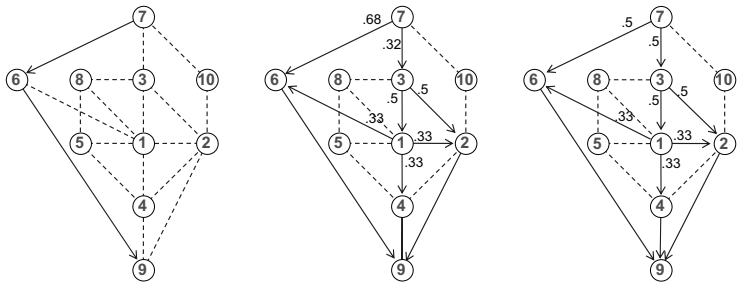


Fig. 2. Examples of routing policies obtained for an on-line game. When the probability of traversing a link is not zero or one, the probability is labeled near to the source end of the link. Dashed links are not used by the policies. The destination is the bottom node labeled 9 and the total percentage of packets p_k scanned at each node k was set to 10%. The left diagram shows the routing policy obtained when $T_\ell \approx 0$, $\forall \ell \in \mathcal{L}$, i.e., when interception poses no penalty. In this case, one obtains exactly Equal-Cost Multi-Path routing [12] with hop count as the metric. In the right diagram, we see the case $T_\ell = 100$, $\forall \ell \in \mathcal{L}$. In this case, there is a heavy penalty incurred when a packet is intercepted and therefore routing favors maximal flooding, utilizing all possible paths. The middle diagram shows the intermediate case ($T_\ell = 45$, $\forall \ell \in \mathcal{L}$). Note that the link from node 7 to node 10 is not utilized while the link from node 7 to node 3 is. The reason for this is that while node 3 is no closer to the destination than node 10, there are many alternate paths from node 3 to the destination, while there is only one path from node 10 to the destination. Therefore, from the defender's perspective, node 3 is more appealing than node 10 because the presence of alternate paths makes the attacker's task more difficult. The path $\{7, 6, 9\}$ is similar to the path $\{7, 10, 2, 9\}$ in that once node 6 or node 10 is selected, the rest of the path is deterministic and an online attacker could easily catch the packets. However, the path $\{7, 6, 9\}$ is shorter. Hence, under no circumstances would the path $\{7, 10, 2, 9\}$ be selected. These results should be compared to the off-line example in the next section.

set of links $\mathcal{L}$, where a_ℓ denotes the probability that the attacker will scan link ℓ. We denote by $\mathcal{A}$ the set of all stochastic attack policies, i.e., the set of distributions over $\mathcal{L}$.

In off-line games the router selects a cycle-free stochastic routing policy $R \in \mathcal{R}_{\text{no-cycle}}$ so as to minimize the probability of a packet being intercepted, whereas the attacker selects a stochastic attack policy $A \in \mathcal{A}$ so as to maximize it. Denoting by $\boldsymbol{\chi}$ a random variable that takes the value one when a packet is intercepted and zero otherwise, the problem just formulated is a zero-sum game for which we will attempt to find an optimal saddle pair of policies $R^* \in \mathcal{R}_{\text{no-cycle}}$, $A^* \in \mathcal{A}$ for which

$$\mathrm{E}_{R^*,A^*}[\boldsymbol{\chi}] = \min_{R \in \mathcal{R}_{\text{no-cycle}}} \max_{A \in \mathcal{A}} \mathrm{E}_{R,A}[\boldsymbol{\chi}] = \max_{A \in \mathcal{A}} \min_{R \in \mathcal{R}_{\text{no-cycle}}} \mathrm{E}_{R,A}[\boldsymbol{\chi}], \tag{8}$$

Note that the expected value of $\boldsymbol{\chi}$ is precisely the probability of a packet being intercepted. We will see shortly how this quantity can be computed from the values of the policies A and R.

The cost in (8) places no penalization on the number of links that a packet will cross from

the source to the destination. However, in some cases one may want to favor shorter paths. This can be done by introducing a new random variable $\boldsymbol{\chi}_\epsilon$, $\epsilon \geq 0$ that is equal to zero in case the packet is not intercepted and equal to $(1+\epsilon)^{\mathtt{t}-1}$ when the packet is intercepted at the $\mathtt{t}$ hop. Suppose now that we consider the zero-sum game

$$\mathrm{E}_{R^*,A^*}[\boldsymbol{\chi}_\epsilon] = \min_{R\in\mathcal{R}_{\text{no-cycle}}} \max_{A\in\mathcal{A}} \mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon] = \max_{A\in\mathcal{A}} \min_{R\in\mathcal{R}_{\text{no-cycle}}} \mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon] \tag{9}$$

For $\epsilon = 0$, $\boldsymbol{\chi}_\epsilon = \boldsymbol{\chi}$ and the cost is the same as in (8). However, for $\epsilon > 0$, (9) penalizes longer paths since the cost incurred increases as the number of hops increases. In fact, as $\epsilon \to \infty$ the potential burden of an extra hop is so large that the optimal solution will only consider paths for which the number of hops is minimal, leading to shortest path routing but not necessarily single-path. In the remainder of this section we consider the game in (9), as (8) is a special case of the former for $\epsilon = 0$.

To compute $\mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon]$, we need to construct the matrix C that encodes the network connectivity. We define C to be a square matrix with one column/row per link, with the (ℓ_1, ℓ_2) entry equal to one if the link $\ell_2 = i\vec{j}$ ends at the node j where link $\ell_1 = j\vec{k}$ starts, and zero otherwise. We also define a column-vector s with one entry per link, which is equal to one if the corresponding link exits the source node n_{src} and zero otherwise. In the construction of C and s we should exclude all links that enter the source node n_{src} and exit the end node n_{end} as these links will never be used in cycle-free routing policies. The order in which the links are associated with the rows/columns of C and s must be consistent. The following result, proved in the Appendix, allows one to explicitly compute the value of $\mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon]$.

Lemma 1: For given $\epsilon \geq 0$, $R \in \mathcal{R}_{\text{no-cycle}}$, $A := \{a_\ell : \sum_\ell a_\ell = 1, \ell \in \mathcal{L}\} \in \mathcal{A}$,

$$\mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon] = \mathrm{row}[A]\, x, \tag{10}$$

where $\mathrm{row}[A]$ denotes a row-vector with one entry per link and $p_\ell a_\ell$ in the entry corresponding to the link $\ell \in \mathcal{L}$; x is the unique solution to

$$x = (1+\epsilon)\,\mathrm{diag}[R]Cx + \mathrm{diag}[R]s, \tag{11}$$

and $\mathrm{diag}[R]$ is a square diagonal matrix with the r_ℓ in the main diagonal. □

The main difficulty in solving the routing game (9) is that the cost (10) is generally not convex on $R \in \mathcal{R}_{\text{no-cycle}}$. However, the relation specified by (11) between the vectors x and policies R is in some sense one-to-one and will allow us to "convexify" the cost (10) by searching for

"optimal" vectors x instead of policies R. To this effect, let C_{out} and C_{in} be matrices with one row per node and one column per link such that the entry of C_{out} corresponding the node k and link ℓ is equal to one if link ℓ *exits* node k and zero otherwise, and the entry of C_{in} corresponding the node k and link ℓ is equal to one if link ℓ *enters* node k and this is not the destination node n_{end} and equal to zero otherwise, i.e., $C_{\text{out}} := [\, c_{k,\ell}\,]$ and $C_{\text{in}} := [\, d_{k,\ell}\,]$ with

$$c_{k,\ell} := \begin{cases} 1 & \exists i \in \mathcal{N} : \ell = \vec{ki} \\ 0 & \text{otherwise,} \end{cases} \qquad d_{k,\ell} := \begin{cases} 1 & \exists i \in \mathcal{N} : \ell = \vec{ik},\ k \neq n_{\text{end}} \\ 0 & \text{otherwise.} \end{cases}$$

We further define s_{src} to be a column-vector with one entry per node such that the entry corresponding to the source node n_{src} is equal to one and all others are equal to zero. In the construction of these matrices we should exclude all links that enter the source node n_{src} and exit the end node n_{end}. The following lemma establishes a one-to-one relation between the set of cycle-free stochastic policies (which is not convex in general) and a convex set. This will be the basis to find a solution to the routing game (9).

Lemma 2: Let $\mathcal{X}$ denote the convex set of vectors $x := \{x_\ell \geq 0 : \ell \in \mathcal{L}\}$ that satisfy

$$C_{\text{out}} x = (1+\epsilon) C_{\text{in}} x + s_{\text{src}}. \tag{12}$$

1) Given any $R \in \mathcal{R}_{\text{no-cycle}}$ there exists an $x \in \mathcal{X}$ for which

$$x = (1+\epsilon)\,\text{diag}[R] C x + \text{diag}[R] s \tag{13}$$

and the norms of the vectors $x \in \mathcal{X}$ can be bounded by a constant that is independent of $R \in \mathcal{R}_{\text{no-cycle}}$.

2) Given any $x := \{x_\ell \geq 0 : \ell \in \mathcal{L}\} \in \mathcal{X}$ that is cycle-free in the sense that there is no sequence of links (2) with $x_\ell > 0$, $\forall \ell \in \mathcal{S}$, there exists an $R := \{r_\ell :\in \mathcal{L}\} \in \mathcal{R}_{\text{no-cycle}}$ for which (13) holds, which is given by

$$r_\ell := \frac{x_\ell}{\sum_{\ell' \in \mathcal{L}[\ell]} x_{\ell'}}, \quad \forall \ell \in \mathcal{L}, \tag{14}$$

where the summation is over the set $\mathcal{L}[\ell]$ of links that exit from the same node as ℓ. □

The equation (12) can be interpreted as a *flow-conservation law*, which states that the incoming flow to a node is equal to the outgoing flow from the same node, possibly amplified by $(1+\epsilon)$ when $\epsilon > 0$. Because of this we call the elements of $\mathcal{X}$ *flow vectors*. We are now ready to state the main result of this section (proved in the Appendix), which provides the solution to off-line routing games. We defer to Section III-C.2 details on the computation of the optimal polices.

Theorem 2: For every $\epsilon \geq 0$, the routing game (9) has saddle-point policies. Moreover, the *flow-game* defined by

$$\mathrm{row}[A^*]x^* = \min_{x\in\mathcal{X}} \max_{A\in\mathcal{A}} (\mathrm{row}[A]x) = \max_{A\in\mathcal{A}} \min_{x\in\mathcal{X}} (\mathrm{row}[A]x), \tag{15}$$

always has a saddle-point $(x^*, A^*) \in \mathcal{X} \times \mathcal{A}$ with x^* cycle-free, from which we can construct a saddle-point $(R^*, A^*) \in \mathcal{R}_{\text{no-cycle}} \times \mathcal{A}$ to the original routing game (9), by using (14) to compute R^* from x^*. □

1) Node-attack variations: While so far we focused our discussion on link attacks, one can easily solve the node attack problem by a suitable transformation of the network graph. When one wants to consider the possibility that one node can be attacked, one simply expands the graph by unfolding the original node into two nodes, one that is fed by all the incoming links and another from which all outgoing links exit. The two new nodes are connected by a single link from the former to the latter that will carry all packets that pass through original node (cf. Figure 3). An attack on the so created "bottleneck" link in the new graph is equivalent to an

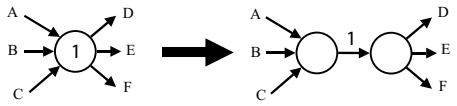


Fig. 3. Converting a node-attack to a link-attack. An attack on node 1 (left) is transformed into an attack on link 1 (right).

attack to the node in the original graph. By choosing a suitable percentage of packets scanned p_ℓ in the new link, we effectively select the percentage of packets that can be scanned in the original node. The percentage of packets that can be scanned in the old links should be set to zero (unless one wants to consider mixed link-node attacks) and therefore the total number of links that are relevant for optimization actually decreases, making the problem computationally simpler.

2) Computational issues and max-flow/load-balancing problems: Theorem 2 allows us to use the formula (14) to compute the optimal routing policy R^* from the saddle-point (x^*, A^*) to the flow-game (15), where x^* is any vector in $\mathcal{X}$ that achieves the following minimum

$$\min_{x\in\mathcal{X}} \max_{A\in\mathcal{A}} (\mathrm{row}[A]x) \tag{16}$$

[26, Theorem 2.4]. It turns out that the computation of x^* in (16) can be reduced to a linear-program optimization: Since the cost is linear on A for fixed x, the inner maximization in (16) is always achieved at a distribution of the form

$$A_\ell = \{ \underbrace{0, 0, \cdots, 0, 1, 0, \cdots, 0}_{\text{single 1 at the } \ell\text{th entry}} \}$$

for some $\ell \in \mathcal{L}$. For such A_ℓ, we have that $\mathrm{row}[A_\ell]x = p_\ell\, x_\ell$ and therefore (16) is equal to

$$\min_{x\in\mathcal{X}} \max_{\ell\in\mathcal{L}} (p_\ell\, x_\ell) = \min_{x\in\mathcal{X}} \min_{p_\ell\, x_\ell \le \mu,\ \forall \ell} \mu = \min_{\substack{0\le p_\ell\, x_\ell \le \mu,\ \forall \ell \\ C_{\text{out}} x = (1+\epsilon) C_{\text{in}} x + s_{\text{src}}}} \mu. \tag{17}$$

In the first equality in (17), we used the fact that $\max\{\alpha_1, \alpha_2, \ldots, \alpha_k\} = \min_{\alpha_i \le \mu, \forall i} \mu$. Although (17) is already in the form of a linear program, it is instructive to reformulate it as a maximization by making the change of variables $\bar{\mu} := \mu^{-1}$ and $\bar{x} := \mu^{-1} x$, which leads to

$$\frac{1}{V^*} = \max_{\substack{0\le p_\ell\, \bar{x}_\ell \le 1,\ \forall \ell \\ C_{\text{out}} \bar{x} = (1+\epsilon) C_{\text{in}} \bar{x} + \bar{\mu} s_{\text{src}}}} \bar{\mu}. \tag{18}$$

One can recognize in this linear program a max-flow optimization, showing that the value of the game is equal to the inverse of the maximum flow $\bar{\mu}$ from source to destination, consistent with the flow conservation law $C_{\text{out}}\bar{x} = (1+\epsilon)C_{\text{in}}\bar{x} + \bar{\mu} s_{\text{src}}$, and subject to the link-bandwidth constraints

$$\bar{x}_\ell \le \frac{1}{p_\ell}, \qquad \forall \ell \in \mathcal{L}. \tag{19}$$

Note also that, since $\bar{x}$ is simply a scaled version of x, the optimal routing policy can be computed directly from $\bar{x}$, using (14). This means that the routing policies that arise from the routing game (9) also *maximize throughput subject to the constraint* (19). For $\epsilon = 0$ this turns out to be a standard max-flow problem, whereas for $\epsilon > 0$ we obtain a generalized max-flow with gain [27]. When this algorithm is applied to the transformed graph that encodes node-attacks, the maximum node-flow is minimized. This means that the policies arising from the routing game (9) also achieve *load-balancing*. If the links and nodes have heterogeneous p_ℓ, then a weighted load-balancing is achieved. Figure 4 shows an example of routing policies obtained for off-line games.

The routing computations in (18) require that each node have knowledge of the entire topology, i.e., this is a link-state algorithm. However, it seems likely that a distributed approach is feasible. In particular, in the case that $\epsilon = 0$, there are several algorithms that solve the max-flow problem

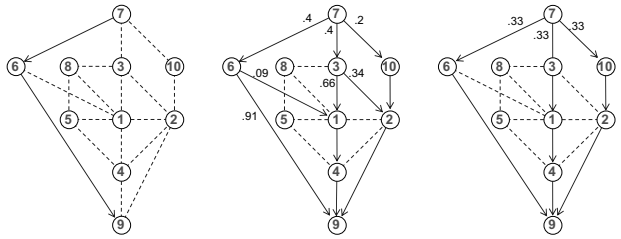


Fig. 4. Examples of routing policies obtained for an off-line game. When the probability of traversing a link is not zero or one, the probability is labeled near to the source end of the link. Dashed links are not used by the policies. The source is the top node labeled 7 and the destination is the bottom node labeled 9. The percentage p_ℓ of packets that the attacker could scan at each link was assumed to be the same for every link ℓ, which is consistent with a network where all links are equally secure. The right diagram shows the routing policy obtained when one selects $\epsilon = 0$ and therefore delay is not penalized, leading to the maximal spreading of packets. In the left diagram, we see that when each extra hop corresponds to a severe cost penalty ($\epsilon = 100$), one obtains minimum-hop routing. The middle diagram shows an intermediate case ($\epsilon = .1$). In contrast to Figure 2, now the link from node 7 to node 10 is in general utilized because it is part of one of the three independent paths from source to destination. However, in on-line games, if a packet was sent through node 10 it was going to be caught with high probability since it necessarily had to be sent towards node 2 so this option was not considered.

in a distributed fashion [28, 29]. The time complexity of these algorithms can be as low as $O(n^2)$ where n is the number of nodes. Furthermore, a hierarchical version of GTSR would likely yield significant computational saving.

3) Routing tables vs. routing matrices: For on-line games without the back-to-start variation, the function T used to compute the optimal cost-to-go and eventually the optimal stochastic routing policies only depend on the destination node n_{end} and not on the source node n_{src}. In practice, this means that the optimal routing policies only depend on the final destination as in the usual deterministic routing algorithms such as RIP, OSPF, etc. However, for off-line games the matrices C_{in} and C_{out} used to compute the optimal routing policies depend both on the source and destination nodes (recall that we need to remove from these matrices the contribution of the nodes entering n_{src} and exiting n_{end}). In practice, this means that the routing tables will actually be matrices, as the choice of the probability distributions for the next-hop depends not only on the final destination node n_{end} but also on the source node n_{src}. A hierarchical approach would result in next-hop probabilities only depending on the source and destination prefix.

Another implementation approach is to use source routing whereby the end-hosts define the path to be taken by each packet and are then responsible for the randomization of paths. The

Dynamic Source Routing (DSR) protocol is an example of a source routing mechanism which has been proposed for use in multi-hop wireless ad-hoc networks (or MANETs) [30]. The use of source routing to enhance security has been previously suggested in [31].

IV. Simulation Results

To evaluate the algorithm proposed in Section III applied to network layer routing, we simulated the network in Figures 2 and 4 using the `ns-2` network simulator [32]. In all simulations presented, all links have propagation delay of 20ms and bandwidth of 10Mbps. Each queue implements drop-tail queuing discipline with maximum queue size set to 200 packets for the case of the CBR simulations and 100 packets for the TCP simulations. All packets are 1000 bytes long. The simulation time for each trial was 100 seconds. Two types of experiments were performed. In the first type only CBR traffic was used. In the second type, TCP-SACK traffic was used. In both types of experiments, the security parameter ϵ is fixed during a trial but varies from trial to trial.

We were interested in determining the effect of stochastic routing on reliability, throughput, and packet transmission delay. To evaluate how secure a particular routing policy is, we determine the maximum fraction of packets that an attacker could see by eavesdropping on one, two, or three links. We assumed here that the attacker chooses the set of links that maximizes the fraction of packets seen (i.e., the "worst" case scenario). This is equivalent to determining how reliable the routing policy is by examining the worst-case fraction of packets lost when one, two, or three links fail. Figure 5 shows the simulation results obtained for off-line games with several values of the parameter ϵ and for on-line games with several values of the delay T_ℓ (the same for every link). Constant bit rate (CBR) traffic was used in all these simulations. As expected, routing is most secure for off-line games with $\epsilon = 0$, since in this case all paths are equally good and the packets will be spread the most across all paths. As ϵ increases, more packets are routed through the shorter paths and therefore an attacker eavesdropping on those will see a larger percentage of packets. In fact, for $\epsilon = 100$, we essentially have minimum-hop routing and by attacking a single link it is possible to see every packet. Because in the network tested there are only three independent paths, when the attacker is allowed to eavesdrop on three links she will be able to see every packet, regardless of which type of routing is used. On-line games generally do not result in maximum flow and in this particular topology explore at most two of

the three independent paths. As mentioned before, this can be explained by the fact that node 10 is heavily penalized by on-line games for having a single output interface.

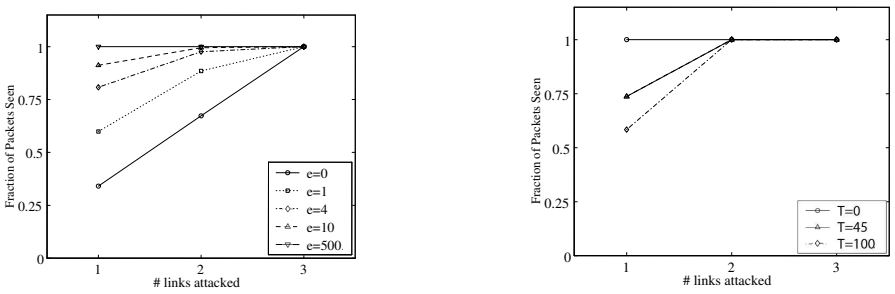


Fig. 5. Worst-case fraction of packets from CBR traffic seen by an attacker eavesdropping on one, two, or three links, using off-line (left) and on-line (right) games.

For off-line games with $\epsilon = 0$ we achieve maximum throughput (from a sender's perspective) since we make use of all available paths (including the minimum cost one). This is supported by the data in Figure 6, where we plot the drop-rate as a function of the source's sending rate. For $\epsilon = 100$, drops start occurring at sending rates around 10Mbps, whereas, for $\epsilon = 0$, drops only become significant for sending rates higher than 30Mbps. The price to pay for security comes

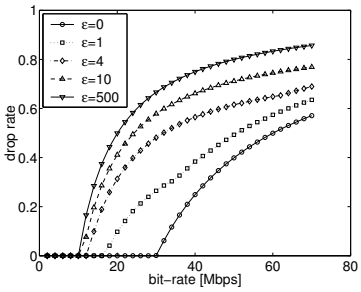


Fig. 6. Drop-rate vs. sending rate.

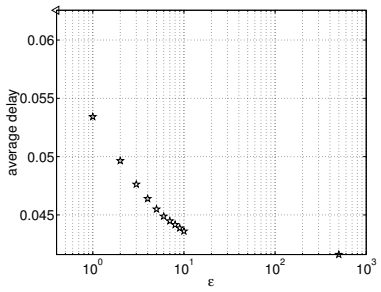


Fig. 7. Average packet transmission delay (CBR traffic). The triangle over the vertical axis corresponds to $\epsilon = 0$ (which could not be represented in log-scale).

in terms of the average latency per packet. This is because to achieve high security one needs to explore all the paths from source to destination, including those with high latency. Figure 7 confirms that the delay does increase as the value of ϵ decreases.

Problems may arise when multi-path routing is used in conjunction with standard TCP (Reno, New Reno, or SACK). The left plot in Figure 8 shows the average throughput of a long-lived

TCP-SACK (circles) and a modified version of TCP called TCP-PR (hash marks), both operating under GTSR. For TCP-SACK, the throughput for $\epsilon = 100$ (essentially minimum-hop routing) is roughly six times larger than that for any other value of ϵ. This is because packets sent through longer paths are often assumed dropped by TCP, as they only arrive after several packets that were sent later but traveled through shorter paths. Because fast-retransmit is only triggered when three duplicate acknowledgments are received, TCP-SACK is able to handle some degree of out-of-order packets. However, in all our simulations this proved insufficient to handle packets traveling through paths with distinct propagation times. The fact that standard TCP performs poorly when used in conjunction with multi-path routing has been observed in [13, 33]. TCP-PR was proposed in [34] and is one of several transport layers capable of coping with out-of-order packet delivery [35]. TCP-PR does not assume a packet is dropped when out-of-order sequence numbers are observed, but only when a time-out occurs. The left plot in Figure 8 shows that for $\epsilon = 0$, the throughput of TCP-PR is larger than all other configurations. That is, the maximum security posture actually increases throughput as a by-product because it explores all alternative paths and, hence, uses all available bandwidth between the source and destination. The right plot in Figure 8 shows the drop rate for the different TCP implementations and different ϵ.

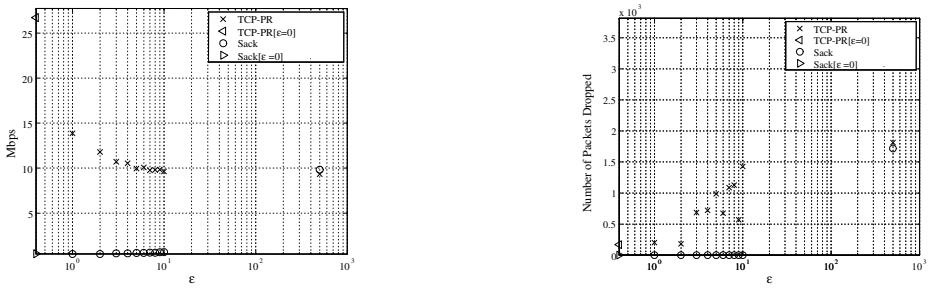


Fig. 8. Transmission rate (left) and drop rate (right) of TCP-SACK and TCP-PR for different security levels ϵ. For $\epsilon < 10$, the TCP-SACK flow experienced no drops. The point $\epsilon = 0$ is indicated with an triangle on top of the y-axis.

A complete discussion of TCP-PR is beyond the scope of this paper. However, it was shown in [34] that TCP-PR and TCP-SACK compete fairly over a network that does not implement multi-path routing. Hence, it is possible to maintain a single transport layer that is compatible with today's network and is also suitable for networks that implement stochastic routing. One implication of this is that multi-path routing could be incrementally deployed. For example,

packets could be marked as being willing to be routed stochastically. Then, flows that use TCP-PR and multi-path routing would compete fairly with flows that request single path routing and use traditional implementations of TCP.

V. Conclusions

We proposed game-theoretic stochastic-routing (GTSR) and demonstrate through simulations that it improves routing security. By proactively forcing packets to probabilistically take alternate paths, stochastic routing mitigates the effects of interception, eavesdropping, and improves fault tolerance. The routing policies proposed proved efficient in achieving statistical flooding at the expense of some increase in average packet delay. A beneficial side-effect of these policies is an increase in throughput, as they explore multiple paths. The algorithms developed are applicable not only to the network layer, but also to application layer overlay networks, and multi-path transport protocols such as SCTP. However, when used at the network layer, a transport layer that is robust to out-of-order packets such as TCP-PR [34] must be employed.

We presented two alternative techniques to generate multi-path routing tables. Routing based on off-line games maximizes the spread of packets across the network, but requires link-state information and generally results in routing matrices. Alternatively, routing based on on-line games does not achieve the same degree of statistical flooding, but the routing computation can be distributed as in distance-vector algorithms.

A direction for future investigation is the development of scalable algorithms to compute next-hop probabilities. One promising approach is to make use of hierarchical decomposition of routing. Since this type of approach generally yields sub-optimal routing policies, its impact on the GTSR performance requires further investigation. Preliminary results on the use of a hierarchical approach to compute routing tables based on off-line games appeared recently in [36]. Alternatively, on-line games can be implemented under distance-vector routing and are scalable. For these algorithms, questions related to convergence need to be investigated.

Appendix

A. Appendix—Derivations for on-line games

Proof of Theorem 1. The routing of a packet can be regarded as a controlled Markov chain, whose state $q_t \in \mathcal{N}$ is a random variable denoting the node where the packet is before the

hop $t \in \{0, 1, 2, \dots\}$. For a given routing policy $R := \{r_\ell : \ell \in \mathcal{L}\} \in \mathcal{R}_{\text{sto}}$ and attack policy $A := \{a_\ell : \ell \in \mathcal{L}\} \in \mathcal{R}_{\text{sto}}$, the transition probability function for the Markov chain is given by $P(q_{t+1} = k' \mid q_t = k) = r_{\vec{kk'}}$, $\forall t \geq 0$. Without loss of generality we assume that the final node n_{end} is an absorbing state, i.e., that $P(q_{t+1} = n_{\text{end}} \mid q_t = n_{\text{end}}) = 1$, $\forall t \geq 0$. The cost to be optimized can be written as $\mathrm{E}_{R,A}[\mathbf{T}^*] = \mathrm{E}_{R,A}\left[\sum_{t=0}^{\infty} c(q_t)\right]$, where

$$c(k) = \sum_{\vec{ki}, \vec{kj} \in \mathcal{L}[k]} r_{\vec{ki}} a_{\vec{kj}} m_{ijk}, \qquad m_{ijk} := \begin{cases} \tau_{\vec{ki}} & i \neq j,\ k \neq n_{\text{end}} \\ \tau_{\vec{ki}} + p_k T_{\vec{ki}} & i = j,\ k \neq n_{\text{end}} \\ 0 & k = n_{\text{end}} \end{cases}$$

The two-person zero-sum game just defined falls in the class of stochastic shortest path games considered in [37]. In particular, it satisfies the SSP assumptions in [37]: [SSP1] There exists at least one proper policy for the minimizer, i.e., a policy that will lead to a finite cost regardless of the policy chosen by the maximizer. This is true because we assume that there exists a sequence of links that connects node n_{src} to node n_{end}. [SSP2] For any policies for which there is a positive probability that the packet will not reach the destination node, the corresponding cost is infinite. This is true provided that $\tau_\ell > 0$, $\forall \ell \in \mathcal{L}$, because every hop that does not reach the final node n_{end} will contribute to the final cost with a positive marginal cost.

Consider now the function T defined by (5) and another function $\tilde{T}$ defined similarly, except that the $\max$ and $\min$ in (5) appear in the reverse order. These functions satisfy the four regularity assumptions in [37]: [R1] The sets over which the controls $\{a_\ell : \ell \in \mathcal{L}[k]\}$, $\{r_\ell : \ell \in \mathcal{L}[k]\}$ take values are compact for every $k \in \mathcal{N}$. [R2] The maps from the controls $\{a_\ell : \ell \in \mathcal{L}[k]\}$, $\{r_\ell : \ell \in \mathcal{L}[k]\}$ to the transition probabilities $r_{\vec{kk'}}$ and the costs $c(k)$ are continuous for every $k \in \mathcal{N}$. [R3-4] The minimum and maximum in the definitions of the functions T and $\tilde{T}$ are achieved and the two functions are actually equal [26, Minimax Theorem]. Statements 1 and 3 then follow from [37, Proposition 4.6] and statement 2 from [37, Proposition 4.7]. ■

B. Appendix—Derivations for off-line games

Proof of Lemma 1. Denoting by $x_\ell(t)$ the probability that a packet is routed through link $\ell \in \mathcal{L}$ at time $t \in \{1, 2, \dots\}$ and has not yet been intercepted, we have that for every $\ell \in \mathcal{L}$

$$x_\ell(1) = \begin{cases} r_\ell & \ell \in \mathcal{L} \text{ exits from node } n_{\text{src}}, \\ 0 & \text{otherwise}, \end{cases} \tag{20}$$

$$x_\ell(t+1) = r_\ell \sum_{\ell' \in \mathcal{L}[\ell]} (1 - a_{\ell'} p_{\ell'}) x_{\ell'}(t) \quad \forall t > 1, \tag{21}$$

where the summation is taken over the set $\mathcal{L}[\ell]$ of links that enter the node from which link ℓ exits. Stacking all the $\{x_\ell : \ell \in \mathcal{L}\}$ into a column-vector x, we can write (20)–(21) as

$$x(1) = \operatorname{diag}[R]\, s, \qquad x(t+1) = \operatorname{diag}[R] C (I - \operatorname{diag}[A]) x(t), \quad \forall t > 1, \tag{22}$$

where $\operatorname{diag}[R]$ and $\operatorname{diag}[A]$ denote diagonal matrices whose main diagonal contains the r_ℓ and the $a_\ell p_\ell$, respectively. From (22), we conclude that, for every $t \geq 1$,

$$x(t) = \big(\operatorname{diag}[R] C (I - \operatorname{diag}[A]) \big)^{t-1} \operatorname{diag}[R]\, s. \tag{23}$$

Since the probability that a packet is intercepted at time t is given by $\operatorname{row}[A] x(t)$ and we can write $\mathrm{E}_{R,A}[\boldsymbol{\chi}_\epsilon] = \sum_{t=1}^{\infty} (1+\epsilon)^{t-1} \operatorname{row}[A] x(t)$. Moreover, because $x(t)$ evolves according to (23), we have that (10) holds with $x := \sum_{t=1}^{\infty} \big((1+\epsilon) \operatorname{diag}[R] C (I - \operatorname{diag}[A]) \big)^{t-1} \operatorname{diag}[R] s$. Since R is cycle-free, the above series only has a finite number of nonzero terms (at most one per hop). Therefore, it must converge and be equal to

$$\sum_{t=1}^{\infty} \big((1+\epsilon) \operatorname{diag}[R] C (I - \operatorname{diag}[A]) \big)^{t-1} = \big(I - (1+\epsilon) \operatorname{diag}[R] C (I - \operatorname{diag}[A]) \big)^{-1}$$

[38]. Thus $x = \big(I - (1+\epsilon) \operatorname{diag}[R] C (I - \operatorname{diag}[A]) \big)^{-1} \operatorname{diag}[R] s$, which is equivalent to (11).

■

Proof of Lemma 2. The convexity of $\mathcal{X}$ is trivial since $\mathcal{X}$ is given by the intersection of the (convex) positive orthant with the (also convex) affine space corresponding to the solution of the linear equation (12). To prove 1, we show that for a given $R \in \mathcal{R}_{\text{no-cycle}}$ we can define

$$x := \sum_{t=1}^{\infty} \big((1+\epsilon) \operatorname{diag}[R] A \big)^{t-1} \operatorname{diag}[R] c. \tag{24}$$

First note that since $R \in \mathcal{R}_{\text{no-cycle}}$, the series converges because it only has a finite number of nonzero terms and therefore

$$\sum_{t=1}^{\infty} \big((1+\epsilon) \operatorname{diag}[R] A \big)^{t-1} = \big(I - (1+\epsilon) \operatorname{diag}[R] A \big)^{-1}$$

[38]. From this and the definition of x we immediately conclude that (13) holds. To verify that the vector x defined by (24) belongs to $\mathcal{X}$ note that every entry of x is nonnegative because it

is the sum of nonnegative numbers and also that left-multiplying (13) by C_{out} we obtain (12) because of the following two equalities that are straightforward to verify:

$$C_{\text{in}} = C_{\text{out}} \operatorname{diag}[R] C, \qquad\qquad s_{\text{src}} = C_{\text{out}} \operatorname{diag}[R] s. \tag{25}$$

Note also that because the number of nonzero terms in the series is always smaller than n and the R are bounded, one can construct a bound on the vectors x defined by (24), which is independent of $R \in \mathcal{R}_{\text{no-cycle}}$.

To prove 2, note that the division in (14) guarantees that the normalization condition (1) holds and therefore that $R \in \mathcal{R}_{\text{sto}}$. Moreover, this definition guarantees that if x is cycle-free then R is also cycle free. To finish the proof it remains to show that (13) holds. To this effect, note that from the definition of R, for every $\ell \in \mathcal{L}$ we have that $x_\ell = r_\ell \sum_{\ell' \in \mathcal{L}[k]} x_{\ell'} = r_\ell C_{\text{out}}[k] x$, where $\mathcal{L}[k]$ denotes that the set of links that exit from the node k from which ℓ exits, and $C_{\text{out}}[k]$ the row of C_{out} corresponding to the node k. But since $x \in \mathcal{X}$, we then have

$$x_\ell = (1+\epsilon) r_\ell C_{\text{in}}[k] x + r_\ell \delta_{k, n_{\text{src}}} \tag{26}$$

where $C_{\text{in}}[k]$ denotes the rows of C_{in} corresponding to the node k and $\delta_{k,n_{\text{src}}}$ is equal to one if $k = n_{\text{src}}$ and zero otherwise. This finishes the proof because (13) is a vector version of (26). ■

Proof of Theorem 2. First we can conclude from von Neumann's Theorem [26] that the flow-game (15) always has a saddle-point $(x^*, A^*) \in \mathcal{X} \times \mathcal{A}$, because the cost of the game is bilinear on the policies x and A, which take values in compact convex sets. Moreover, since removing cycles from a policy never increases the cost, x^* can be assumed cycle-free. The set $\mathcal{X}$ is actually not bounded and therefore not compact. However, in view of 1 in Lemma 2, we can restrict our attention to a bounded subset of $\mathcal{X}$.

Let then $(x^*, A^*) \in \mathcal{X} \times \mathcal{A}$ be the saddle-point whose existence was just proved. We show next that $(R^*, A^*) \in \mathcal{R}_{\text{no-cycle}} \times \mathcal{A}$, with R^* is constructed from x^* using (14), is a saddle-point for the routing game (9). To this effect we need to show that

$$\mathrm{E}_{R^*, A}[\boldsymbol{\chi}_\epsilon] \leq \mathrm{E}_{R^*, A^*}[\boldsymbol{\chi}_\epsilon] \leq \mathrm{E}_{R, A^*}[\boldsymbol{\chi}_\epsilon], \tag{27}$$

for every $R \in \mathcal{R}_{\text{no-cycle}}$ and every $A \in \mathcal{A}$. Because of (10), (27) is actually equivalent to

$$\operatorname{row}[A] x^* \leq \operatorname{row}[A^*] x^* \leq \operatorname{row}[A^*] x, \tag{28}$$

where x is the unique solution to (11). To verify that x belongs to $\mathcal{X}$ note that left-multiplying (11) by C_{out} we obtain (12) because of (25). This means that (28) must hold because (x^*, A^*) is a saddle-point of the flow-game (15). ■

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