Game Theory
Lecture #1

Focus of Lecture:

- What are "Socio-technical Systems?"
- Game theory introduction
- Examples

1 Engineered Systems vs Socio-Technical Systems

Engineering is predominantly concerned with the design and analysis of physical systems, taking into account various considerations including cost, weight, efficiency, environmental impact, performance, among others. The overarching goal when designing such systems is to meet a desired performance specification, subject to a given set of constraints. Examples are numerous and diverse: computer hardware, industrial infrastructure, robotics, and many more.

A socio-technical system is an engineered system that is coupled with a societal system, such as a highway network combined with all of its vehicles and drivers. In this context, the performance and efficiency of the overall system depends on how the members of society utilize the system. For example, while an engineer can design the physical aspects of a transportation system, the engineer in unable to specify how drivers utilize the transportation system to meet their individual demands. Ultimately, even if there are no clear engineering defects, the efficiency of the overall system may suffer if drivers use the system poorly. How should an engineer design the physical and technical aspects of a social-technical system when faced with the uncertainty of how users will use such a system?

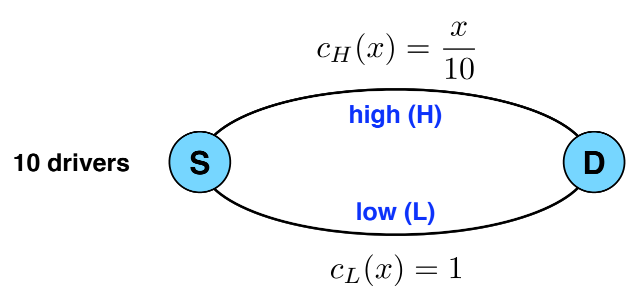
Application 1.1 (Smart Grid) *The performance and efficiency of an electrical power distribution system is highly dependent on the real-time choices of consumers of electricity. Thus, if many users in a single area plug in their electric vehicles and turn on their air conditioners simultaneously, this could stress the power lines and lead to cascading failures and ultimately widespread blackouts. The goal of the smart grid paradigm is to design mechanisms that incentivize individual members of society to adjust their local energy consumption in response to the real-time availability of energy. The engineer is tasked with designing the infrastructure and incentive mechanism to facilitate these interactions, subject to constraints on available energy, aggregate demand, security, and others. How should an engineer assess the quality of a design given the uncertainty of how societal members will behave in such a system?*

Application 1.2 (Medical Resident Matching) *Every year there is a process implement that governs how graduating medical students get assigned to hospitals for their medical* *residency training. There are several challenges associated with this process including (i) a larger number of applicants than spots, (ii) preferences and skills of applicants, (iii) preferences and specialties of programs, (iv) the need to fill all the spots at hospitals, etc. Accordingly, an engineer is responsible with coming up with a system to assign prospective students to medical schools. This system entails getting information from the students and schools, and utilizing this information to make an assignment. The performance is ultimately gauged by social welfare or aggregate happiness from the programs and applicants. How should an engineer assess the quality of such a design given the uncertainty of how prospective residents and medical programs will behave in such a system?*

2 Social Models

Regardless of the setting, an engineer needs to have some kind of mathematical model of social behavior in order to successfully evaluate the design of a socio-technical system. That is, the engineer needs a *social model* – a model that predicts how members of society will utilize the system. Hence, these models will be key in characterizing the performance guarantees associated with a given system design. The following examples demonstrate some of the many intricacies surrounding social models.

Example 2.1 (The Price of Anarchy) *Consider the simple transportation system illustrated below with 10 drivers, denoted by $N = \{1, 2, \ldots, n\}$ seeking to traverse from S (source) to D (destination) across one of two paths, High or Low.*



Each path H or L is associated with a given congestion function, $c_H : \{1, \ldots, 10\} \to \mathbb{R}$ and $c_L : \{1, \ldots, 10\} \to \mathbb{R}$ respectively, that defines the congestion or quality of service on each road as a function of the utilization. Here, $c_H(k)$ defines the congestion on path H when there are $k \geq 0$ drivers on path H. For this specific example, we consider congestion functions of the form: for any $k \in \{1, \ldots, 10\}$

$$c_H(k) = \frac{k}{10} \tag{1}$$

$$c_L(k) = 1. \tag{2}$$

Hence, the congestion on the high road is sensitive to utilization while the congestion on the Low road is not sensitive to utilization. If $k_H \geq 0$ drivers take the High road and $k_L \geq 0$

drivers take the Low road, the total congestion on the network is

$$C(k_H, k_L) = k_H c_H(k_H) + k_L c_L(k_L) = k_H \left(\frac{k_H}{10}\right) + k_L = k_H \left(\frac{k_H}{10}\right) + (10 - k_H)$$

where the last equality comes from the fact that $k_H + k_L = 10$.

- ***System Optimal:*** *Suppose the system operator could divide the traffic, i.e., specify* k_H *and* k_L *as she wishes. The division of traffic that optimizes the total congestion in the network is when* $k_H^* = k_L^* = 5$ *which yields a total congestion of*

$$C(k_H^* = 5, k_L^* = 5) = 5\left(\frac{5}{10}\right) + (10 - 5) = 7.5$$

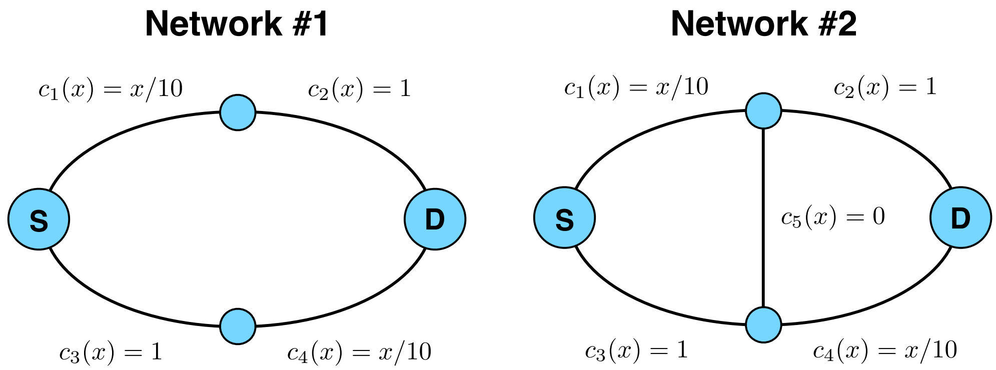
- ***Social Behavior:*** *The above analysis demonstrates that the given infrastructure can meet the societal demands and achieve a performance as measured by total congestion of* 7.5. *Will this traffic pattern emerge if the drivers are self-interested and only care about their experienced congestion? Given a routing profile where* $k_H = 5$ *and* $k_L = 5$, *note that all drivers on the high road are experiencing a congestion of* 0.5 *while all driver on the low road are experiencing a congestion of* 1. *Accordingly, if drivers are self-interested, drivers on the Low road will switch to the High road until the delay on the two roads equalizes, i.e., the benefit of switching no longer exists. This is achieved when* $k_H = 10$ *and* $k_L = 0$, *which yields a total congestion of*

$$C(k_H = 10, k_L = 0) = 10\left(\frac{10}{10}\right) = 10$$

This example serves to illustrate that the performance resulting from self-interested behavior is often far worse than the optimal conceivable performance. Here, the engineer might be tempted to make the assumption that "if each driver is seeking the fastest travel time, this must result in the fastest travel times for socity." However, this reasoning is a fallacy and leads us to an important observation: ***individual optimization need not lead to collective optimization.***

The following example further illustrates the importance of social models in determining how to evaluate the quality of a given system. Here, we will investigate a subtle and counterintuitive phenomenon which results from social behavior.

Example 2.2 (Braess's Paradox) *Consider the two simple transportation systems illustrated below with 10 drivers, denoted by* $N = \{1, 2, \ldots, 10\}$ *seeking to traverse from S (source) to D (destination).*



For Network #1, each driver is given two different path options, i.e., $\mathcal{P}^1 = \{\{1,2\},\{3,4\}\}$, *which entail either taking the top two edges or the bottom two edges. The congestion function associated with each of the links is illustrated. For Network #2, there is an additional edge* 5 *with highlighted congestion function and all other edges are the same. In Network#2, each driver is given four different path options, i.e.,* $\mathcal{P}^2 = \{\{1,2\},\{3,4\},\{1,5,4\},\{2,5,3\}\}$, *where the last two options result from using the new edge* 5*. Now, the cost associated with each driver will be additive over the paths. At first glance, Network #2 looks superior to Network #1 due to the fact that there is additional infrastructure. Analyzing the resulting social behavior yields a vastly different conclusion.*

- ***Network #1:*** *What is the resulting behavior that emerges from self-interested behavior in Network #1? Following the same reasoning as in the previous example, having* 5 *users take the top path* $\{1,2\}$ *and* 5 *users taking the bottom path* $\{3,4\}$, *yields a congestion of* 1.5 *on each path and is a reasonable prediction of behavior since no driver could switch paths and be better off. Accordingly, the total congestion associated with this distribution of traffic is*

$$5c_1(5) + 5c_2(5) + 5c_3(5) + 5c_4(5) = 15$$

- ***Network #2:*** *Is the resulting behavior that emerges from self-interested behavior in Network #1 still stable in Network #2? Note that each user is experiencing a congestion of* 1.5*. Further, note that if any user switched to the path* $\{1,5,4\}$ *her congestion would be* 1.2*. Following this same logic would demonstrate that this emergent collective behavior for Network #2 would correspond to all users taking the path* $\{1,5,4\}$, *which would yield a total congestion of*

$$10c_1(10) + 10c_5(10) + 10c_c(10) = 20,$$

meaning that the congestion of this "improved" networked resulted in a total congestion of 20 *compared to the previous network which yielded a total congestion of* 15*; hence, a* 33% *degradation in performance. Further, note that every driver's cost is now* 2 *instead of* 1.5 *as before.*

These examples serve to illustrate the following key points:

- The optimal design **must** account for social behavior.
- Predicting social behavior is non-trivial.
- Social behavior can be far from optimal.
- Emergent social behavior is often non-intuitive.

3 Strategic Information-Gathering

One of the main difficulties in developing accurate social models is the problem of obtaining information about users' preferences. In many cases, obtaining this information requires very careful attention to strategic aspects of this problem. A central focus of this class will be the design of mechanisms to facilitate the emergence of desirable behavior. Mechanisms have the following properties:

- **Users:** There is a set of users $N = \{1, 2, \ldots, n\}$ seeking to participate in the system.
- **Private information:** Each user possesses some private information regarding her preferences/utility.
- **System objective:** A social planner would like to optimize an system-level objective that depends on the private information of the users. The social planer does not have access to this information.
- **Information exchange:** The societal planner may ask the users to reveal their information in order to make a given decision. Whether or not the users reveal accurate information is unknown.

Mechanisms are deployed in several societal domains including auctions, kidney exchanges, university admissions, and many others. We now present the following real-world example to illustrate the potential impact of complications of designing mechanisms for societal applications:

Example 3.1 (Boulder Valley School District – Open Enrollment) *Boulder Valley School District (BVSD) allows students the opportunity to open enroll to any BVSD school provided that there is space available. Since demands for certain schools far exceeds space, BSVD implements the following mechanism for determining who gets allocated to which schools. The primitives of the problem are as follows:*

- *Schools:* $\{1, \ldots, m\}$.
- *Number of open spots in each school:* $\{n_1, \ldots, n_m\}$, $n_i \geq 0$
- *Students:* $\{s_1, \ldots, s_n\}$

- *School ranking for each applicant s:* $\{q_1^s, \ldots, q_m^s\}$. *Interpretation:* $q_i^s > q_j^s$ *means school* i *is preferred to school* j *by student* s

The goal of BVSD is to assign students to schools to maximize social benefit (happiness). The mechanism employed by BVSD to place students in schools is a lottery based mechanism of the following form:

- ***Information Exchange:*** *Each student is required to submit an ordered list of their top three schools*
- ***Mechanism:*** *The mechanism proceeds as follows:*
  - Round #1: *Randomly pick each student. Assign the student to their top choice if a spot is available.*
  - Round #2: *Randomly pick each student not assigned in Round #1. Assign the student to their second choice if available.*
  - Round #3: *Randomly pick each student not assigned in Rounds #1 or #2. Assign the student to their third choice if available.*

  If a student is not assigned after Round #3, then assign the student to their neighborhood school.

At first glance, it would appear that this would be a completely reasonable approach for allocating students to schools. However, taking into account the self-interested behavior of the applicants potentially changes our conclusions. Will students report truthfully? What is a model of the emergent behavior? How do you ensure desirable behavior if users do not provide accurate information? Are there alternative mechanisms that yield better behavior? What does yield better behavior mean?

4 Conclusions

Regardless of the specific setting (e.g., transportation networks or school assignments) we have the following core elements:

- **Decision Makers**: There are a set of users N seeking to participate in the system. We will utilize the common language or users, players, decision-makers, agents, etc.
- **Choices**: Each user $i \in N$ is associated with a set of choices X_i. A choice $x_i \in X_i$ could correspond to the route taken in a transportation network or the information conveyed in an assignment problem
- **Preferences**: Each user $i \in N$ is associated with a preference function (or utility function) of the form $U_i : X \to \mathbb{R}$ where $X = X_1 \times \cdots \times X_n \to \mathbb{R}$. That is, each user has a preferences over the set of joint choices $(x_1, \ldots, x_n) \in X$. We will call this function $U_i(\cdot)$ a utility function, payoff function, reward function, among others.

- **System Objective**: The system is associated with some performance metric of the form $W : X \to \mathbb{R}$ where $X = X_1 \times \cdots \times X_n \to \mathbb{R}$. This could represent total congestion on a transportation system or aggregate happiness with a given collective assignment.

Game theory provides us with mathematical tools to analyze the social/strategic behavior arising from these structures, which can then be used to inform the design of socio-technical systems. This course serves as a basic introduction to some of the core concepts of game theory which can be applied to designing socio-technical systems.