Univeristy of California, Santa Barbara
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Discussion: Friday, 30 May 2008
Homework Due: Friday, 6 June, 4:00PM

ECE 148 Applications of Signal Analysis and Processing
Spring 2008
Prof. Michael Liebling
Electrical & Computer Engineering

Homework 4

Problem 4.1 Spectral Unmixing

Three fluorescent dyes are present in a cell: DAPI (blue), FITC (green), and TR (Red). We wish to measure their respective concentration, $a_1, \ldots, a_3$.
The cell is therefore illuminated and fluorescence emission is sequentially measured through four different filters, each of them sensitive to a different wavelength range, which yields the following measurements:

$$I_{450\text{–}500\text{ nm}} = 0.18, \quad I_{500\text{–}560\text{ nm}} = 0.30, \quad I_{550\text{–}600\text{ nm}} = 0.41, \quad I_{590\text{–}800\text{ nm}} = 0.32.$$

Previously, in a calibration experiment, the fluorescence signal for each of the filters and for each of the dyes separately (all measured at the same, fixed concentration) has yielded the following intensity measurements:

	DAPI	FITC	TR
450–500 nm	0.8	0.05	0
500–560 nm	0.2	0.8	0.05
550–600 nm	0.001	0.2	0.7
590–800 nm	0	0.05	0.6

Estimate the concentrations $a_1, \ldots, a_3$ of, respectively, DAPI, FITC, and TR present in the cell.

Problem 4.2: B-Splines

The B-spline of degree n, $\beta^n(x)$, is defined by

$$\beta^n(x) = \underbrace{\beta^0 * \beta^0 * \cdots * \beta^0}_{n+1 \text{ terms}}(x),$$

where

$$\beta^0(x) = \begin{cases} 1 & |x| < 1/2 \\ 0 & |x| \geq 1/2. \end{cases}$$

The Fourier transform or β^n is given by

$$B^n(\omega) = \left(\frac{\sin(\omega/2)}{\omega/2}\right)^{n+1}.$$

a) What is the support of $\beta^n(x)$ for $n = 0, 1, 2, 3$? And for and arbitrary $n \in \mathbb{N}$?
b) By using

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) \overset{\mathscr{F}}{\leftrightarrow} j\omega F(\omega).$$

show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\beta^n(x) = \beta^{n-1}\left(x + \frac{1}{2}\right) - \beta^{n-1}\left(x - \frac{1}{2}\right).$$

c) Sketch $\frac{\mathrm{d}}{\mathrm{d}x}\beta^2(x)$.