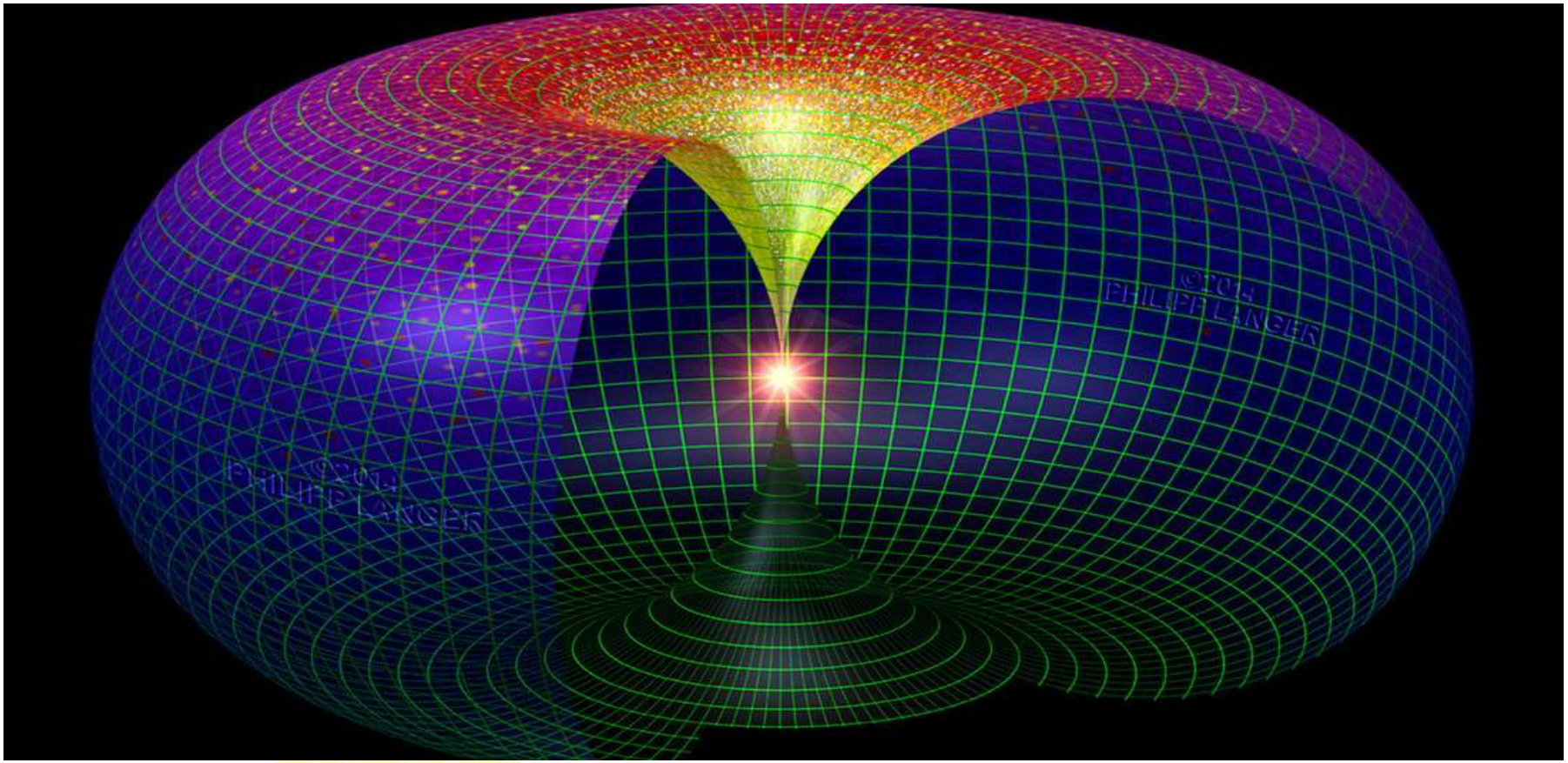
Computational Geometry

A Lecture in the Freshman Seminar Series:
Puzzling Problems in Science and Technology



About This Presentation

Edition	Released	Revised	Revised	Revised	Revised
First	Nov. 2016	Nov. 2018			

What Is Computational Geometry

Study of algorithms which can be stated in terms of geometry

Digital / Discrete geometry: Drawing lines, circles, …

Smallest bounding box/circle or sphere

Largest empty box/circle or sphere

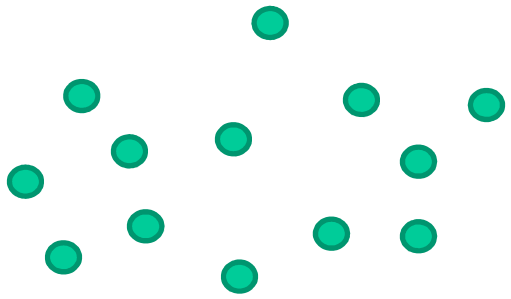
Line-segment intersection problems

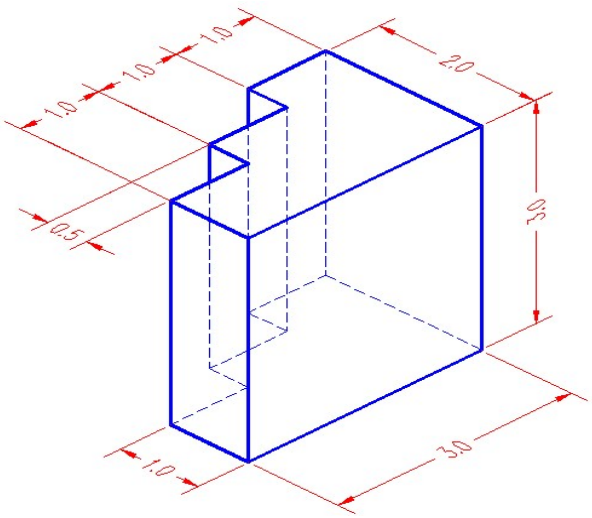
Closest / Furthest pair of points

Hidden surface / line determination

Shading and texture

Robot path planning with obstacles

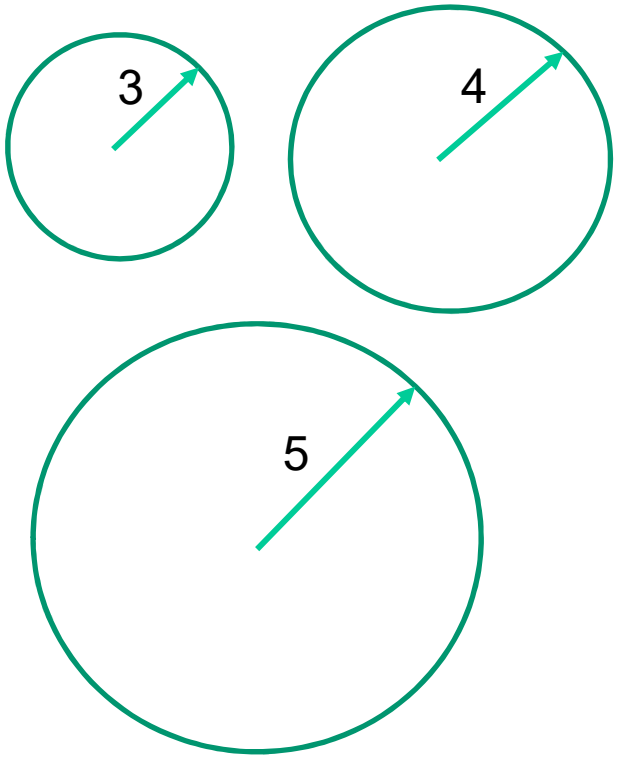


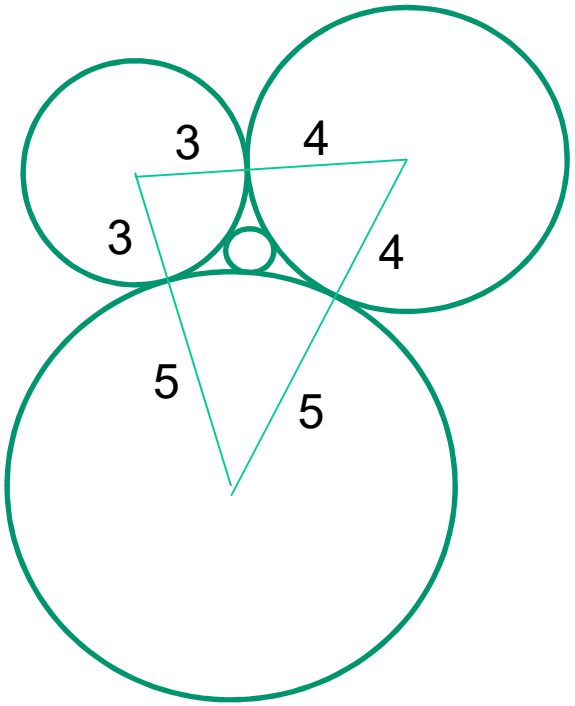


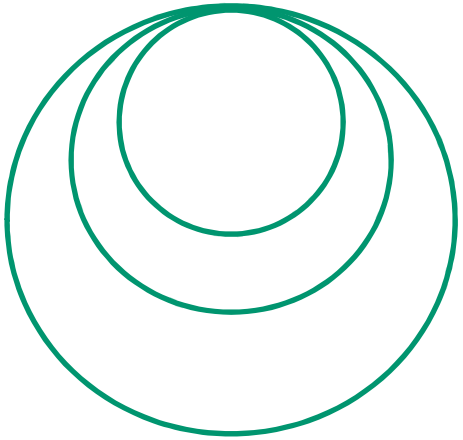
Warm-Up Puzzle

Four circles on a plane are drawn so that each touches the other three:

If the radii of three of them are 3, 4, and 5, what is the largest possible size for the fourth circle:







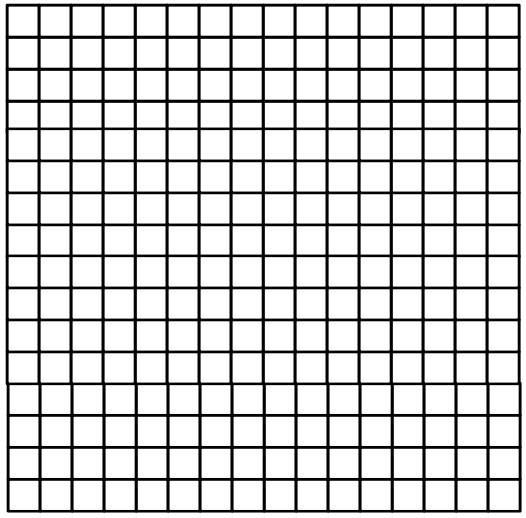
Digital / Discrete Geometry: Objectives

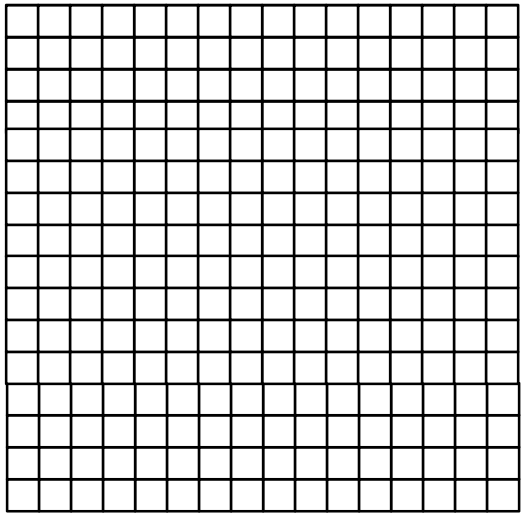
Geometric shapes formed from pixels (drawing or recognizing):

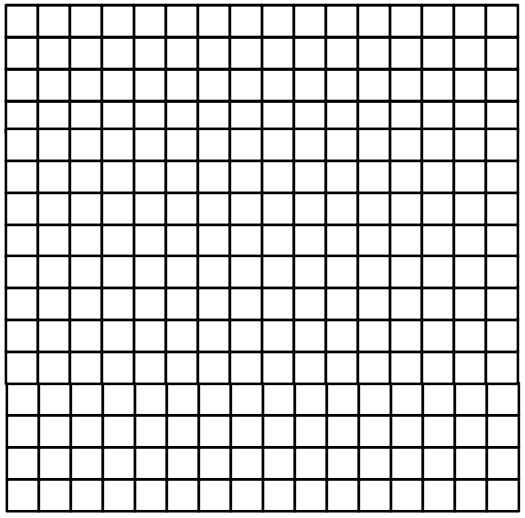
Digital straight lines of different slopes (1 or 1/3) and thicknesses; drawing a straight line between two given points

Digital circle of a given radius (5) and line thickness; Digital disk

Various other shapes: Square; Triangle; Heart; Diamond; US map







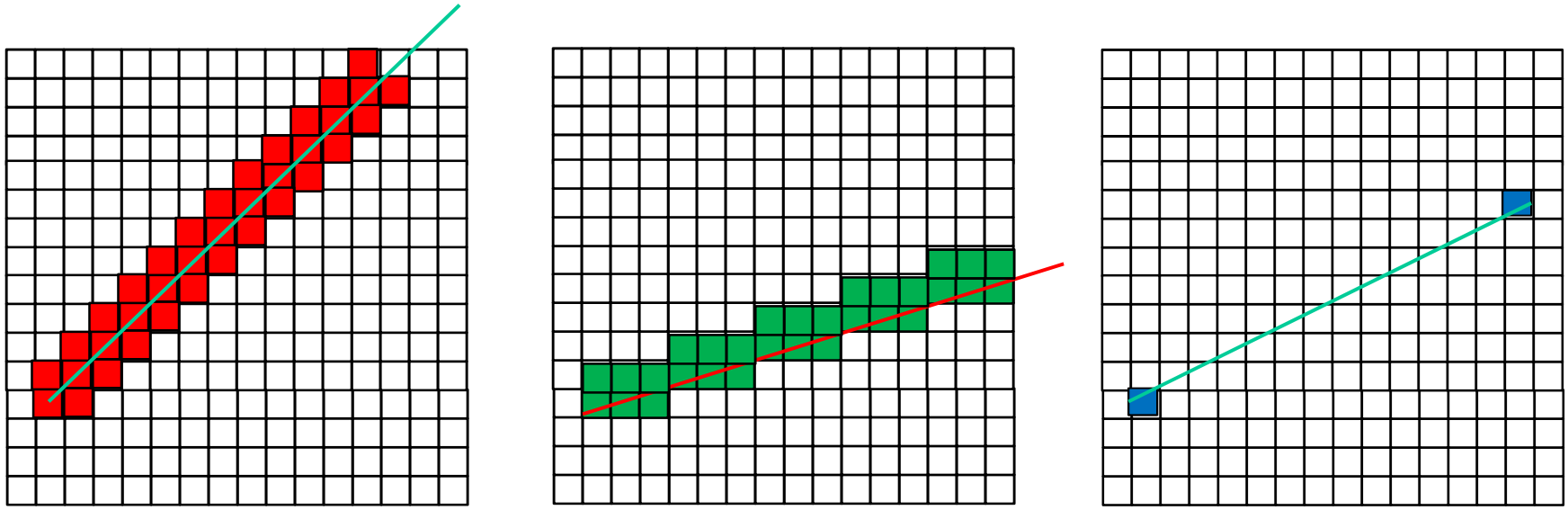
Digital / Discrete Geometry: Straight Lines

The digital geometry of straight lines and line segments:

Digital straight lines of different slopes (1 or 1/3) and thicknesses

Drawing a digital straight line between two given points

Intersections and other problems for digital lines and line segments



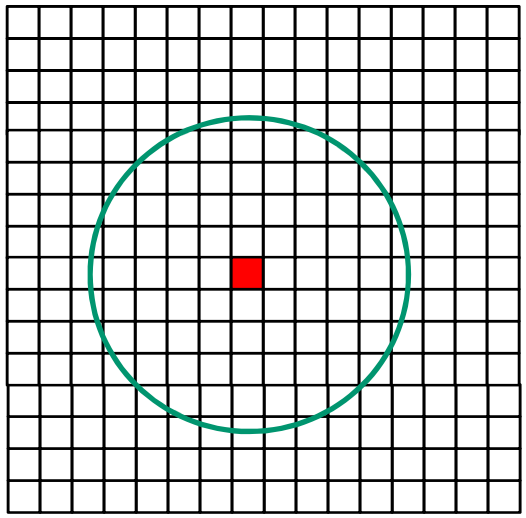
Digital / Discrete Geometry: Circles, Disks, …

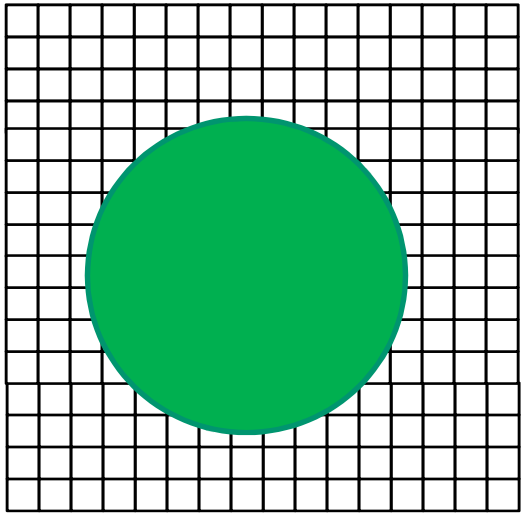
Geometric shapes formed from pixels:

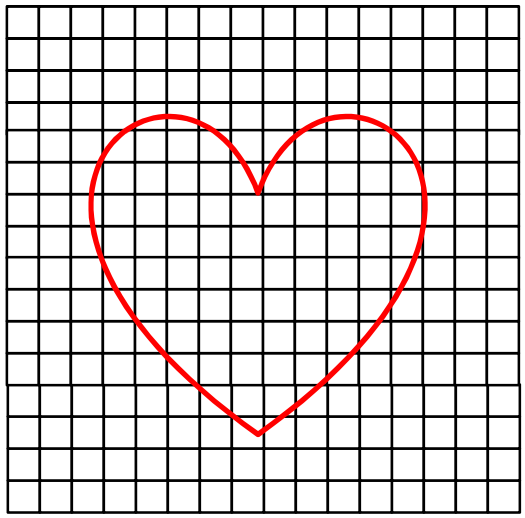
Digital straight lines of different slopes (1 or 1/3) and thicknesses

Digital circle of a given radius (5) and line thickness; Digital disk

Various other shapes: Square; Triangle; Heart; Diamond; US map





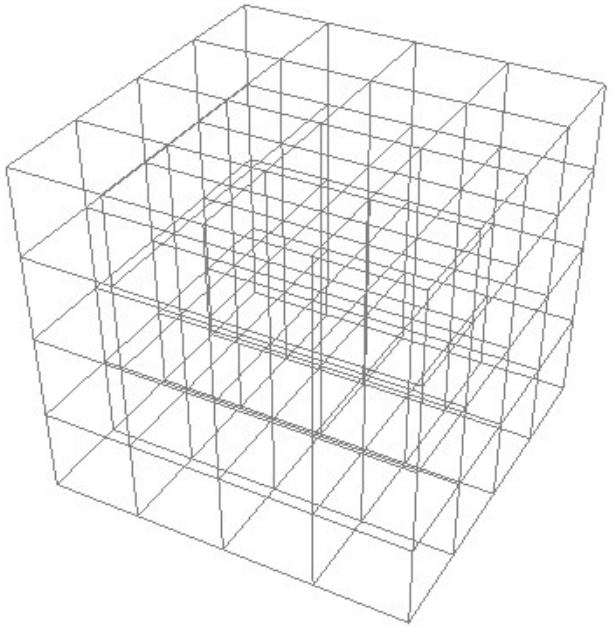


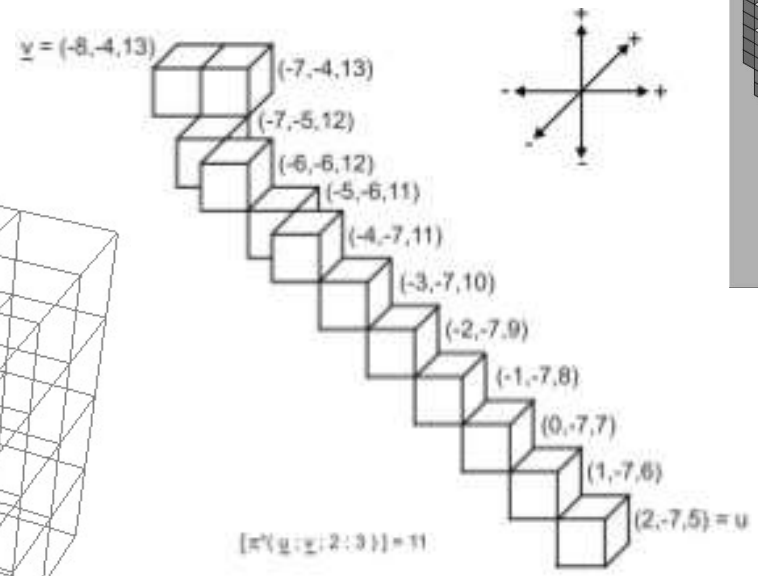
Digital / Discrete Geometry in 3D

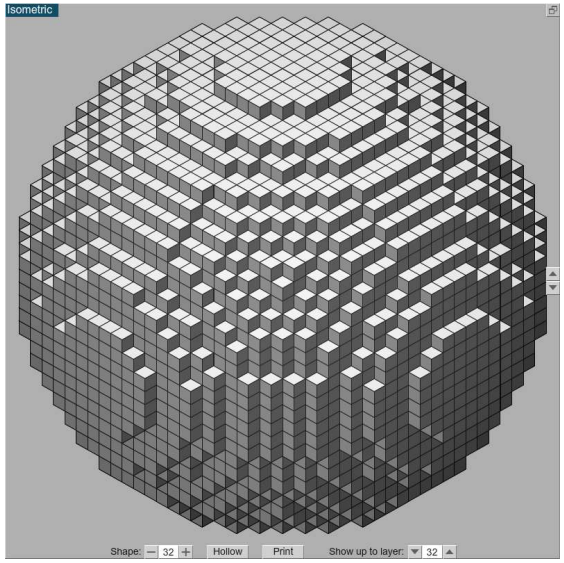
Geometric shapes formed from 3D pixels:

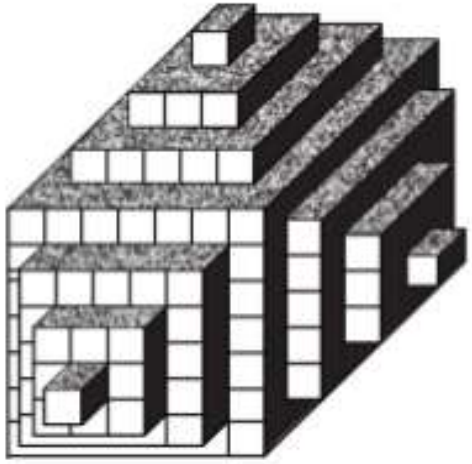
Digital straight lines and curved paths

Digital 3D shapes: Cube; Sphere; Cone; …

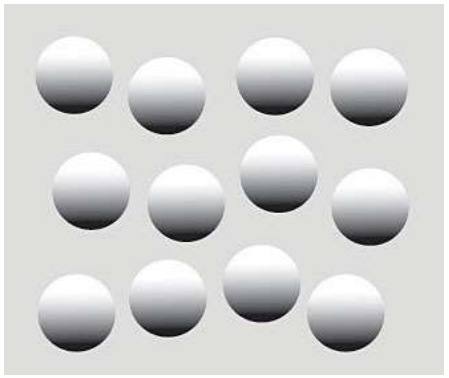
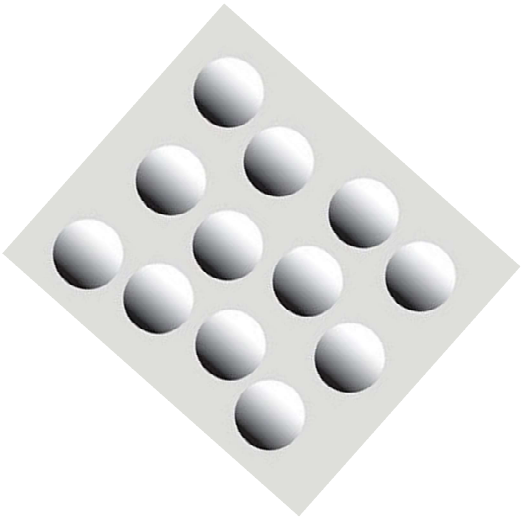
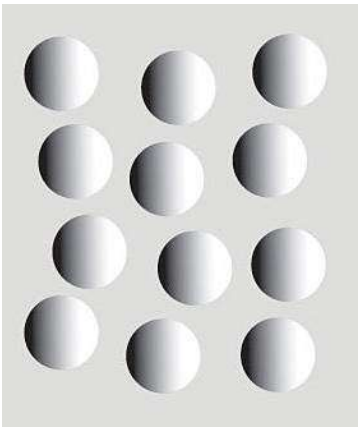
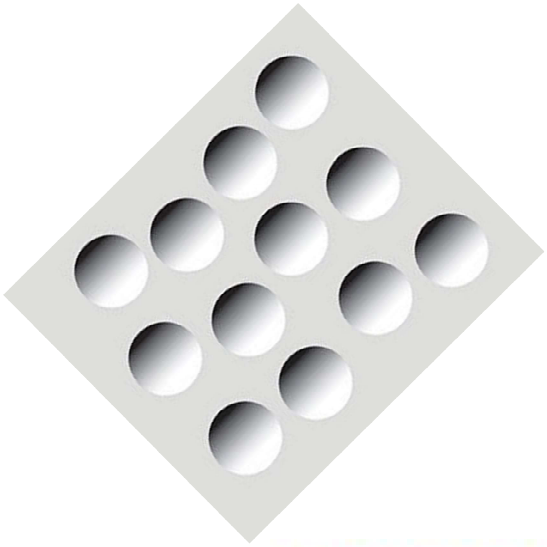
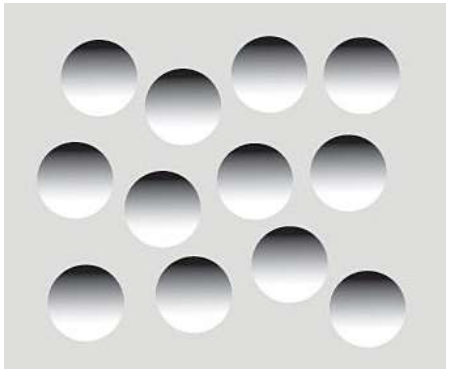
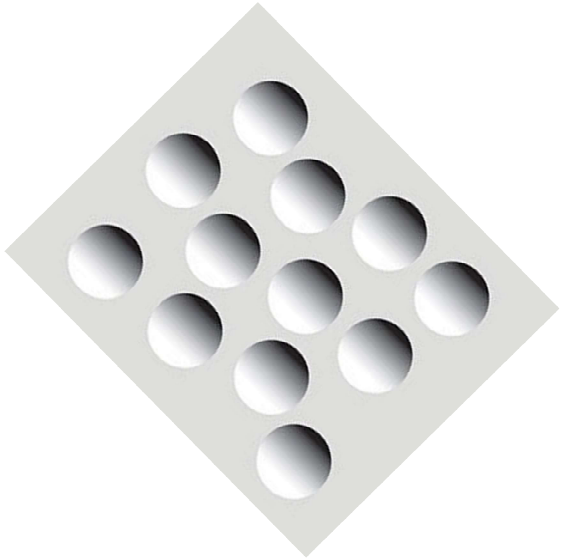




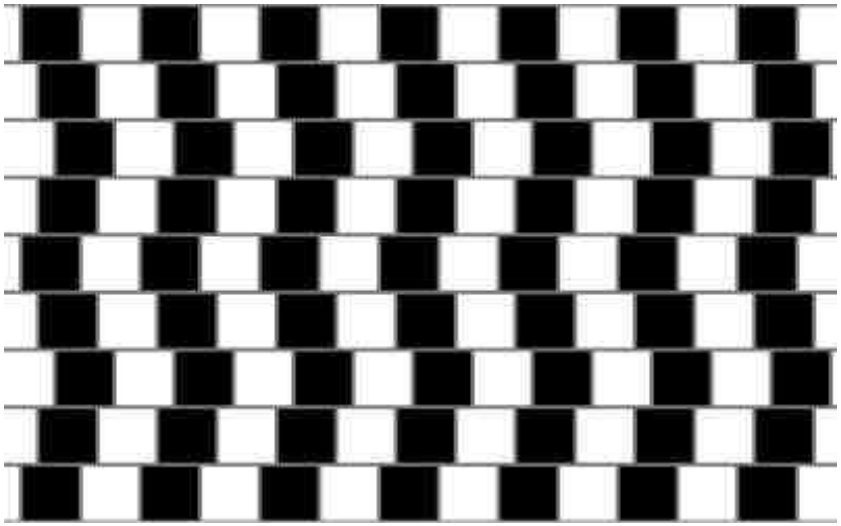


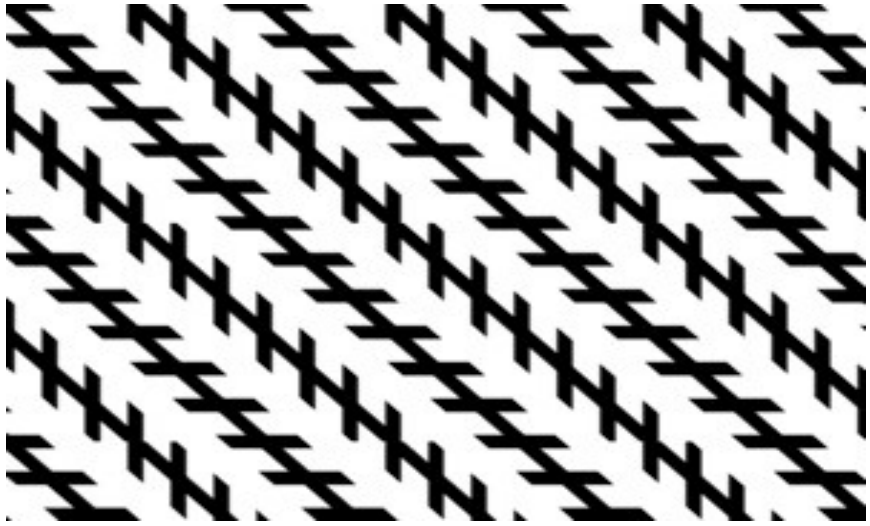


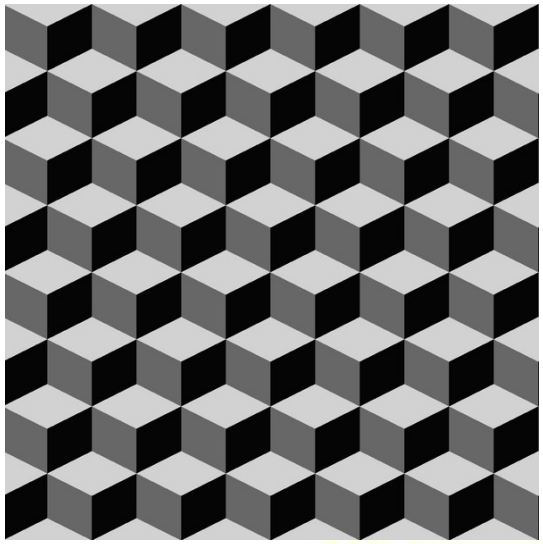
Optical Illusions via Shading

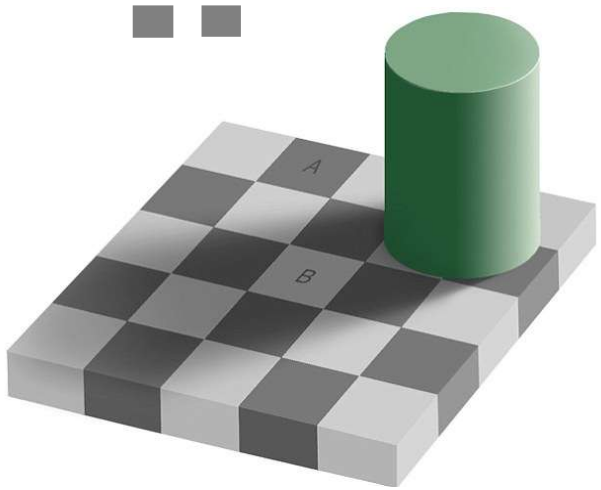







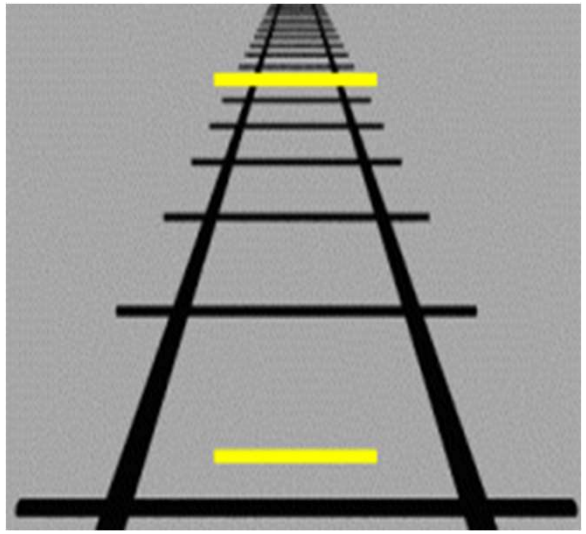
Don’t Believe Your Eyes



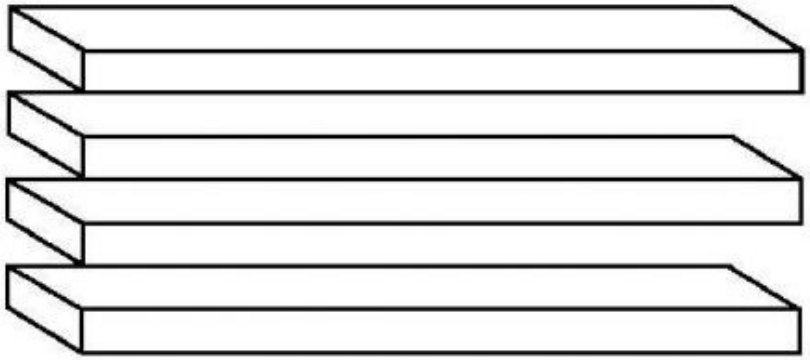


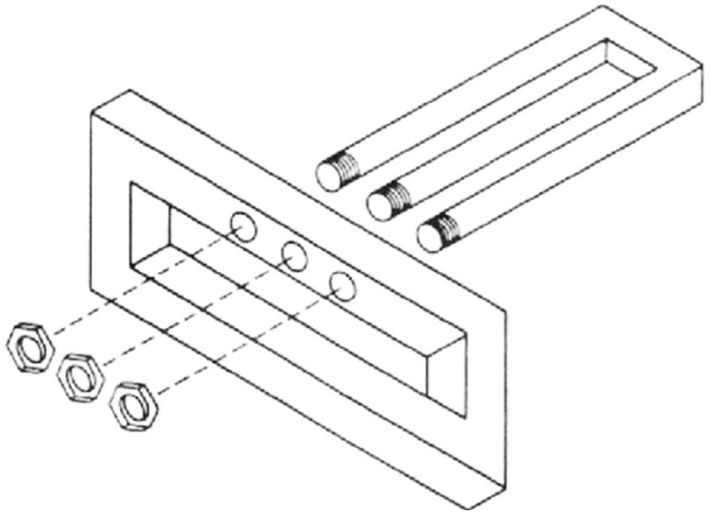


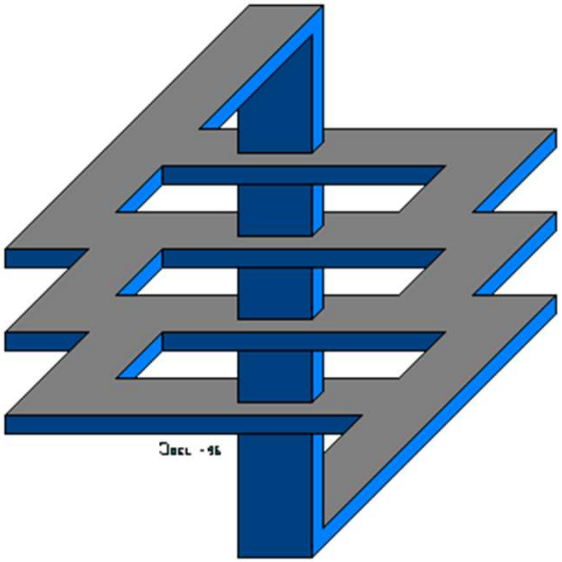


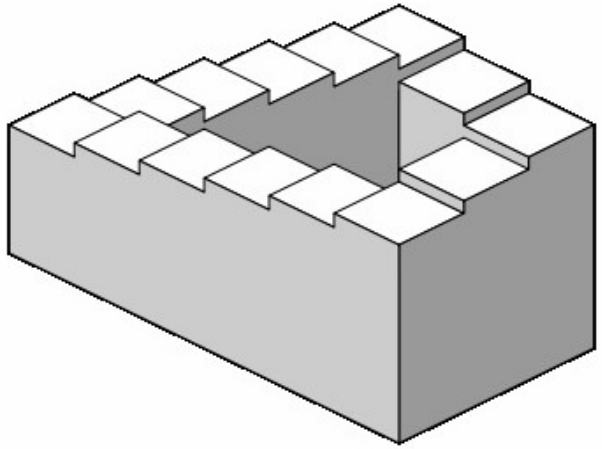


Impossible Objects

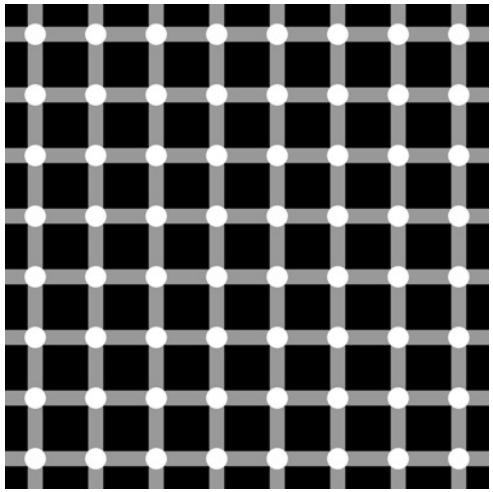


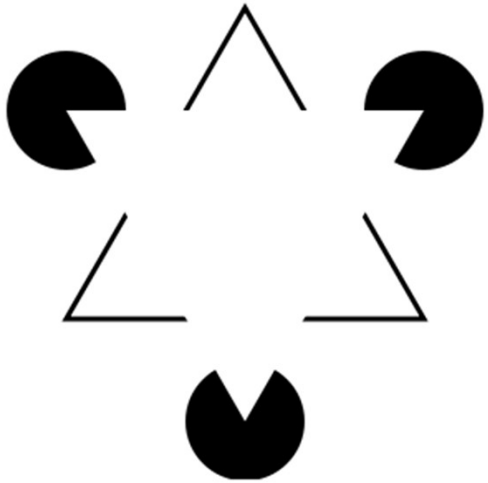


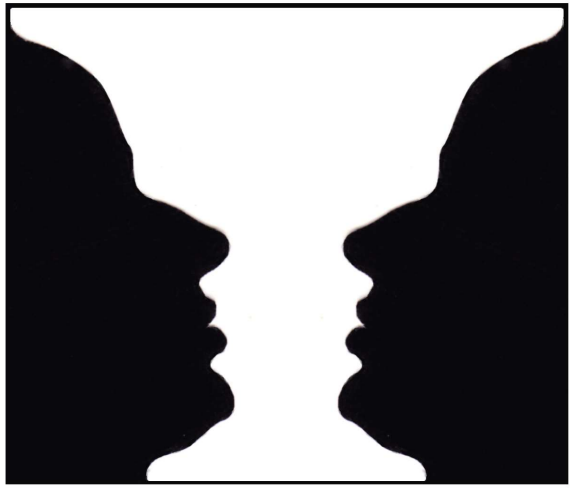


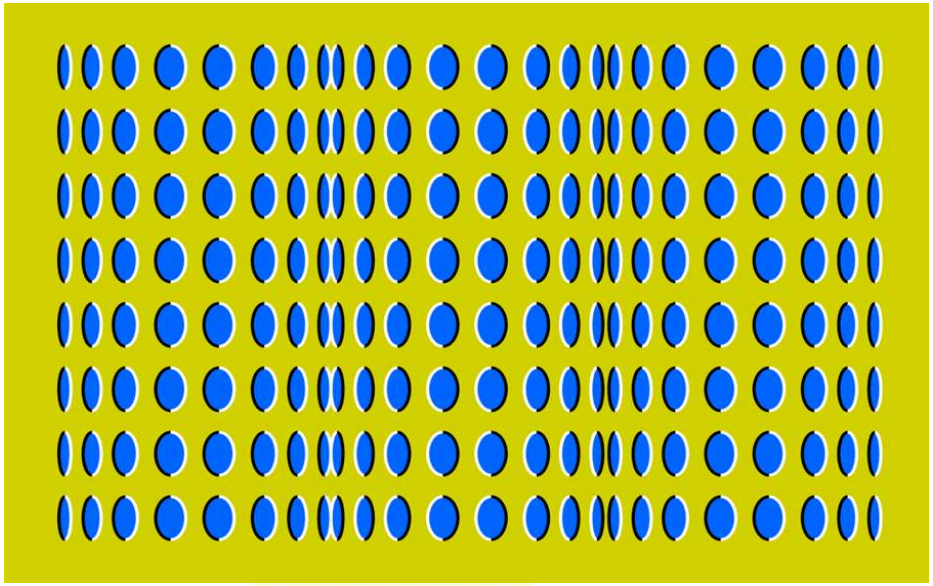


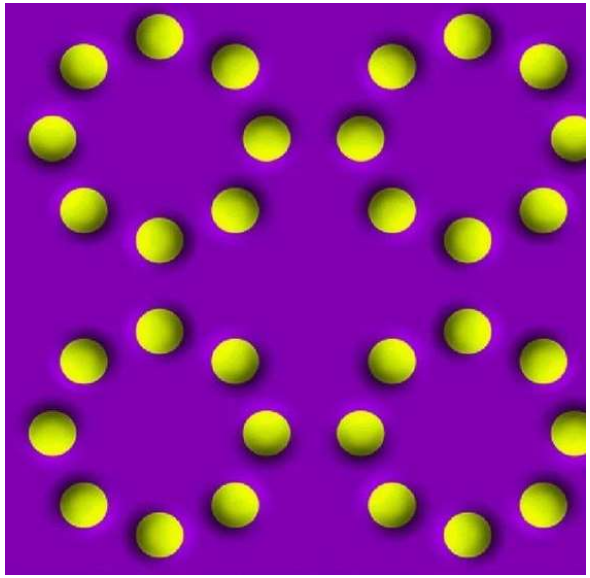
Seeing What Isn't There



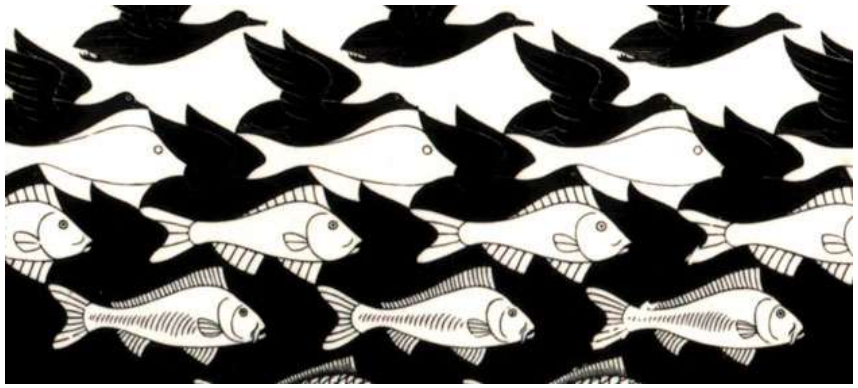


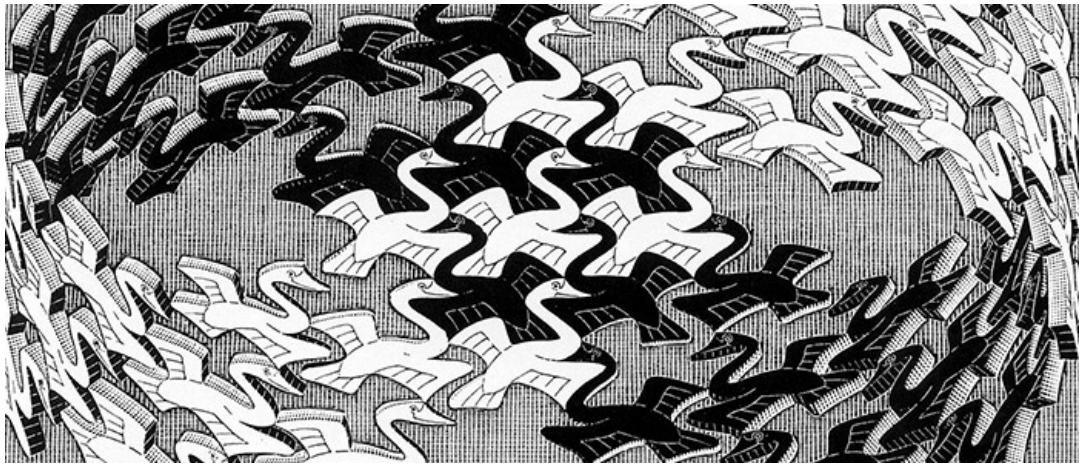




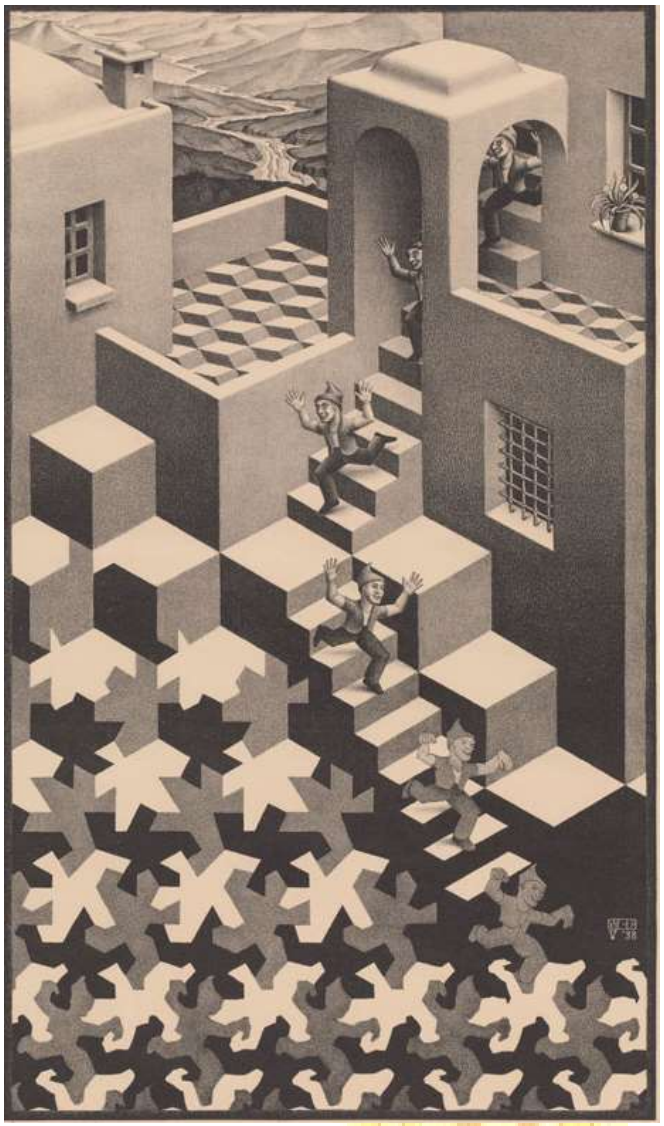


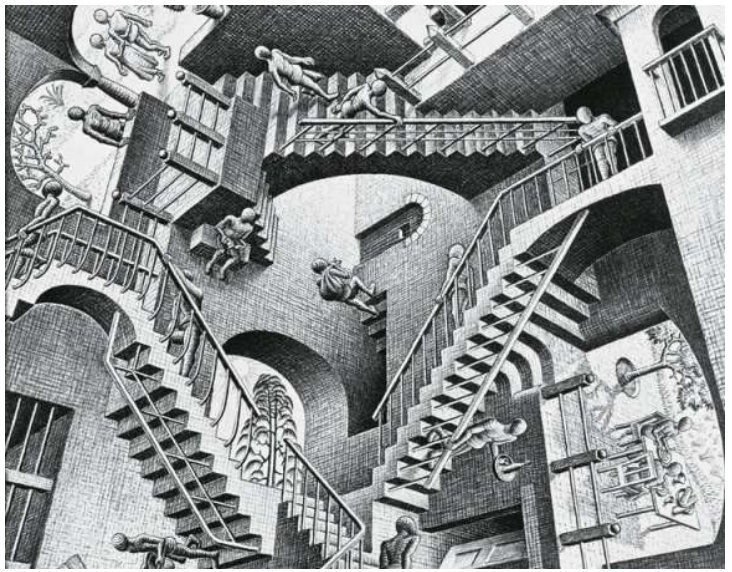
The Graphic Designs of M. C. Escher

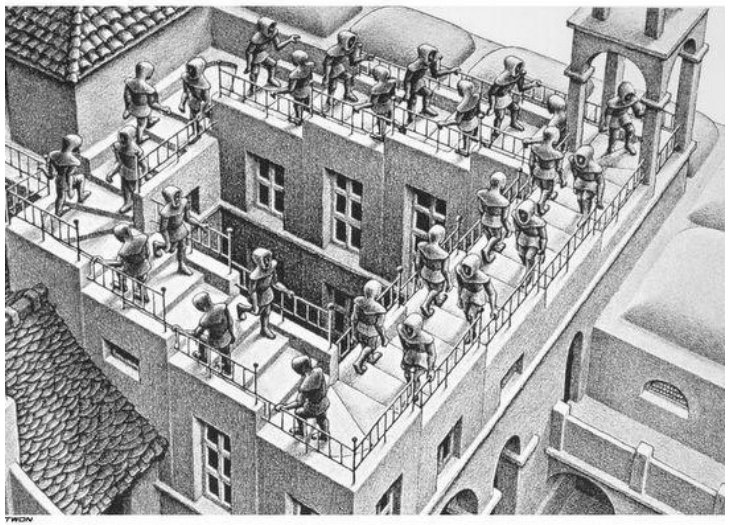




More Works by M. C. Escher

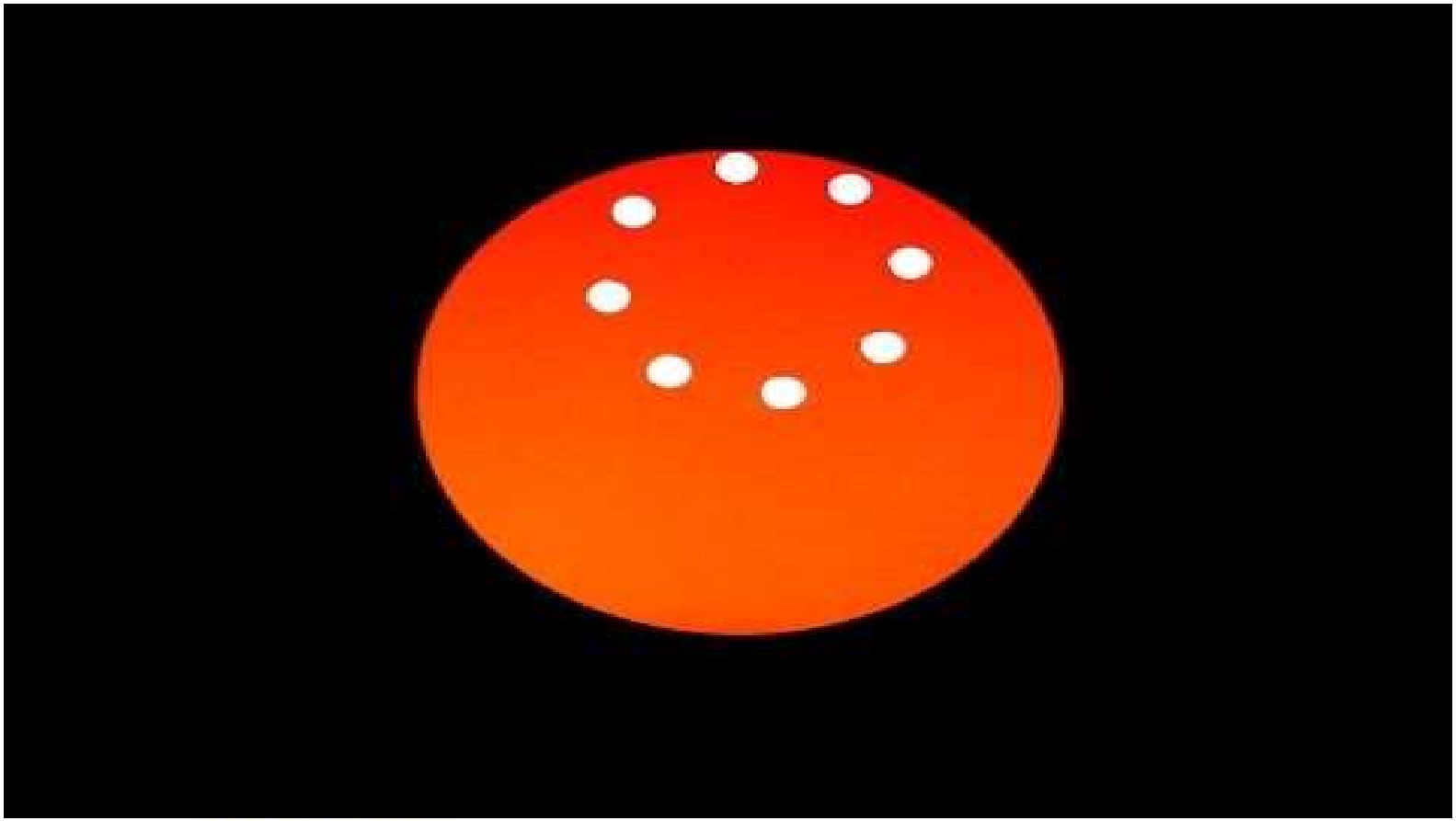






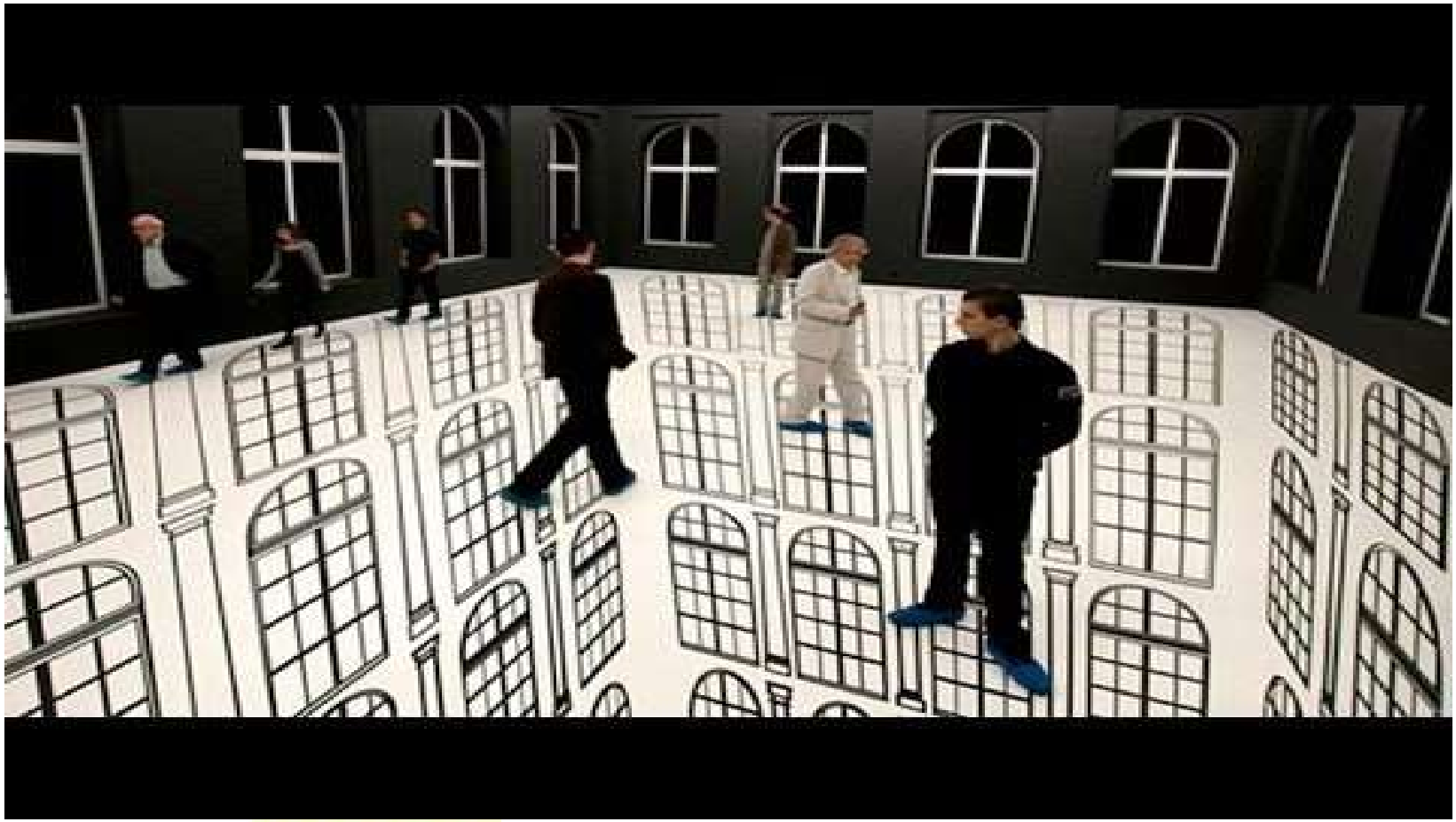
Illusary Motion and Other Neat Visual Tricks

Straight-line motions create the illusion of circular motion:
2-minute video: http://www.youtube.com/watch?v=pNe6fsaCVtI



More Visual Tricks and Optical Illusions

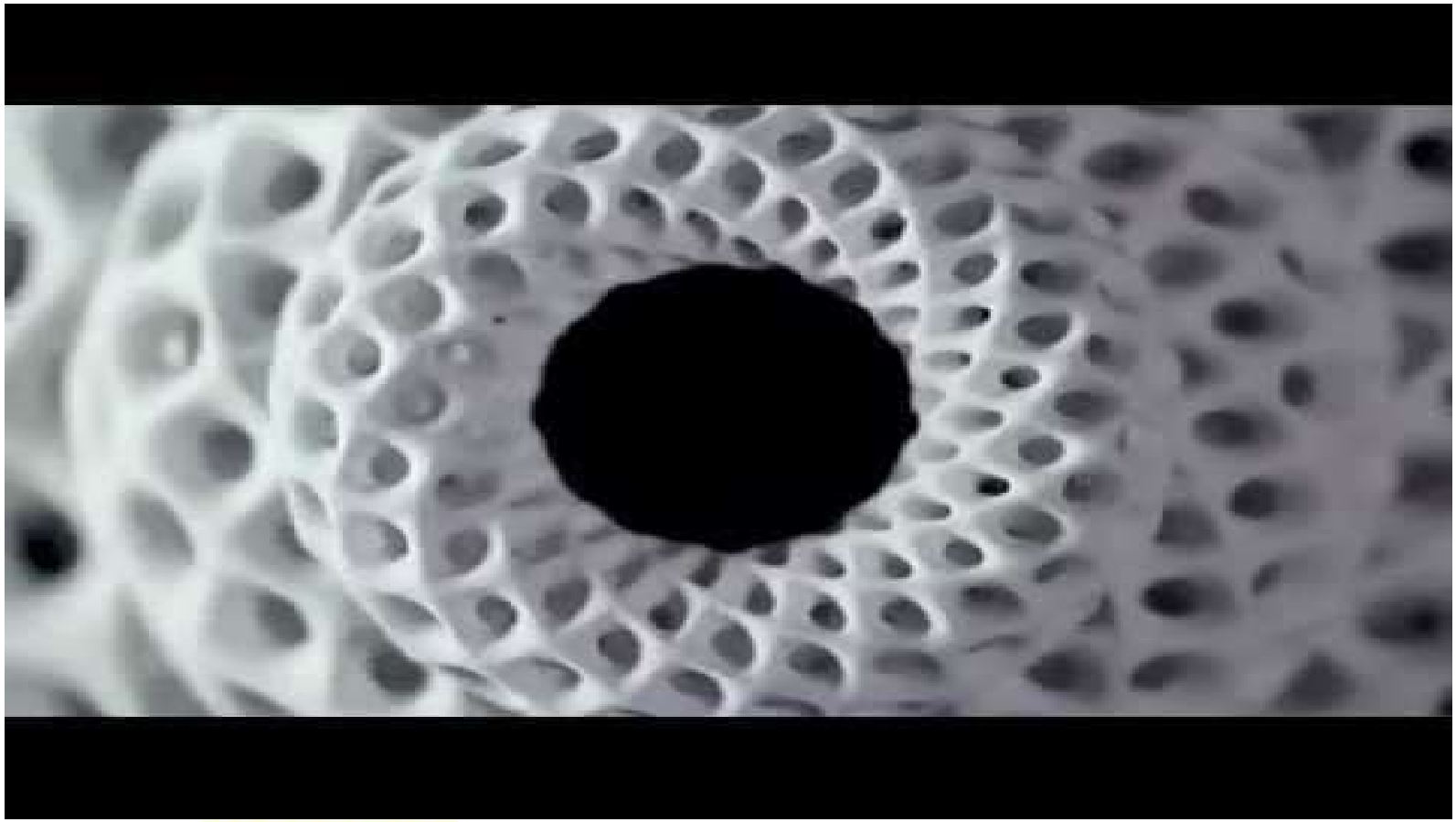
A collection of 10 neat visual tricks and optical illusions:
4-minute video: http://www.youtube.com/watch?v=-IWk5NkxQF8



Patterns Seen Under a Strobe Light

3D-printed sculptures come to life when spun under a strobe light

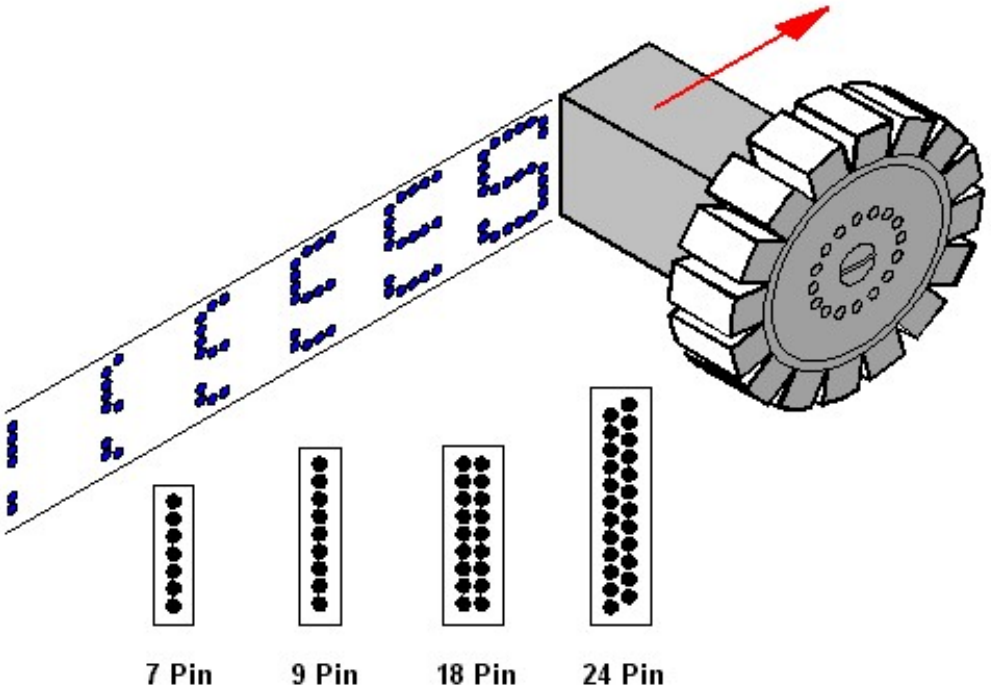
3-minute video: http://vimeo.com/116582567

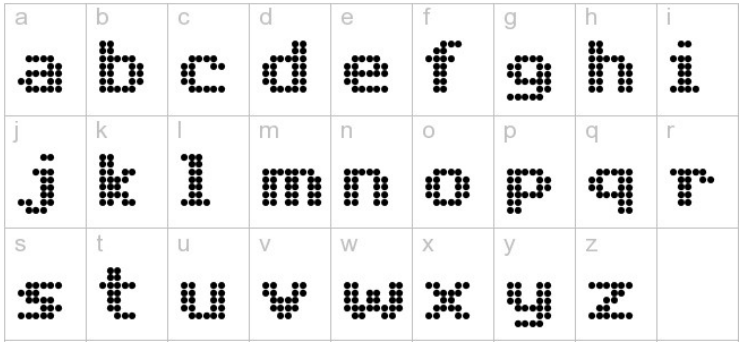


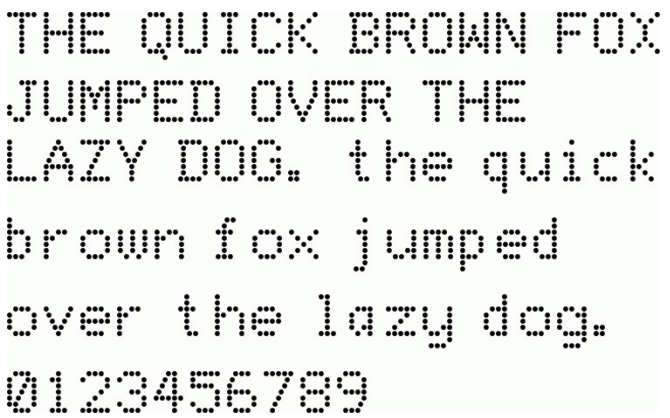
Dot-Matrix Printing and Display

The surface of paper or monitor is viewed as a huge dot matrix:

Black or colored dots are places at appropriate positions to form letters, shapes, or images of interest





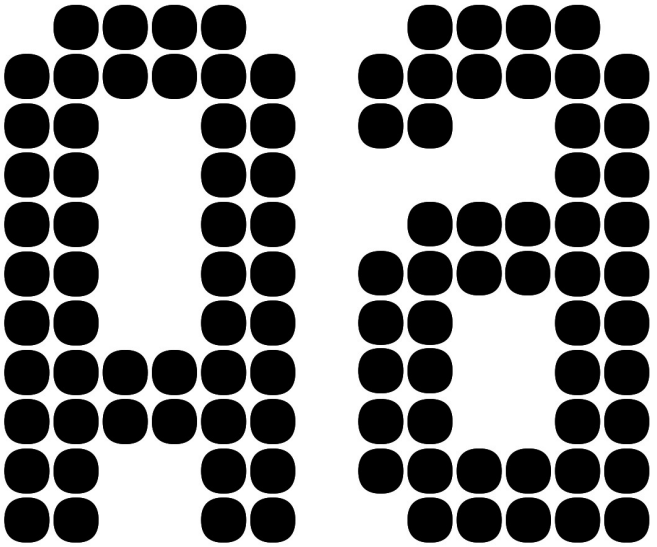


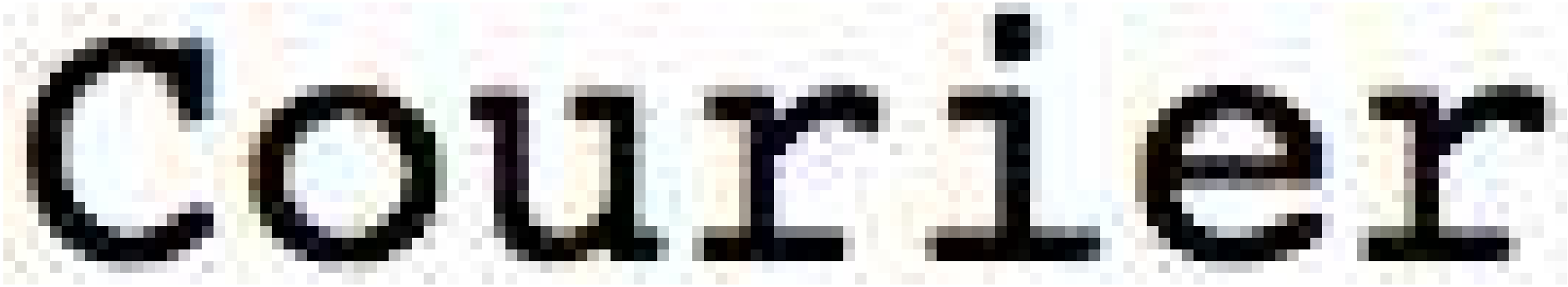
Design of Fonts and Font Families

A font is a set of geometric dot patterns for letters and symbols:

Arial
Helvetica
Times New Roman
Courier
Palatino
Garamond
Bookman

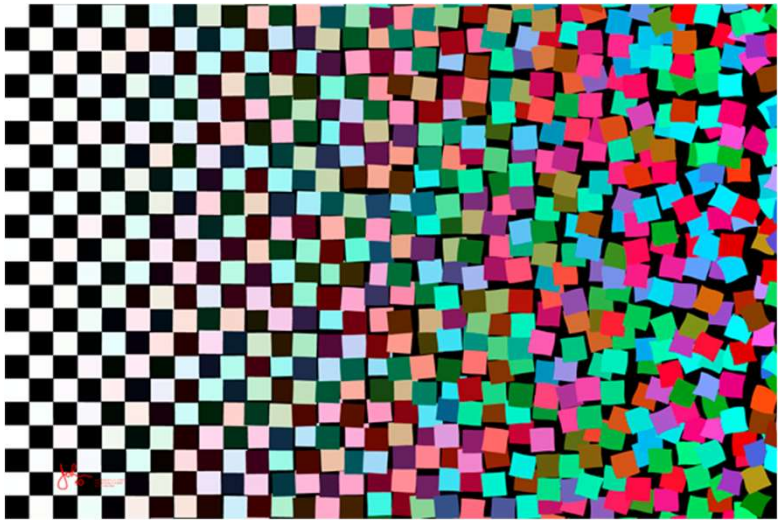
Verdana
Georgia
Comic Sans
Trebuchet
Arial Black
Impact

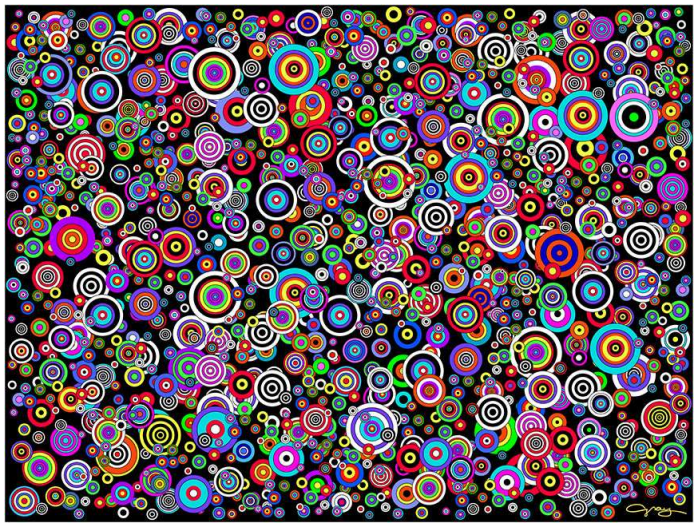


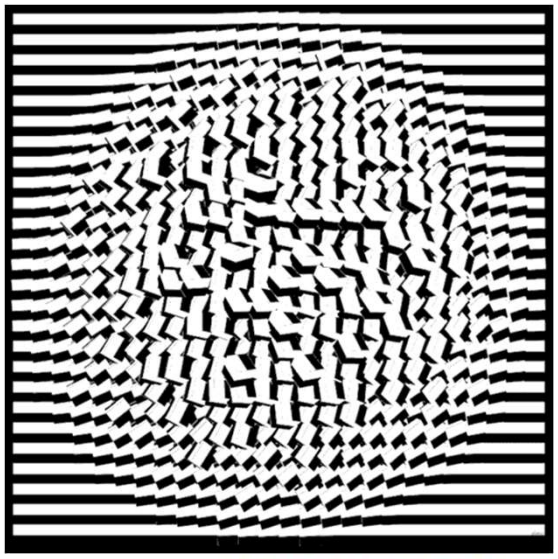


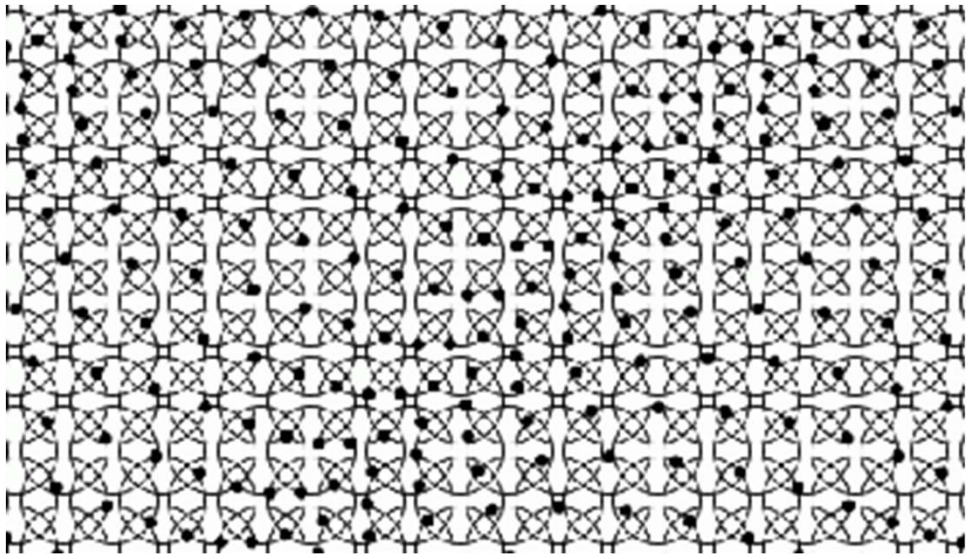
Dot-Matrix Displays Are Everywhere

Artful Computational Geometry

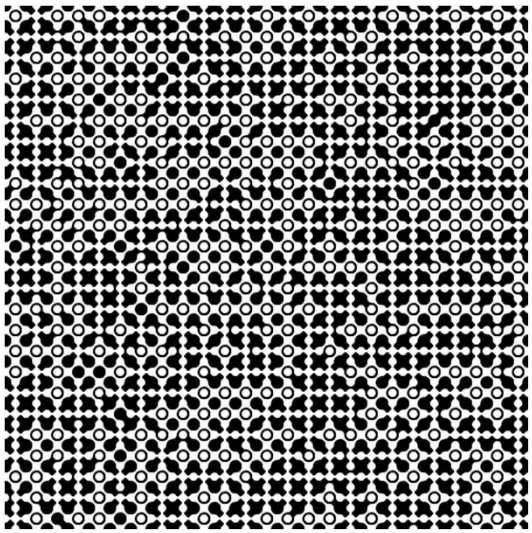


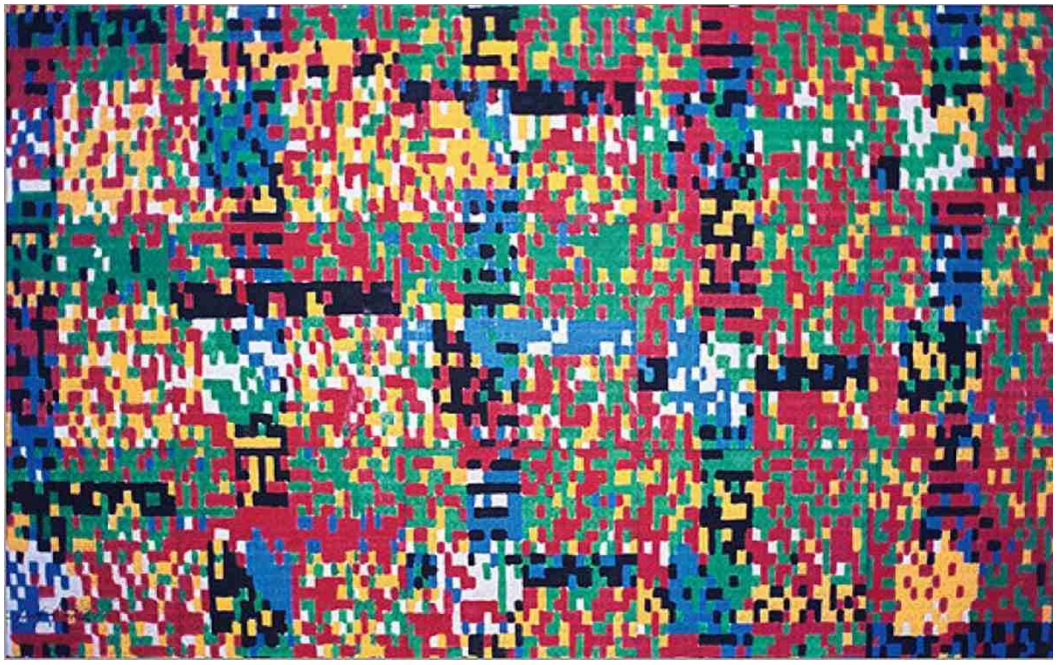


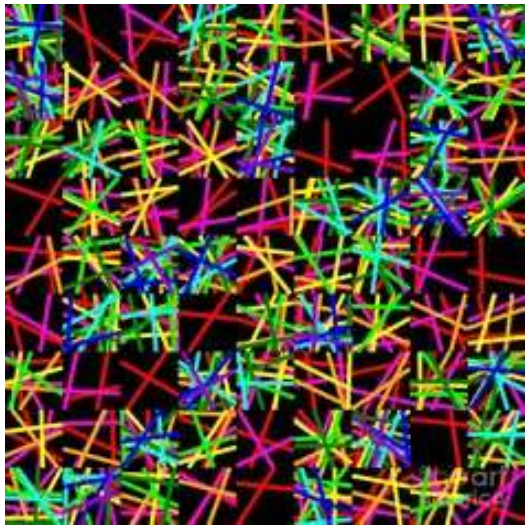




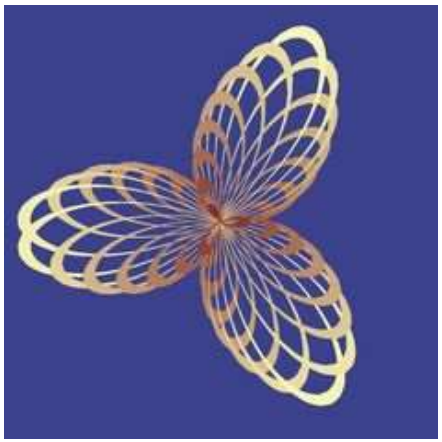
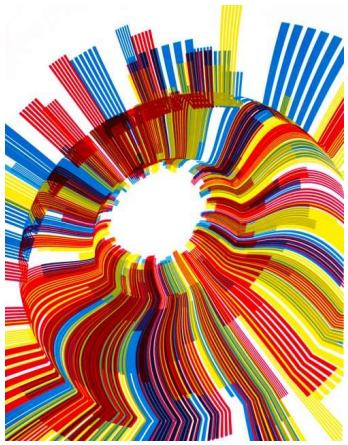
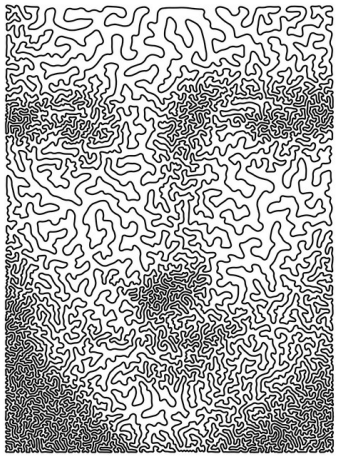
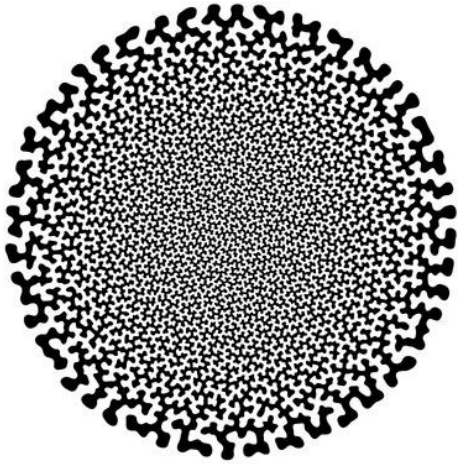
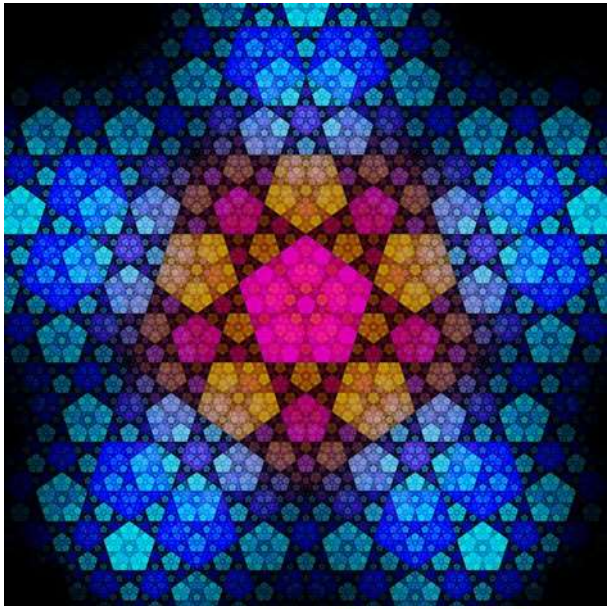
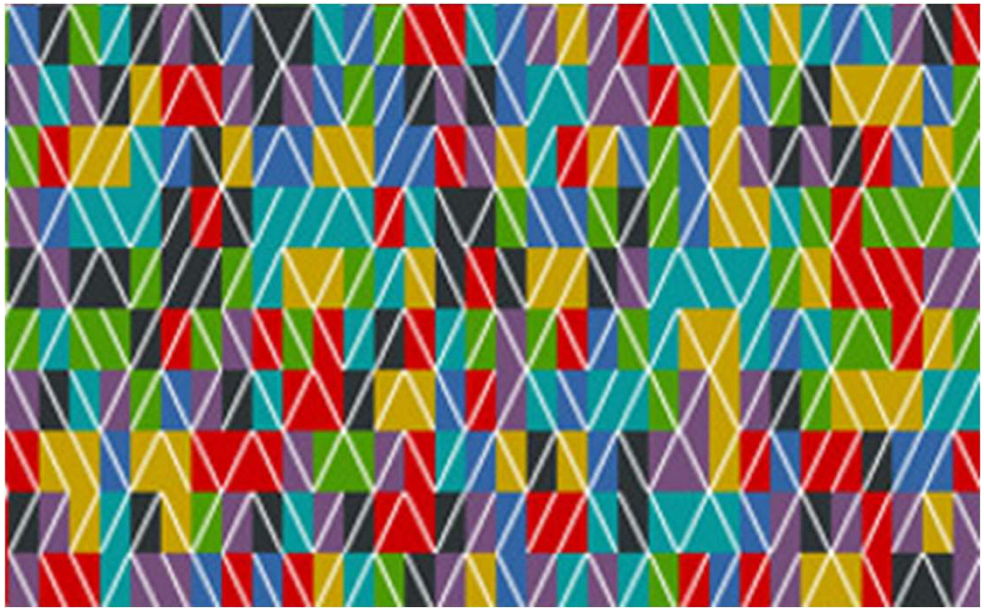
Randomly-Generated Digital Art





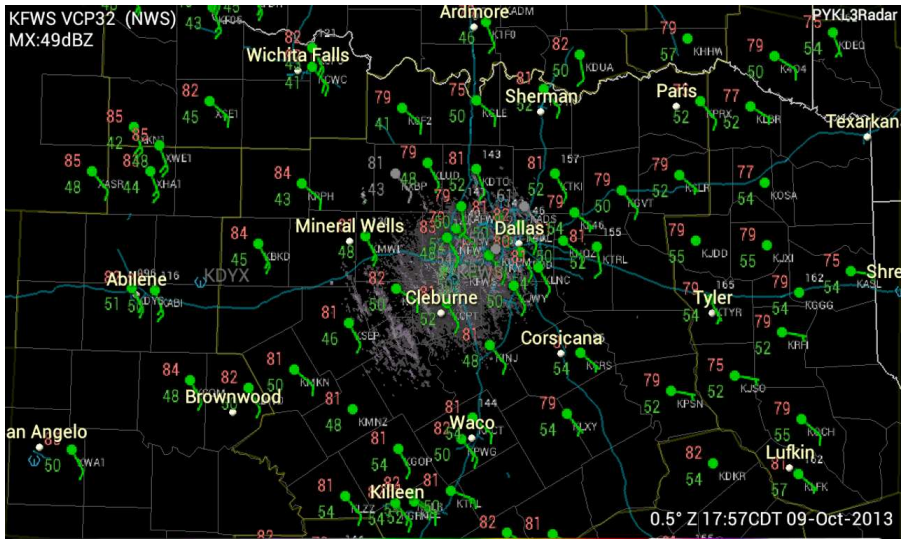


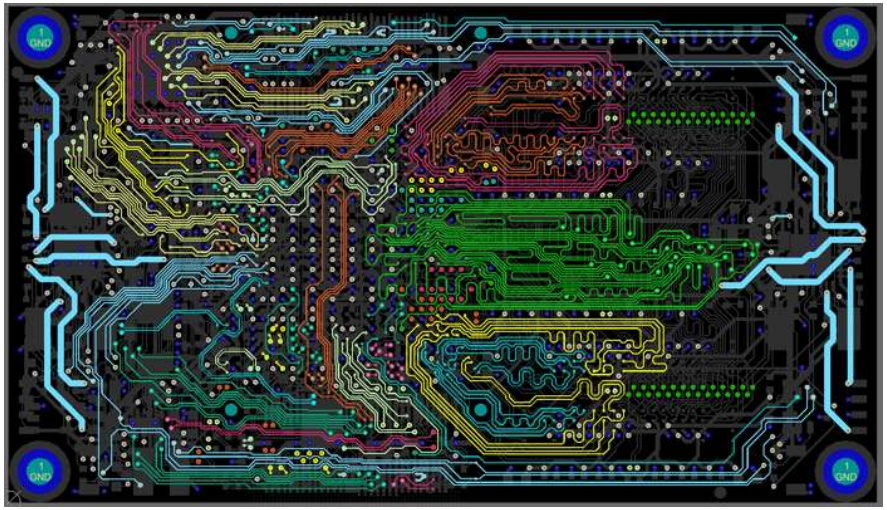
Computational or Algorithmic Art

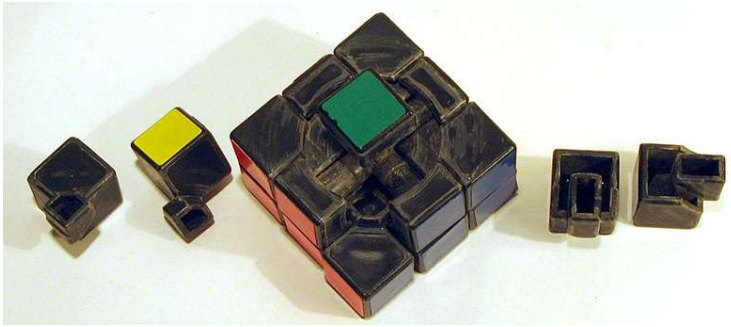







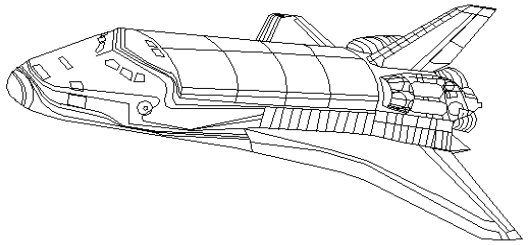
Some Applications of Computational Geometry

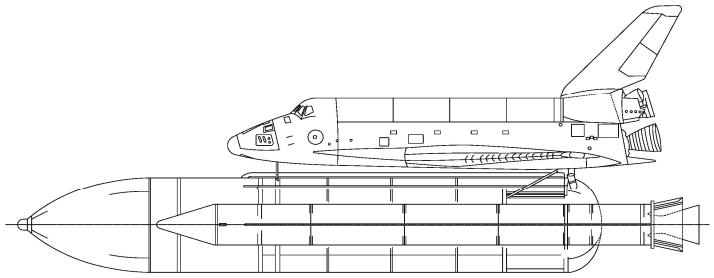
Maps & navigation (Google Maps)
Airspace design & air-traffic control
Protein structure prediction
Computer-aided design





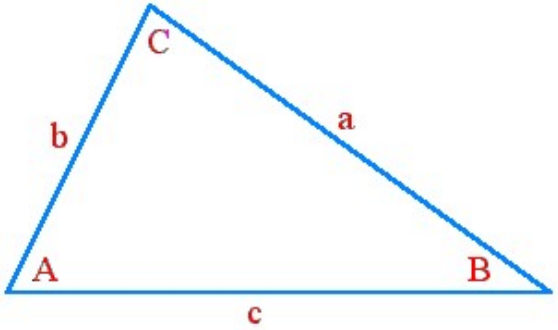


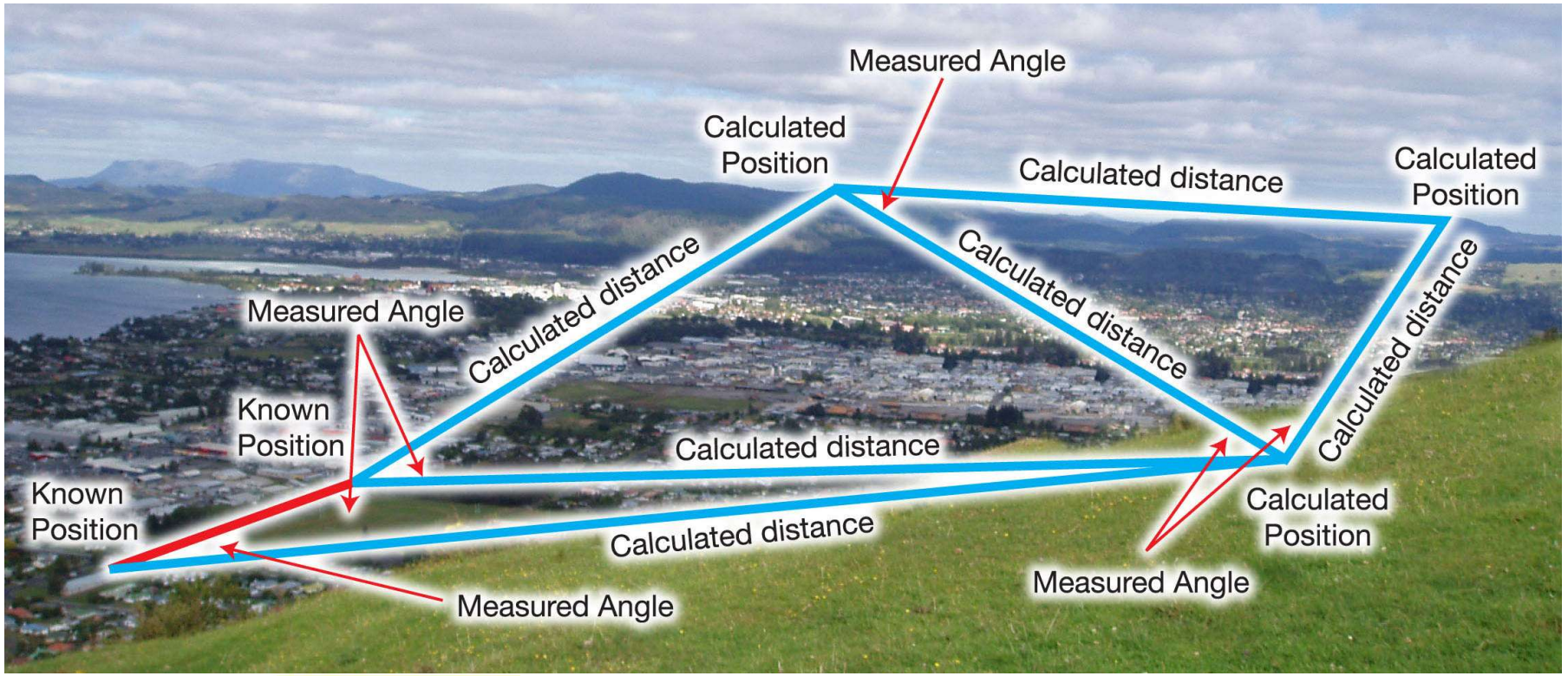




Triangulation and Its Use in Map-Making

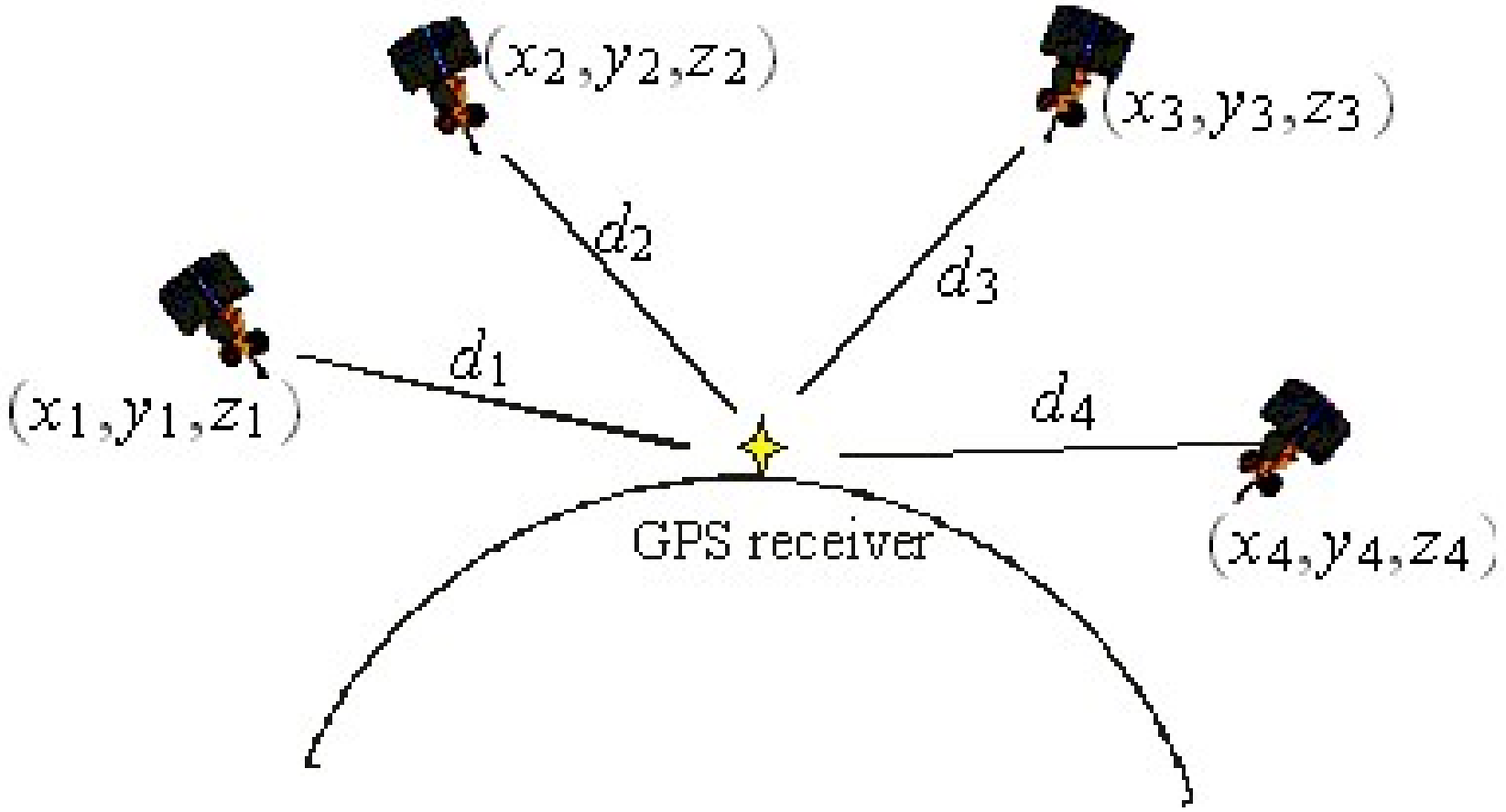
A triangle is fully specified by:
Lengths of its 3 sides
One side and its two adjacent angles
Two sides and the angle between them





Triangulation for GPS Calculations

Distance and angle to 4 GPS satellites used to compute position



Robot Motion Planning

Robot is first taught a task such as spooning or stirring:
It then executes the task will dealing with unforeseen circumstances.
3-minute video: http://www.youtube.com/watch?v=oY1FfytaD-c