Malfunction Diagnosis

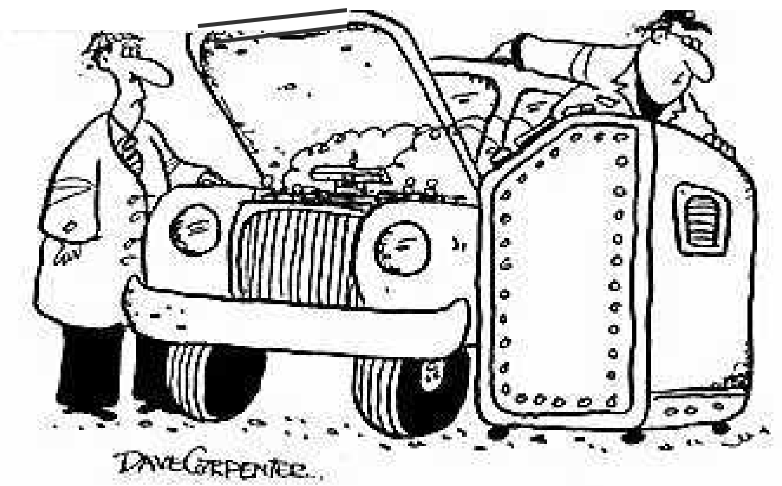
A Lecture in CE Freshman Seminar Series:
Ten Puzzling Problems in Computer Engineering

About This Presentation

Edition	Released	Revised	Revised	Revised	Revised
First	June 2007	May 2008	May 2009	May 2010	May 2012
	May 2016				

"Sizzling...smoking...flames shooting from the hard drive. OK...that IS a malfunction and you'd best unplug it."

"You made that diagnosis just to be mean."



" ACCORDING TO OUR DIAGNOSTIC COMPUTER, YOUR CAR WANTS TO HAVE $800 SPENT ON IT."

"I'm afraid your problem is a serious misprint in your home-medical guide."

Finding an Impostor

There are three people of a certain profession (say, medical doctors) in a room, but one of them may be an impostor. Each person asks the other two a question that can determine whether the person is real. The six yes (pass) / no (fail) opinions are presented to a judge who must decide whether an impostor is present and, if so, who it is. How would the judge go about deciding?

Somewhat similar to the fake coin puzzle

1 2 3

A real person always arrives at the correct judgment about another one, but an imposter may render an incorrect judgment

Assessment matrix A

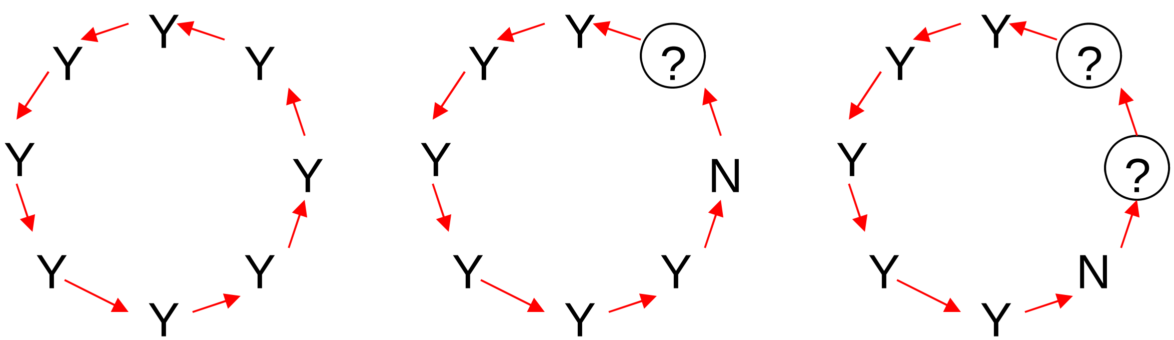
Assessor \ Person assessed	1	2	3
1	--	N	Y
2	N	--	N
3	Y	N	--

Assessment matrix B

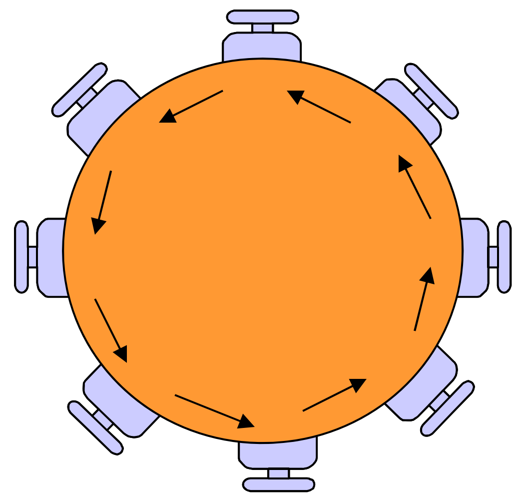
Assessor \ Person assessed	1	2	3
1	--	N	Y
2	Y	--	N
3	Y	N	--

Impostors Around a Dinner Table

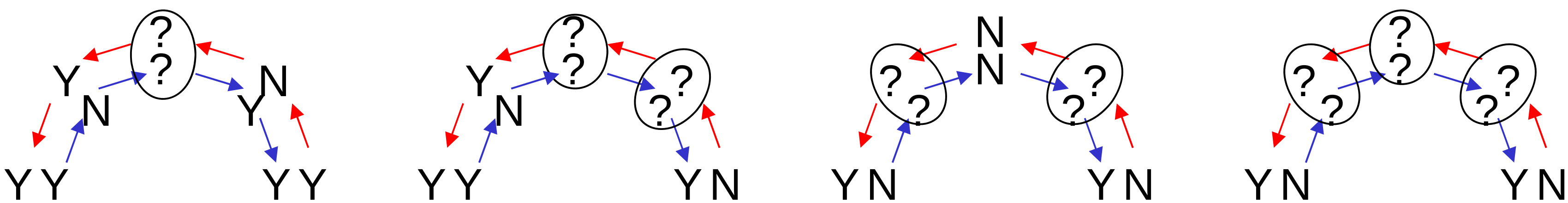
At a round dinner table, *n* people of a certain profession (say, computer engineers) try to determine if there are impostors among them. Each asks the person to his or her right a question and renders a judgment. Assumptions are identical to the previous puzzle.
How many impostors can be correctly identified?



No impostor   One impostor   Two impostors

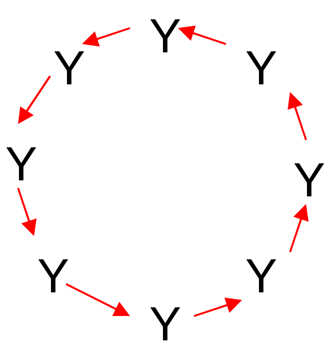


Repeat the puzzle above, but this time assume that each person asks a question of his/her neighbor on both sides



Analysis for a Given Set of Outcomes

In each of the cases shown below, determine the smallest possible number of impostors that would be consistent with the shown outcome



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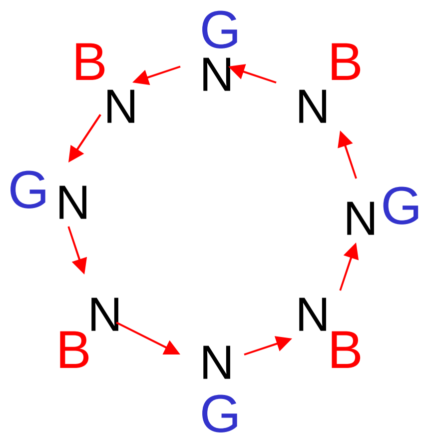
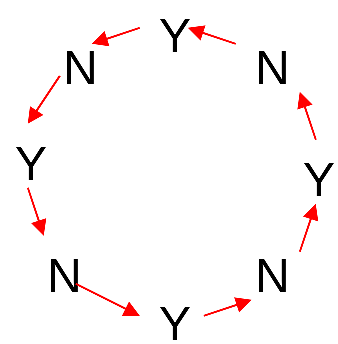


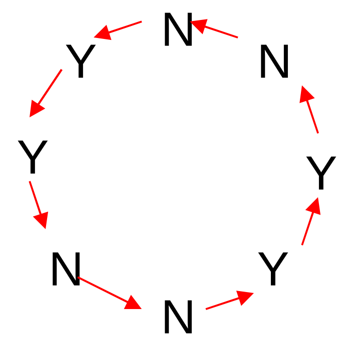
What is the largest possible number of non-impostors?

Generalize to ring of arbitrary size, even or odd



All Ns good
All Ys bad:
Is 4 smallest possible?

Generalize to ring of arbitrary even size



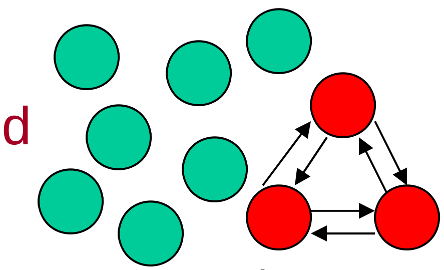
???

Finding Impostors with Limited Questioning

At a party, 10 people of a certain group (say, science-fiction writers) try to determine if there are impostors among them. Each person is asked a question by 2 different people and there are at most 3 impostors. Can the impostors be always correctly identified from the outcomes of the 20 questions? Solve the puzzle in the following two cases:

Case 1: It is possible for persons *A* and *B* to ask each other questions

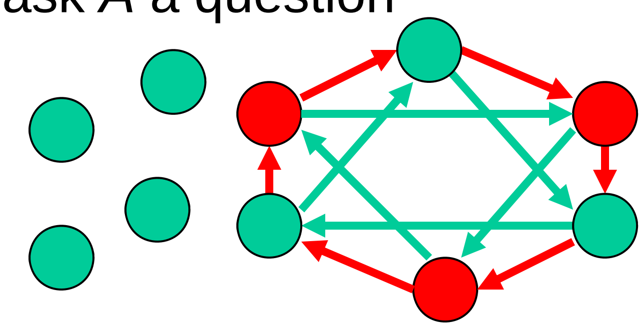
Can't be done. Ten people around a dinner table is a special case of this, because each person is questioned by his/her two neighbors. In that case, we determined that no more than 2 impostors can be identified



Another reason

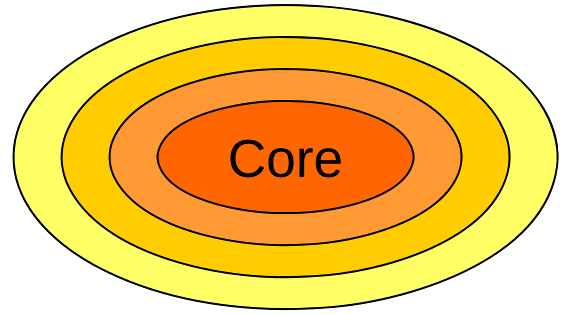
Case 2: If *A* asks *B* a question, then *B* will not ask *A* a question

Can't be done. If you switch the reals and impostors in the 6-person cluster, exactly the same syndrome may be observed

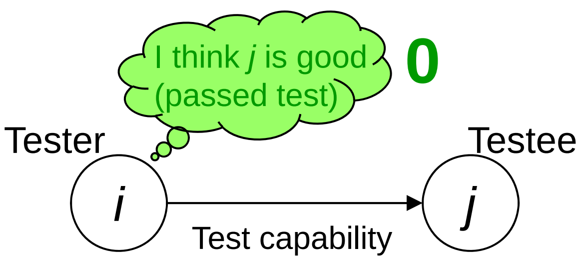


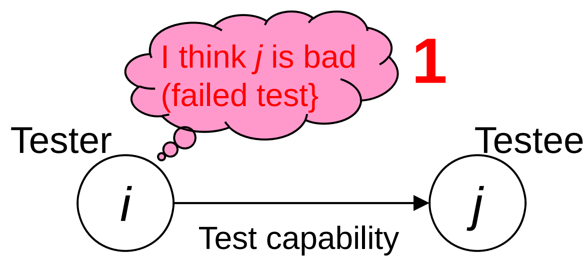
Malfunction Diagnosis Model

Layered approach to self-diagnosis
- A small core part of a unit is tested
- Trusted core tests the next layer of subsystems
- Sphere of trust is gradually extended



Diagnosis of one unit by another
- The tester sends a self-diagnosis request, expecting a response
- The unit under test eventually sends some results to the tester
- The tester interprets the results received and issues a verdict



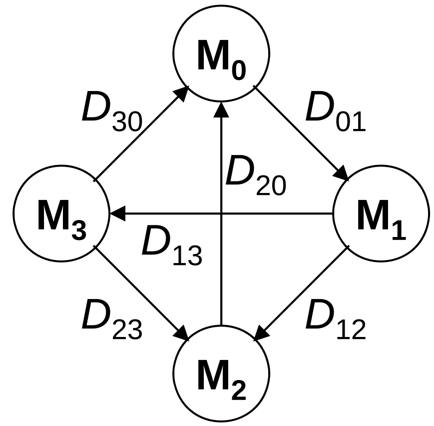


Testing capabilities among units is represented by a directed graph

The verdict of unit *i* about unit *j* is denoted by $D_{ij} \in \{0, 1\}$
All the diagnosis verdicts constitute the $n \times n$ diagnosis matrix D

One-Step Diagnosability Example

Consider this system, with the test outcomes shown



Malfunction syndromes (x means 0 or 1)

Malfn	D_{01}	D_{12}	D_{13}	D_{20}	D_{30}	D_{32}
None	0	0	0	0	0	0
M_0	x	0	0	1	1	0
M_1	1	x	x	0	0	0
M_2	0	1	0	x	0	1
M_3	0	0	1	0	x	x
M_0,M_1	x	x	x	1	1	0
M_1,M_2	1	x	x	x	0	1

Syndrome dictionary:

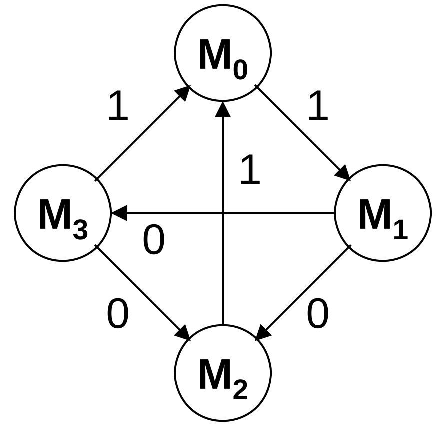
0	0	0	0	0	0	OK
0	0	0	1	1	0	M_0
0	0	1	0	0	0	M_3
0	0	1	0	0	1	M_3
0	0	1	0	1	0	M_3
0	0	1	0	1	1	M_3
0	1	0	0	0	1	M_2
0	1	0	1	0	1	M_2
1	0	0	0	0	0	M_1
1	0	0	1	1	0	M_0
1	0	1	0	0	0	M_1
1	1	0	0	0	0	M_1
1	1	1	0	0	0	M_1

The system above is 1-step 1-diagnosable (we can correctly diagnose up to one malfunctioning unit in a single round of testing)

Simple O(n^3)-Step Diagnosis Algorithm

Find a labeling of nodes (each designated G or B) that is consistent with the given test results

Input: The diagnosis matrix
Output: Every unit labeled G or B
while some unit remains unlabeled repeat
    choose an unlabeled unit and label it G or B
    use labeled units to label other units
    if the new label leads to a contradiction
    then backtrack
    endif
endwhile



1-step 1-diagnosable

M_0 is G (arbitrary choice)
M_1 is B
M_2 is B (contradiction, 2 Bs)

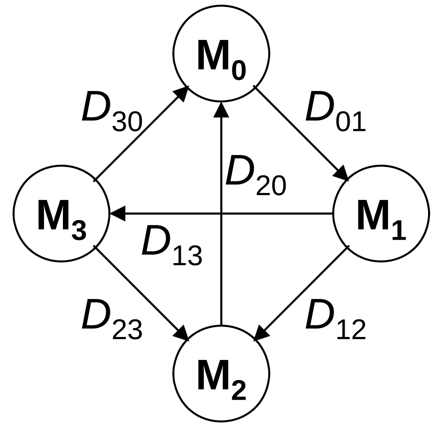
M_0 is B (change label)
M_1 is G (arbitrary choice)
M_2 is G
M_3 is G

More efficient algorithms exist

$$\begin{bmatrix} \text{--} & D_{01} & \text{--} & \text{--} \\ \text{--} & \text{--} & D_{12} & D_{13} \\ D_{20} & \text{--} & \text{--} & \text{--} \\ D_{30} & \text{--} & D_{32} & \text{--} \end{bmatrix}$$

Analysis versus Synthesis Problems

Analysis problem 1
Given an n-node directed graph defining the test links, find the extent of diagnosability (t)

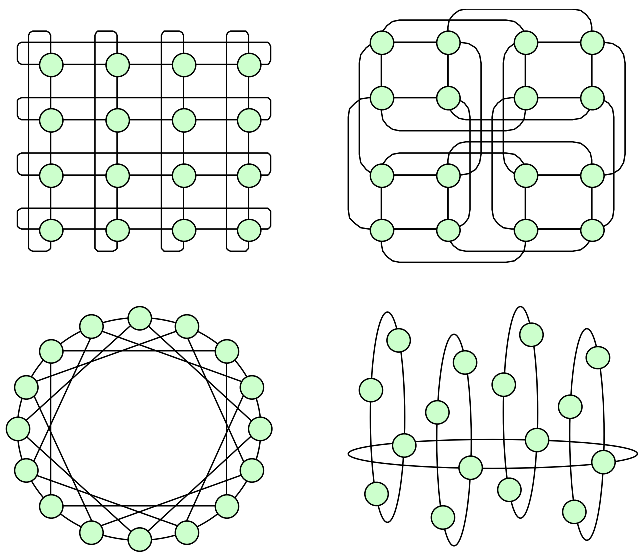


Analysis problem 2
Given an n-node directed graph and its $n \times n$ diagnosis matrix, identify the malfunctioning units

$$\begin{bmatrix} -- & D_{01} & -- & -- \\ -- & -- & D_{12} & D_{13} \\ D_{20} & -- & -- & -- \\ D_{30} & -- & D_{32} & -- \end{bmatrix}$$

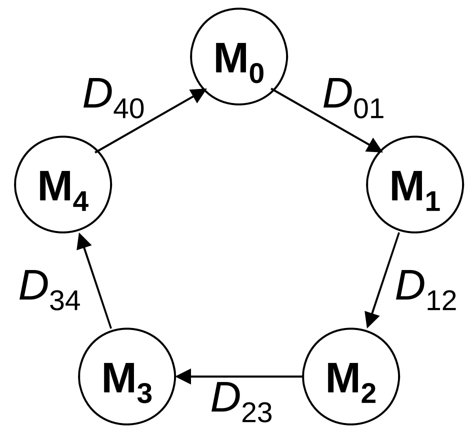
Synthesis problem 1
Given an n-node (un)directed graph defining a system and potential test links, identify a minimal number of test links to make the system t-diagnosable

Synthesis problem 2
How should we connect the n nodes of a system for best diagnosability?



Sequential t-Diagnosability

An n-unit system is sequentially t-diagnosable if the diagnosis syndromes when there are t or fewer malfunctions are such that they always identify, unambiguously, at least one malfunctioning unit



This is useful because some systems that are not 1-step t-diagnosable are sequentially t-diagnosable, and they can be restored by removing the identified malfunctioning unit(s) and repeating the process

This system is sequentially 2-diagnosable

In one step, it is only 1-diagnosable

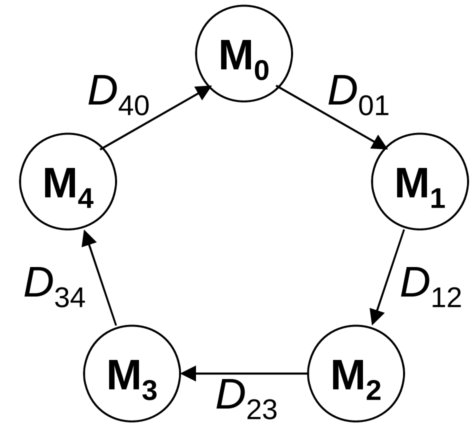
Necessary condition:

$n \geq 2t + 1$; i.e., a majority of units must be good

Sequential diagnosability of directed rings:
An n-node directed ring is sequentially t-diagnosable for any t that satisfies $\lceil (t^2 - 1)/4 \rceil + t + 2 \leq n$

Sequential 2-Diagnosability Example

Consider this system, with the test outcomes shown



Malfunction syndromes (x means 0 or 1)

Malfn	D_{01}	D_{12}	D_{23}	D_{34}	D_{40}
M_0	x	0	0	0	1
M_1	1	x	0	0	0
M_2	0	1	x	0	0
M_3	0	0	1	x	0
M_4	0	0	0	1	x
M_0,M_1	x	x	0	0	1
M_0,M_2	x	1	x	0	1
M_0,M_3	x	0	1	x	1
M_0,M_4	x	0	0	1	x

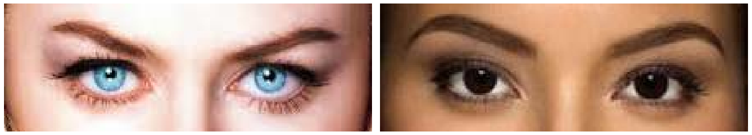
Syndromes for M_0 bad:

0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	1	0	1

The system above is sequentially 2-diagnosable (we can correctly diagnose up to two malfunctioning units, but only one at a time)

A Related Puzzle: Blue-Eyed Islanders

Inhabitants of an island are blue-eyed or brown-eyed, but none of them knows the color of his/her own eyes, and must commit ritual suicide the next day at noon if s/he ever finds out.



[Okay, this is silly, but don’t argue with the premises, such as there not being any mirrors, etc.; just view it as an exercise in logical reasoning.]

The islanders are quite proficient in logical reasoning and won’t miss a chance to deduce their eye color, should there be enough info to do so.

Unaware of the islanders’ traditions, which make discussing eye colors a taboo, a visitor giving a speech on the island begins his speech thus: “It’s so good to see someone else with blue eyes on this island.”

What are the consequences of this faux-pas?

Hint: Begin by thinking about what would happen if there were just one blue-eyed islander and build up to larger numbers of blue-eyed people.