Maps and Graphs

A Lecture in the Freshman Seminar Series:
Puzzling Problems in Science and Technology

About This Presentation

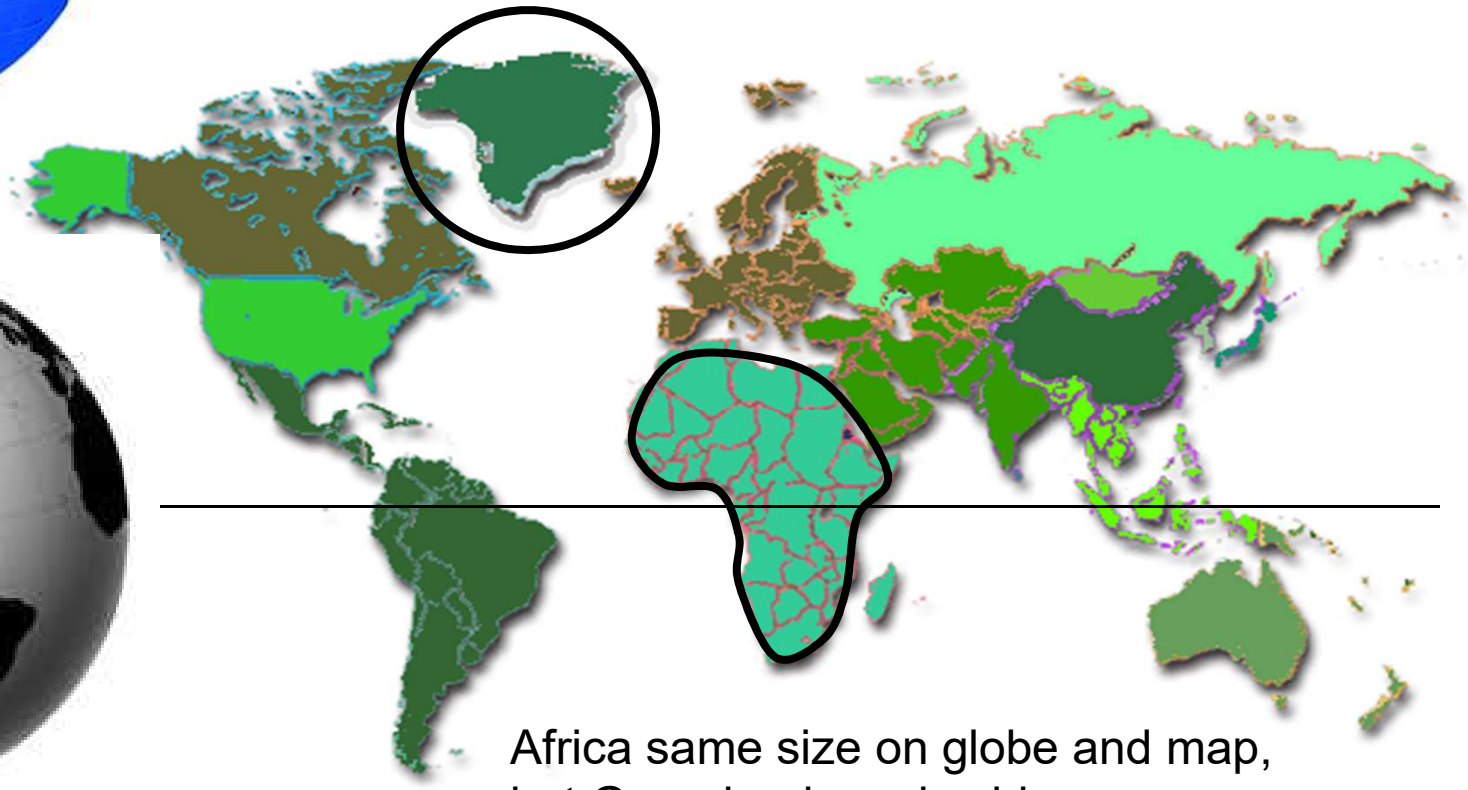
This presentation belongs to the lecture series entitled "Puzzling Problems in Science and Technology," devised for a ten-week, one-unit, freshman seminar course by Behrooz Parhami, Professor of Computer Engineering at University of California, Santa Barbara. The material can be used freely in teaching and other educational settings. Unauthorized uses, including any use for financial gain, are prohibited.

Edition	Released	Revised	Revised	Revised	Revised
First	Nov. 2016	Nov. 2018			

Ptolemy's Map of the World (150 AD)

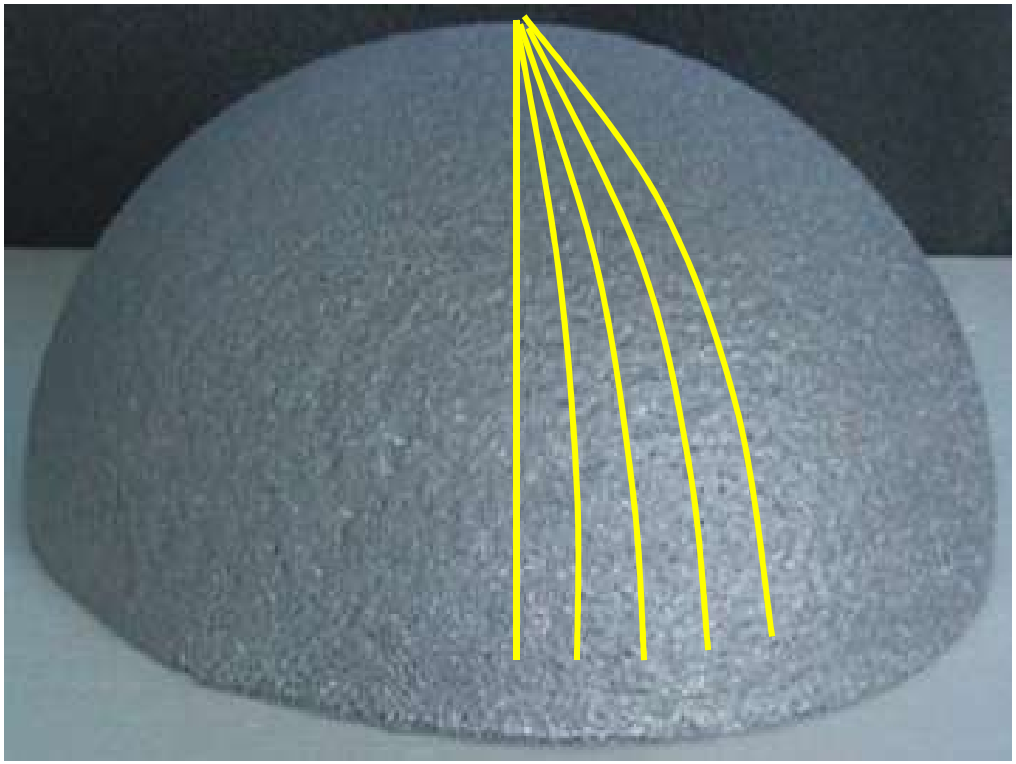
Globes and Flat Maps

Continents and countries appear to scale on a globe but they get distorted when drawn on some flat maps



Africa same size on globe and map, but Greenland much wider on map

The Globe Can Be Flattened

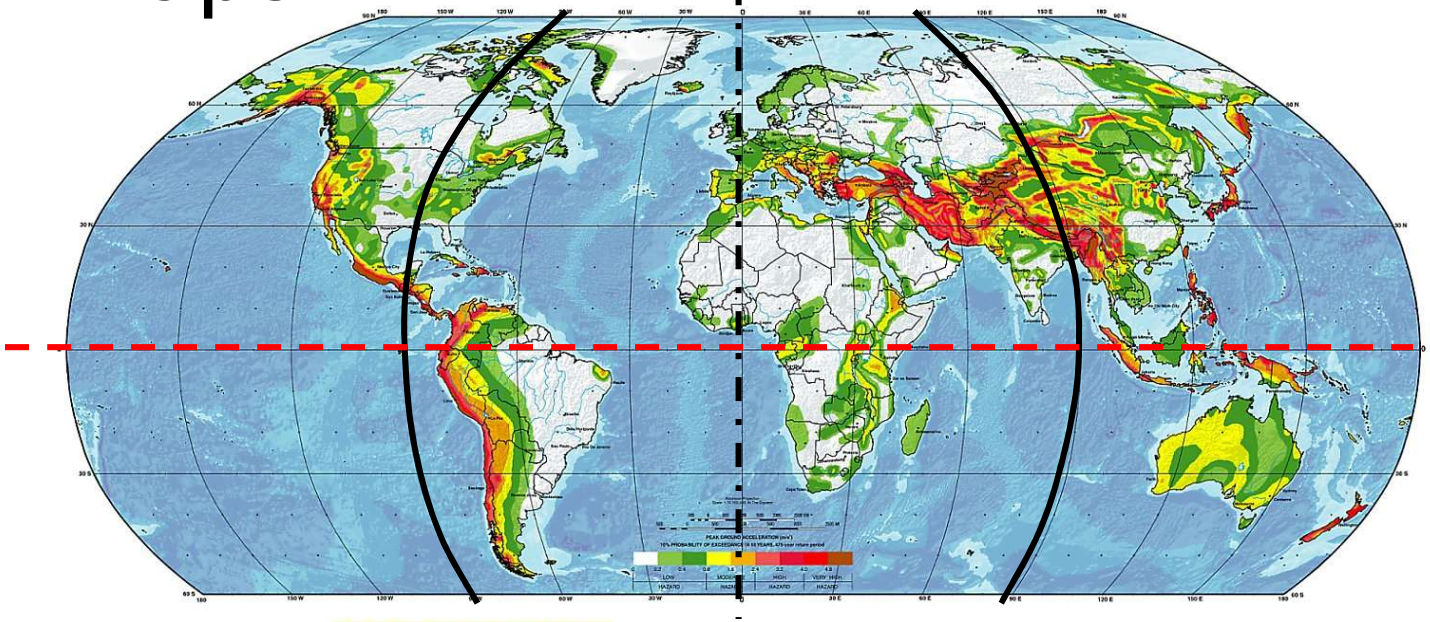


Segment in a photographic map of Mercury, with gaps inserted to preserve scale

Flattened Maps

Vertical longitudinal lines

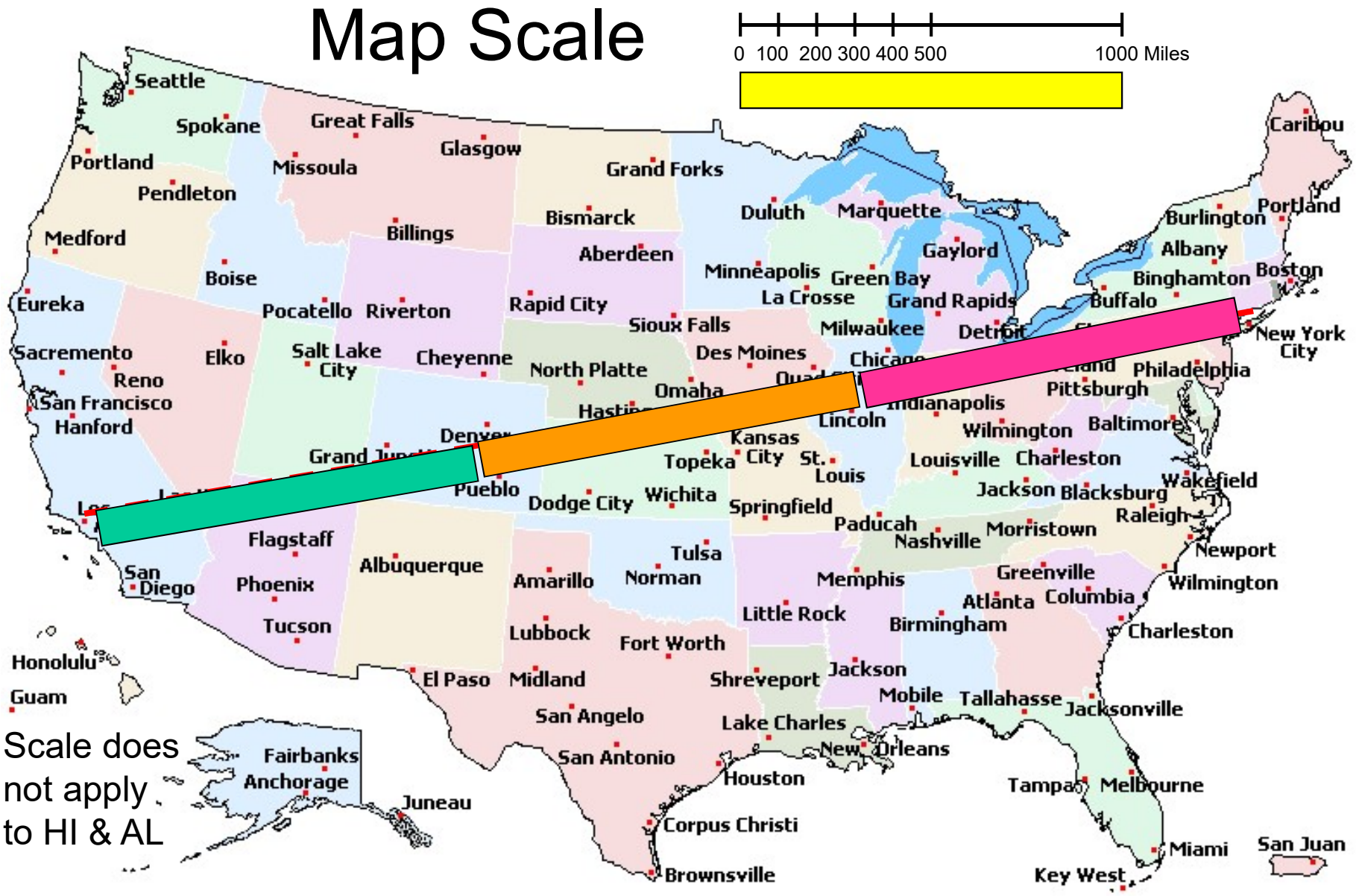
The Equator



Curved longitudinal lines

The Equator

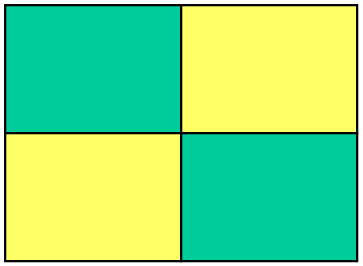
Map Scale



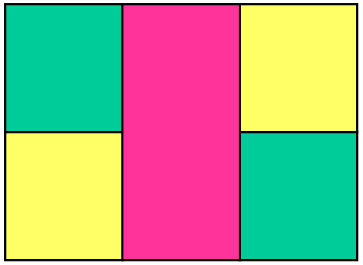
Scale does not apply to HI & AL

The Map Coloring Problem

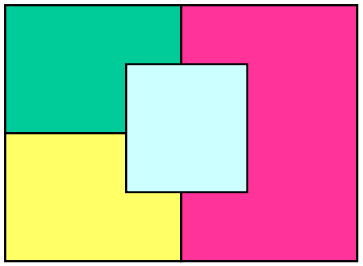
We want to color countries, oceans, lakes, and islands on a map so that no two adjacent areas have the same color.



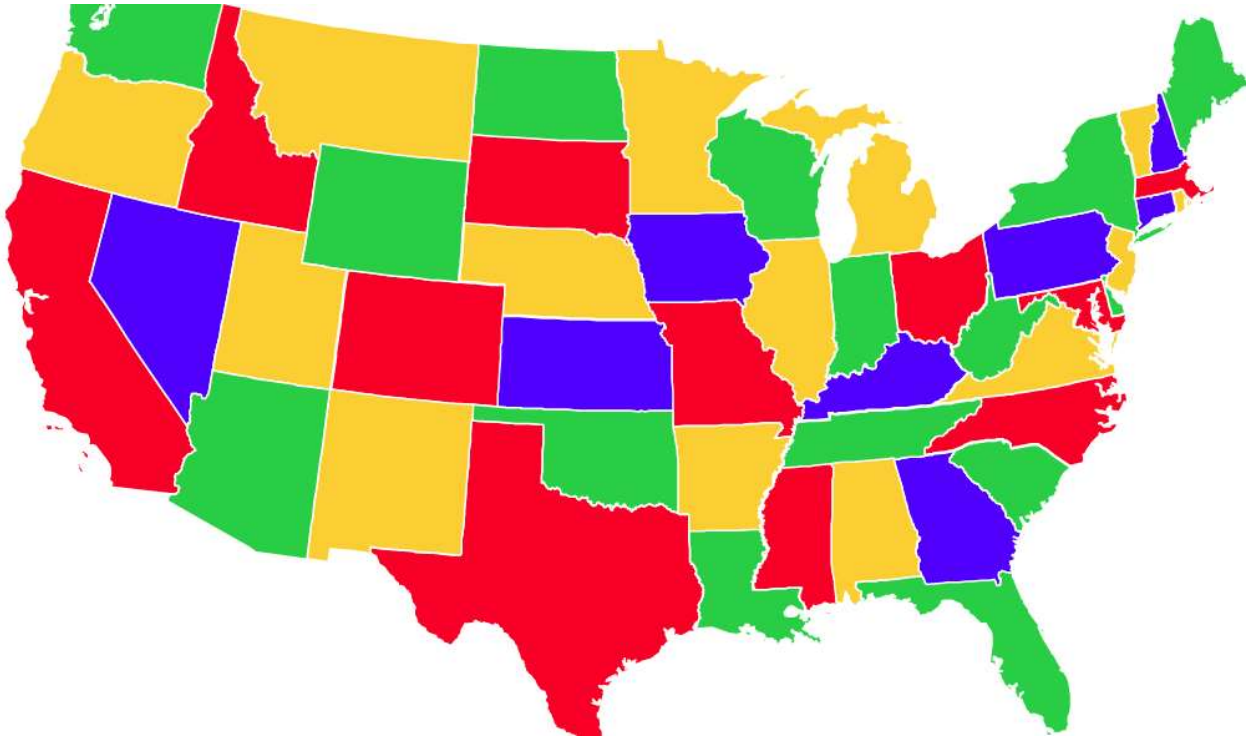
Two colors



Three colors

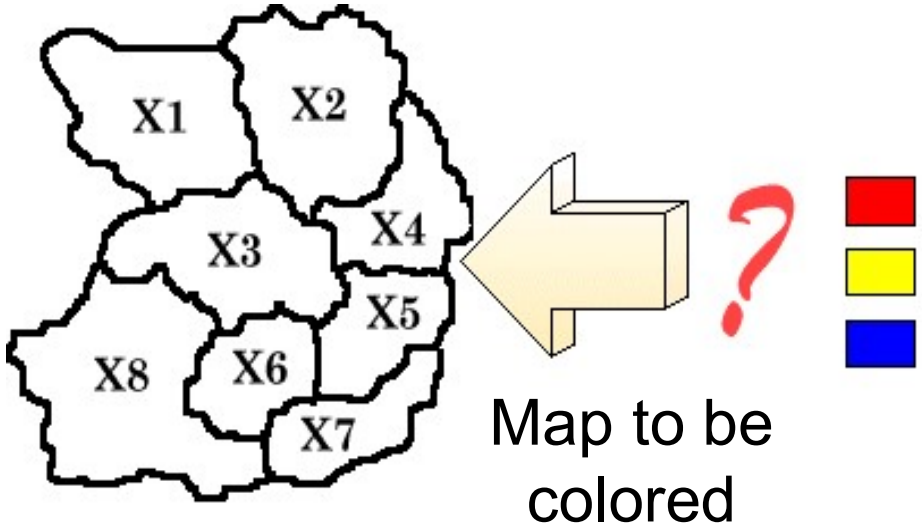


Four colors

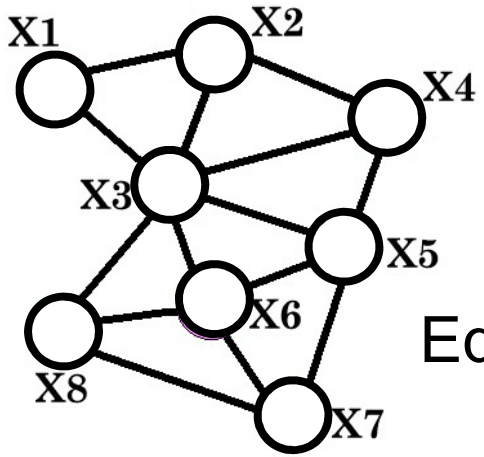


We know that four colors always suffice

Map and Graph Coloring

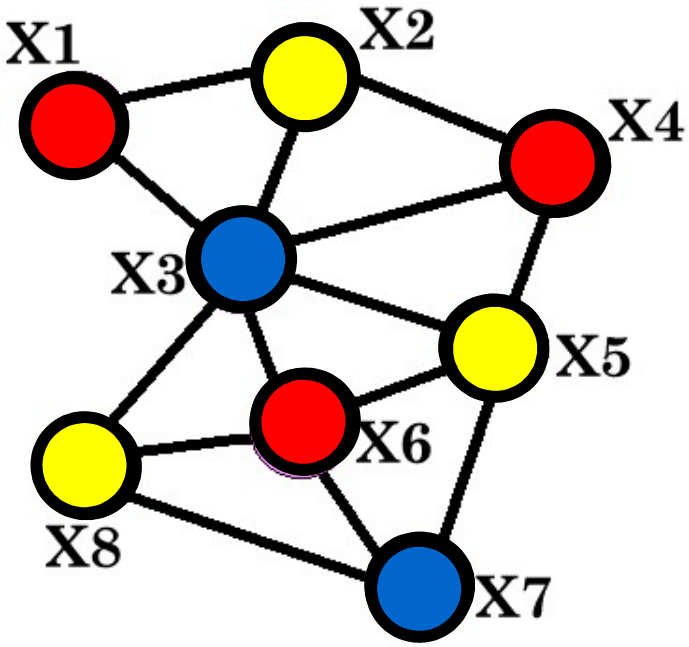


Map to be colored



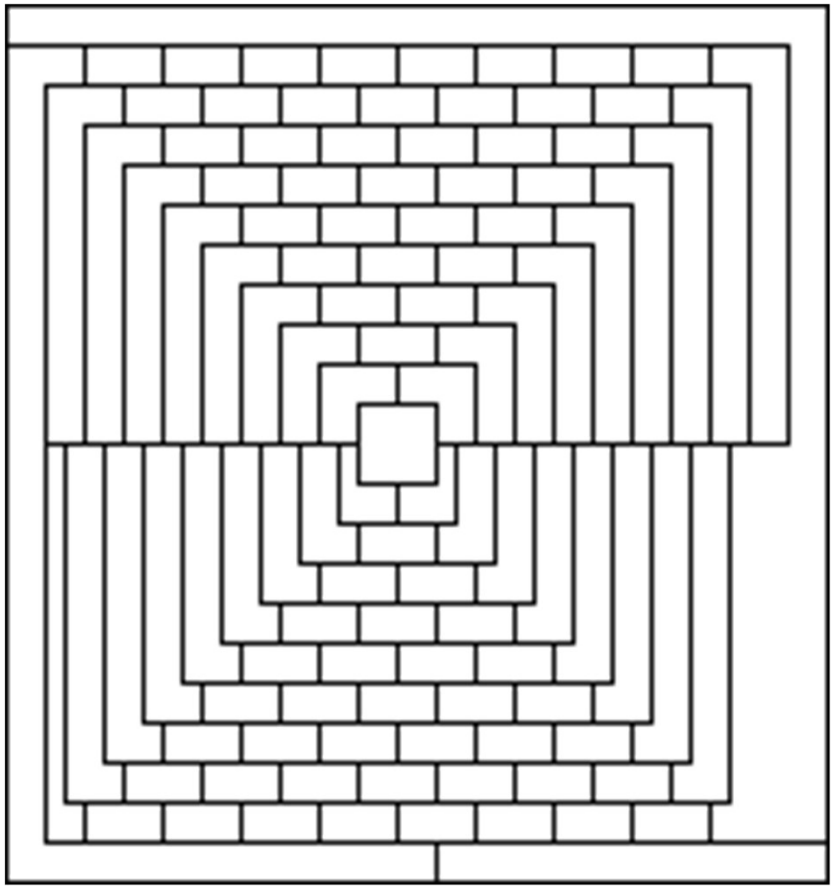
Equivalent graph

A map can be converted to a planar graph that can be drawn with no edges crossing

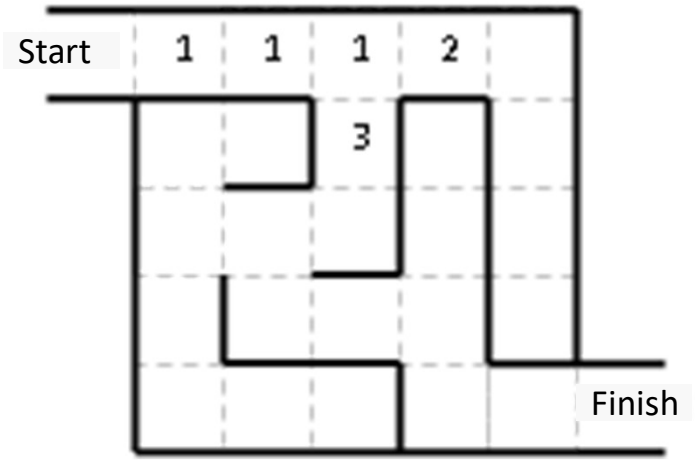


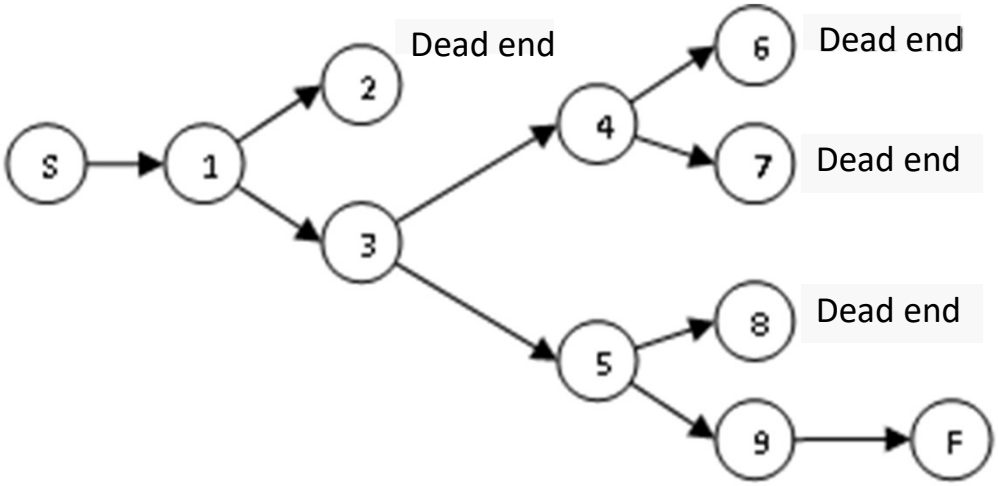
Color These Maps with Four Colors

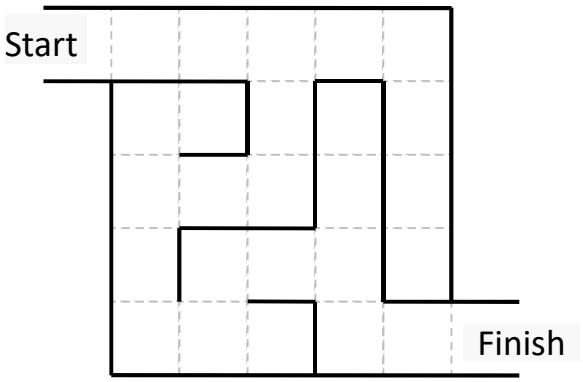
Color the following natural and artificial maps so that no two adjacent areas have the same color

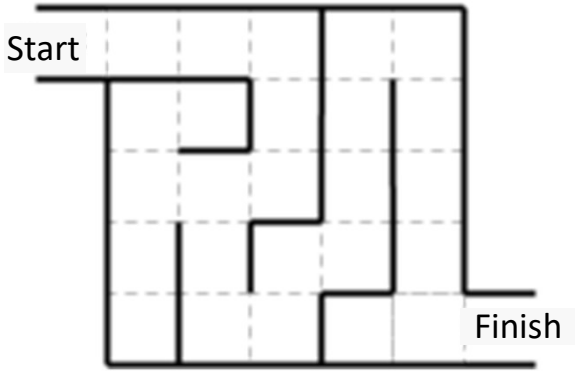


Mazes Represented as Graphs



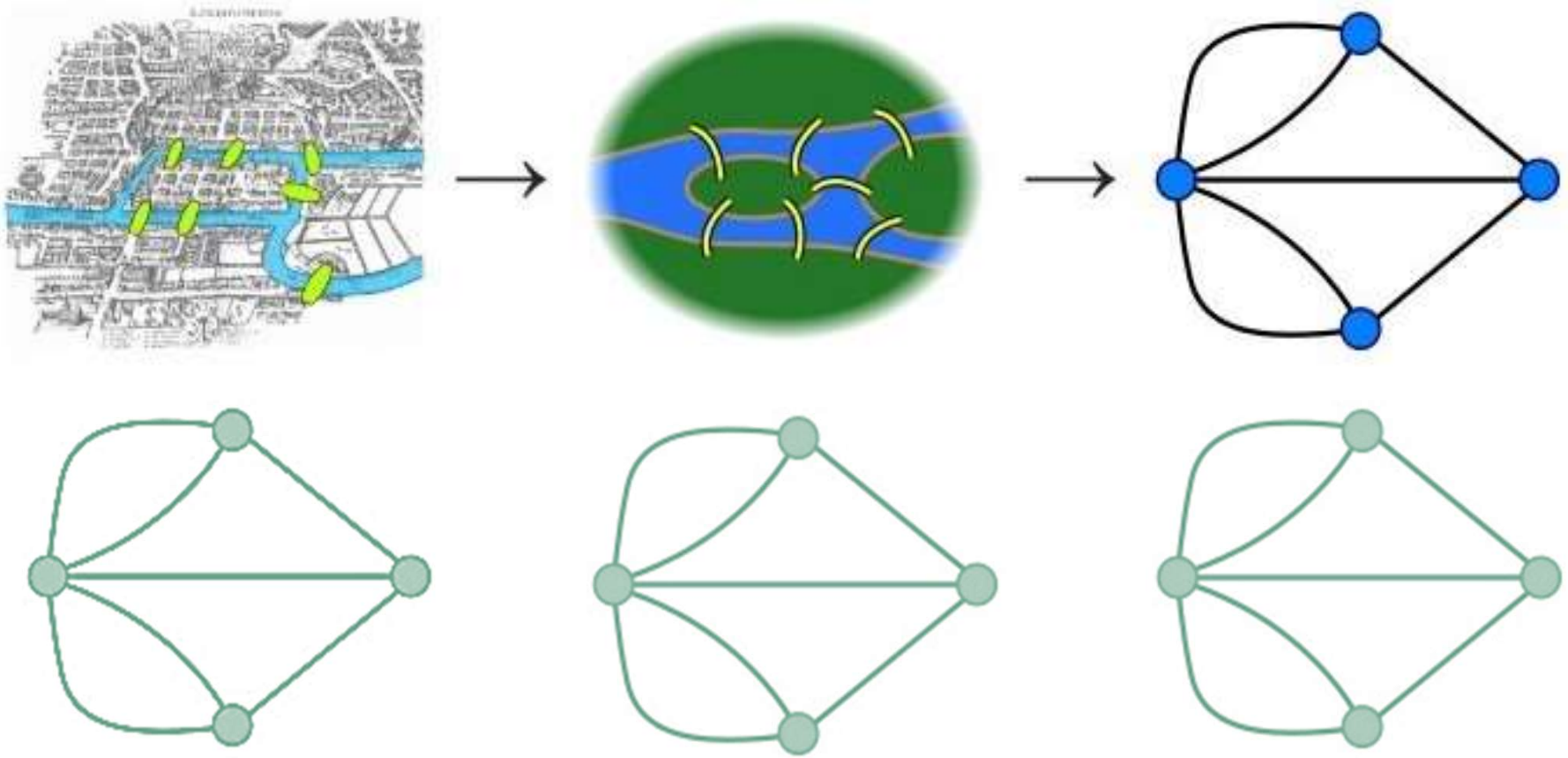






The Bridges of Konigsberg Puzzle

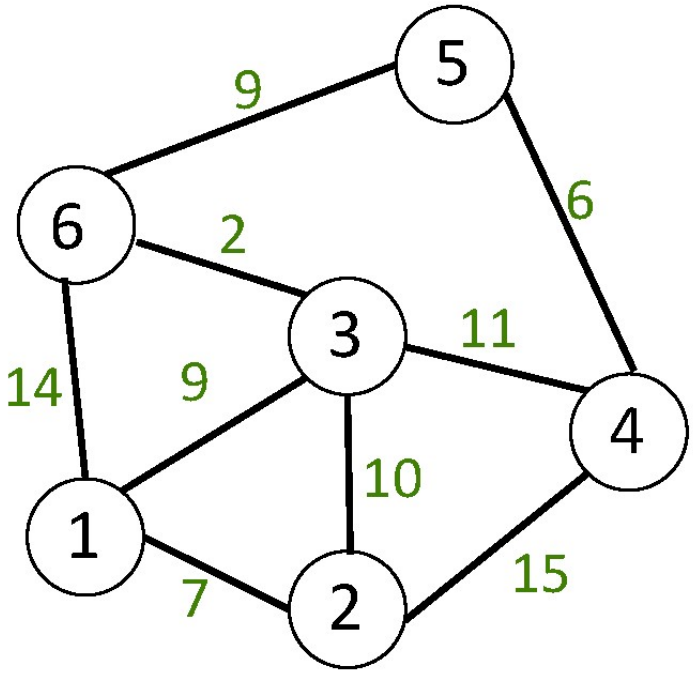
Can you walk in the city of Konigsberg in such a way that you cross each bridge once and only once (in either direction)?

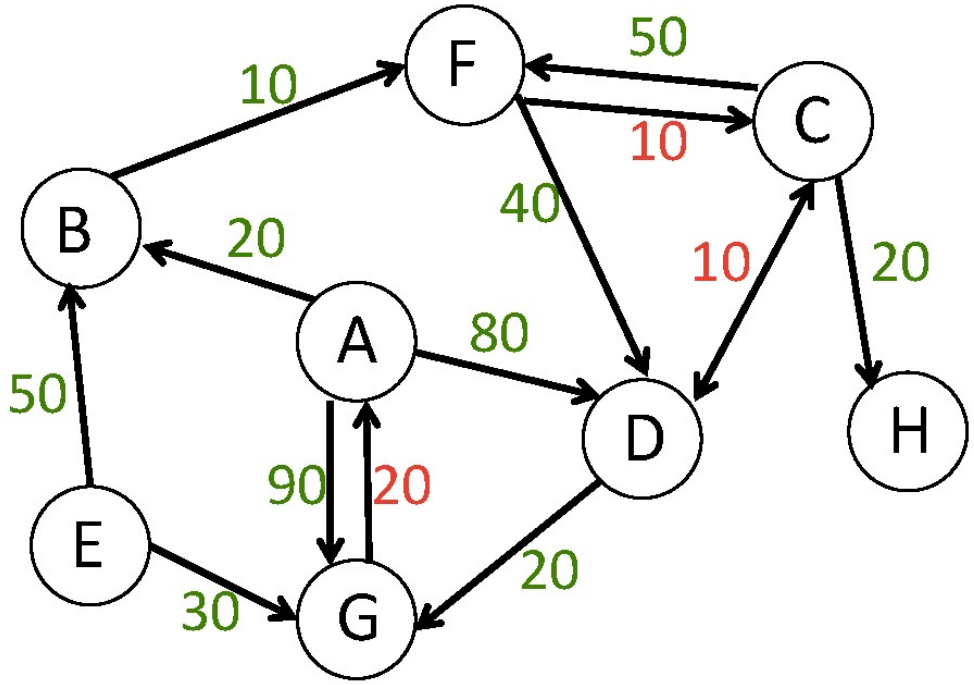


Shortest Path in a Graph

If the number on an edge represents travel distance, time, or cost, what is the shortest/fastest/cheapest way from point 2 to point 5?

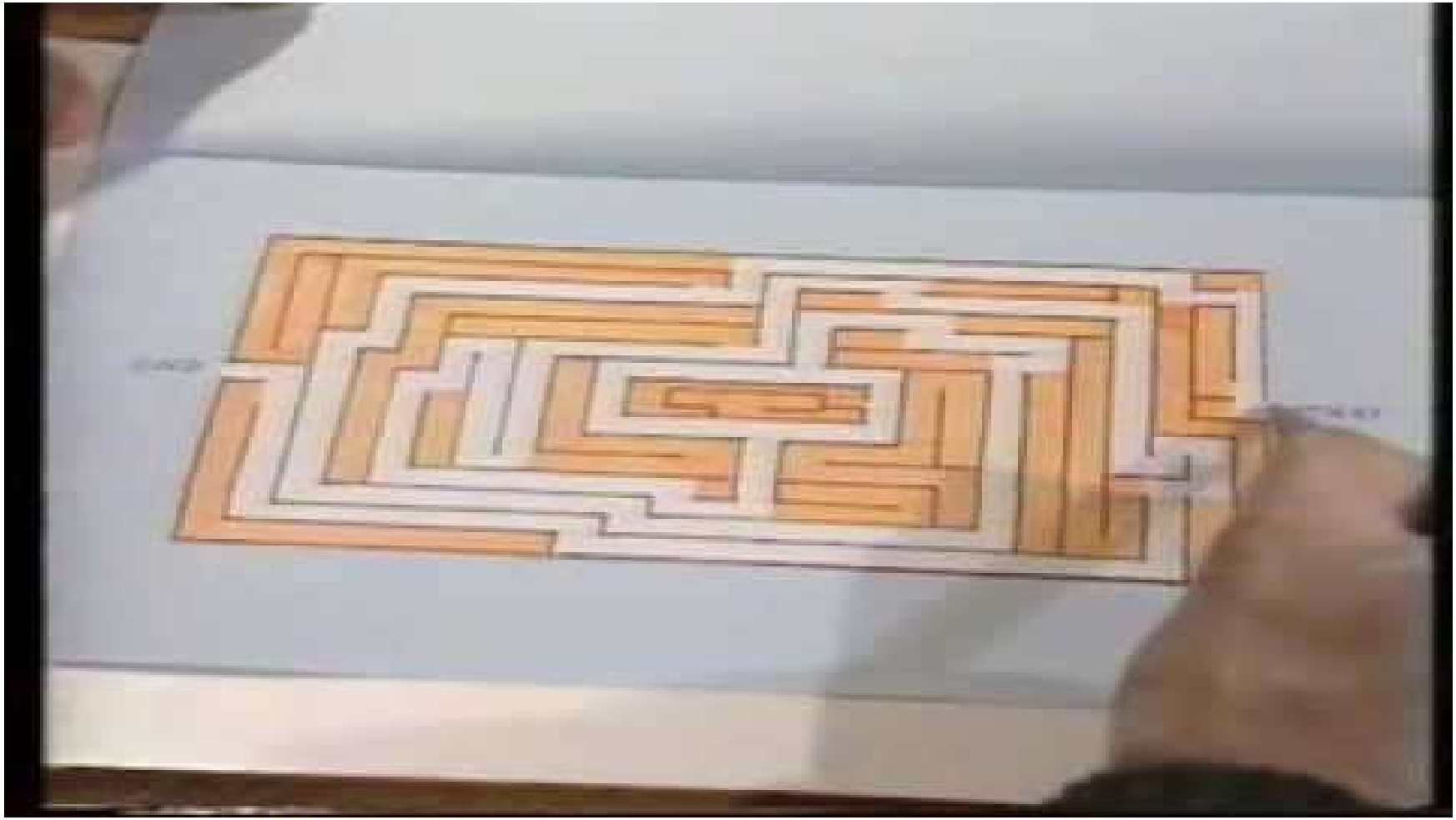
What about from point G to point C in the graph on the right?





Solving a Maze

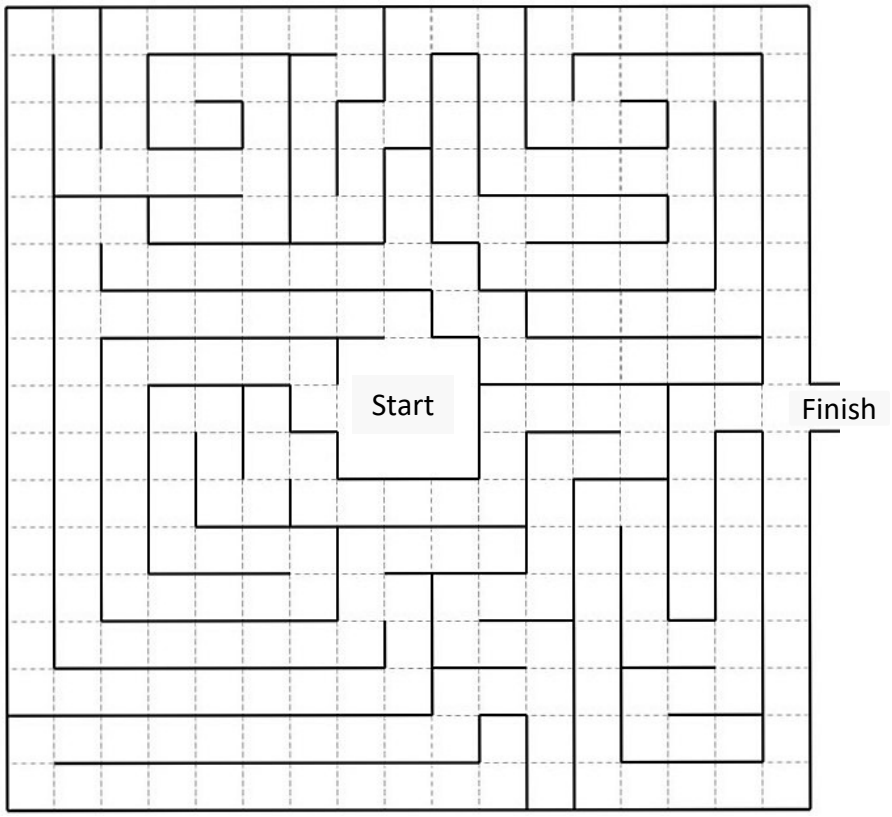
2-minute video: http://www.youtube.com/watch?v=mM10gQJJ7aM

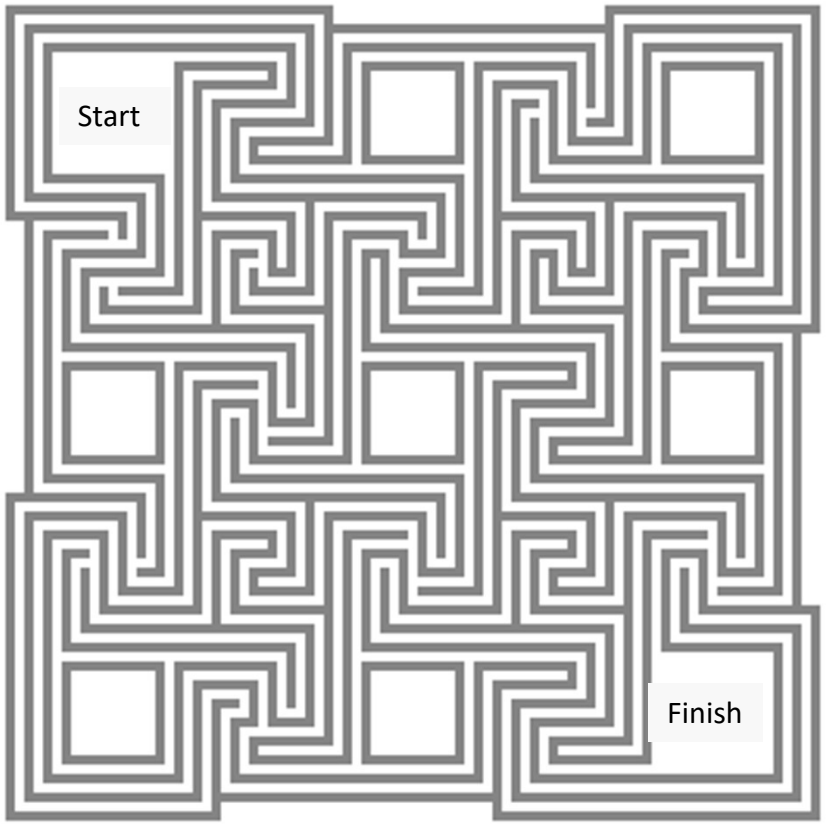


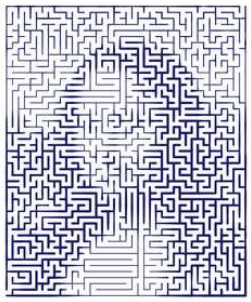
Maze-Solving Robots

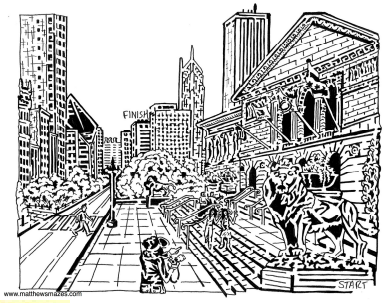
4-minute video: http://www.youtube.com/watch?v=MLHeUEPLSAY

Solve These Two Large Mazes



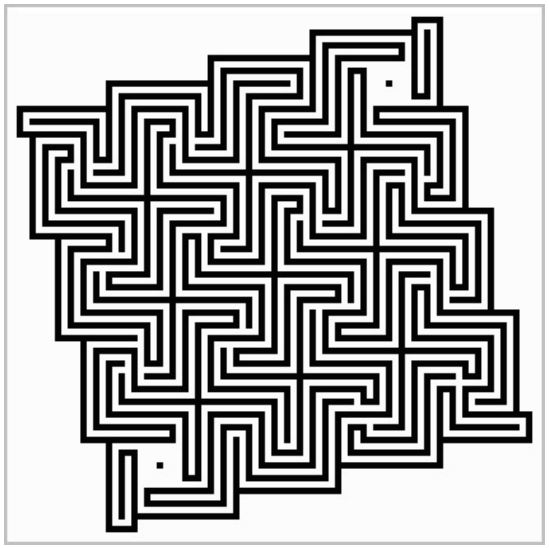


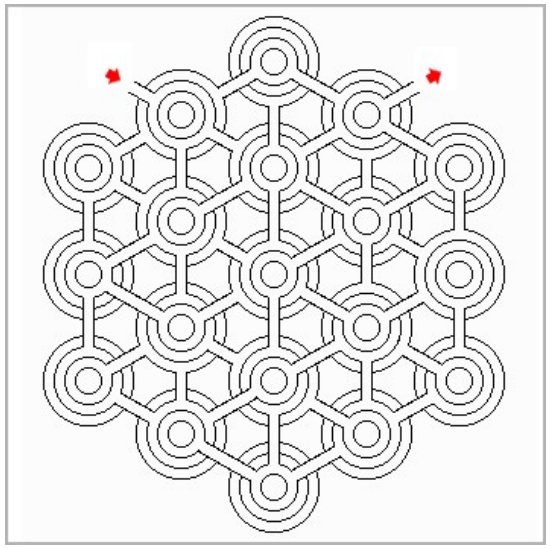


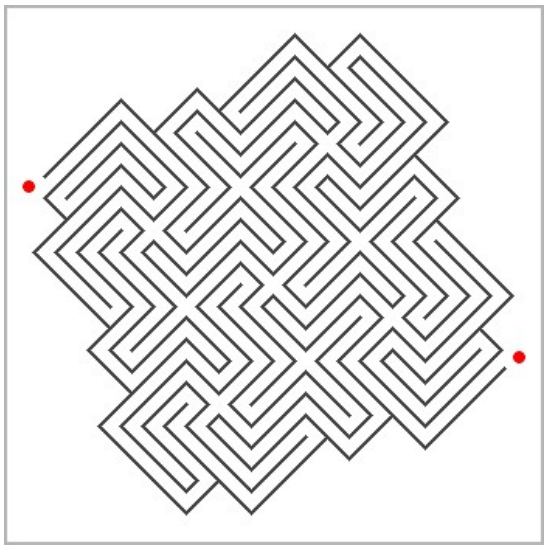


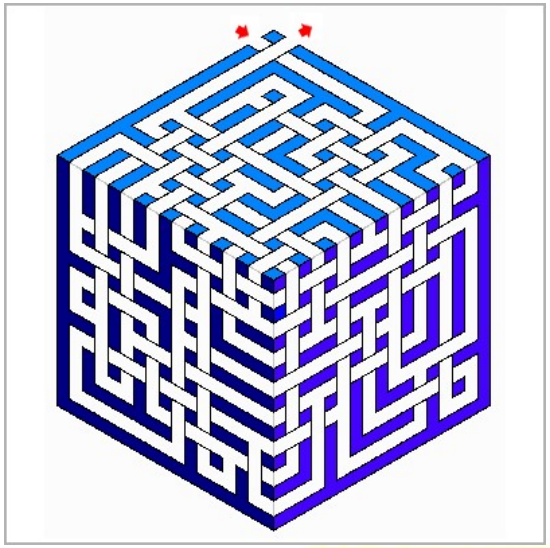
Examples of maze art

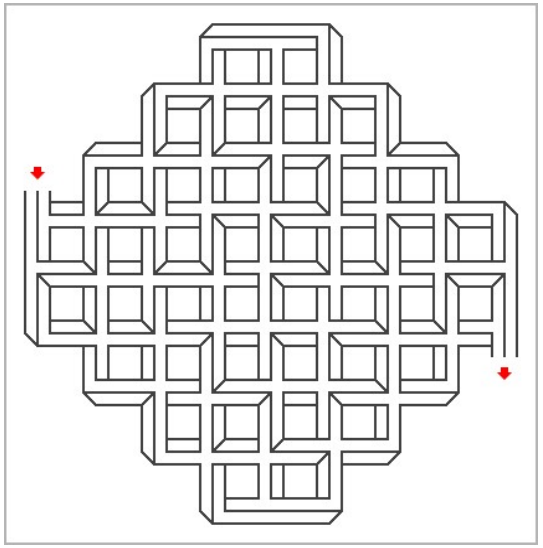
More Mazes to Solve

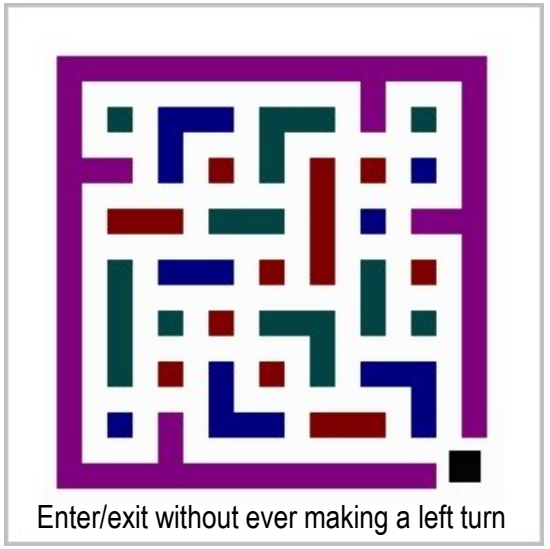












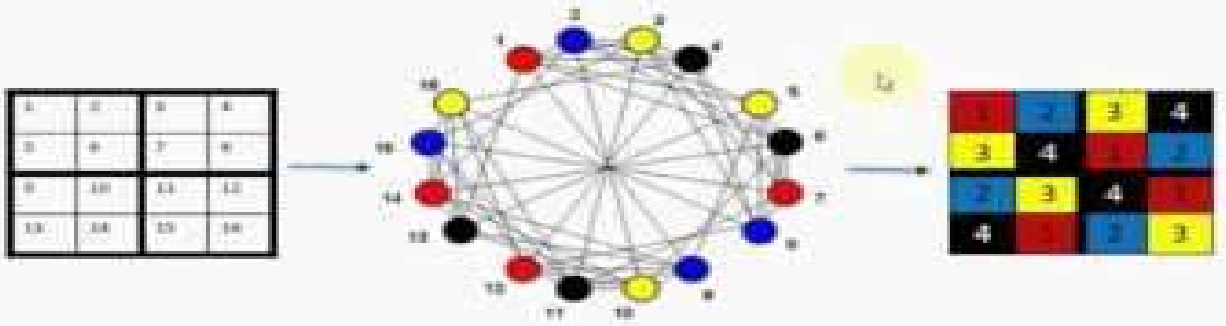
Enter/exit without ever making a left turn

Applications of Graph Coloring

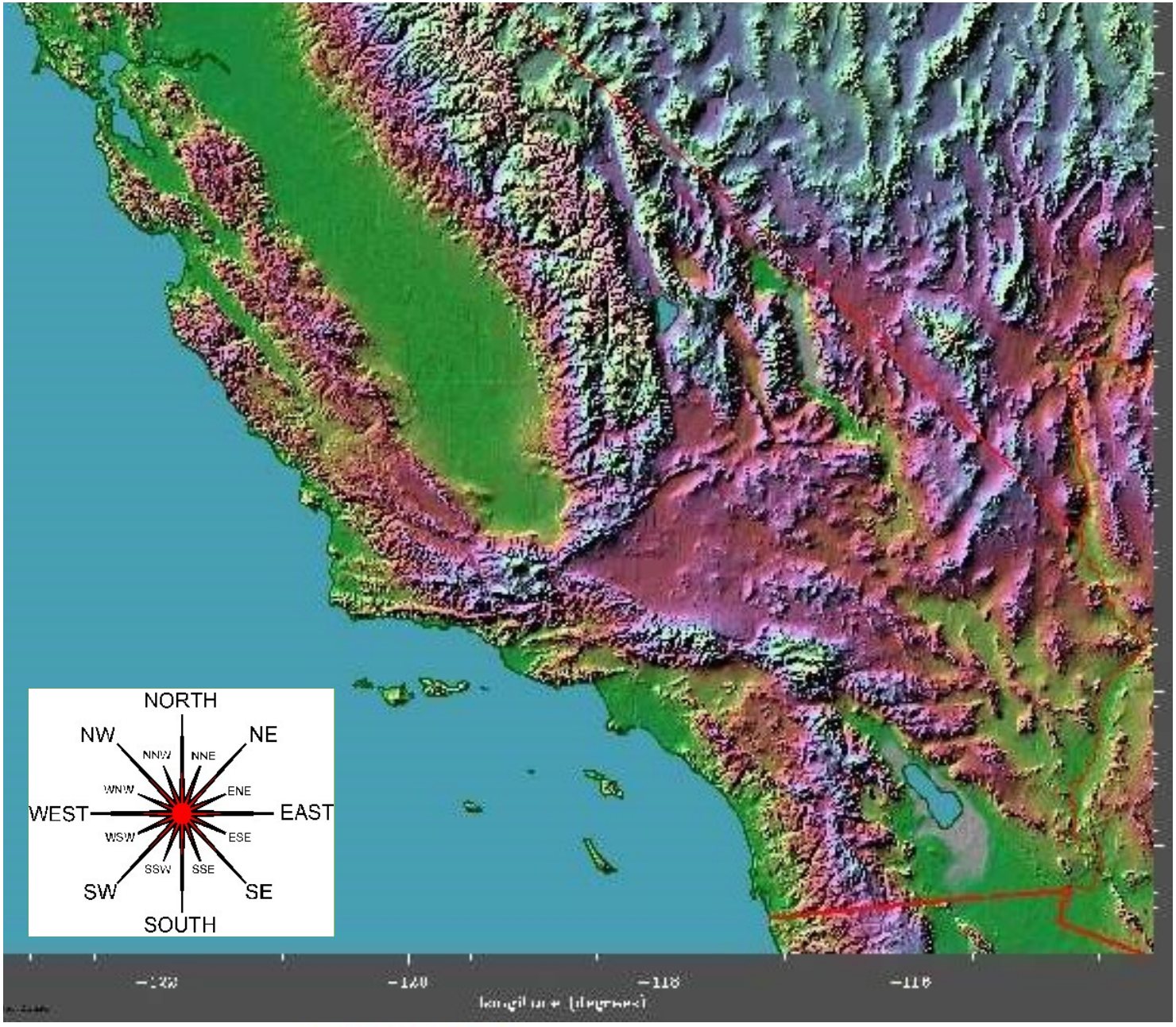
9-minute video: http://www.youtube.com/watch?v=y4RAYQjKb5Y

Solving Sudoku Puzzles

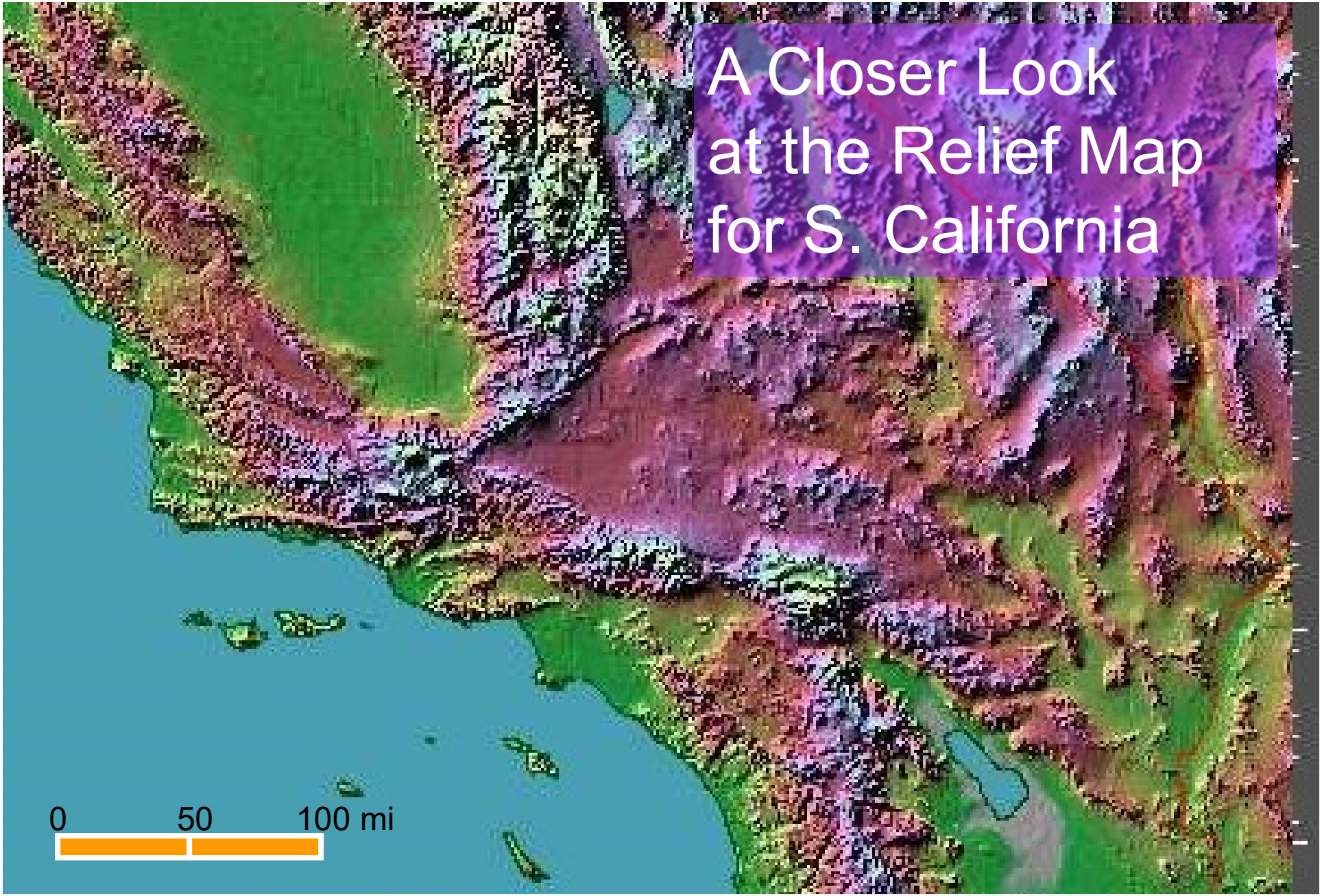
- Fill in the blank cells so that each row, column and 2x2 box has the characters 1 to 4 exactly once.

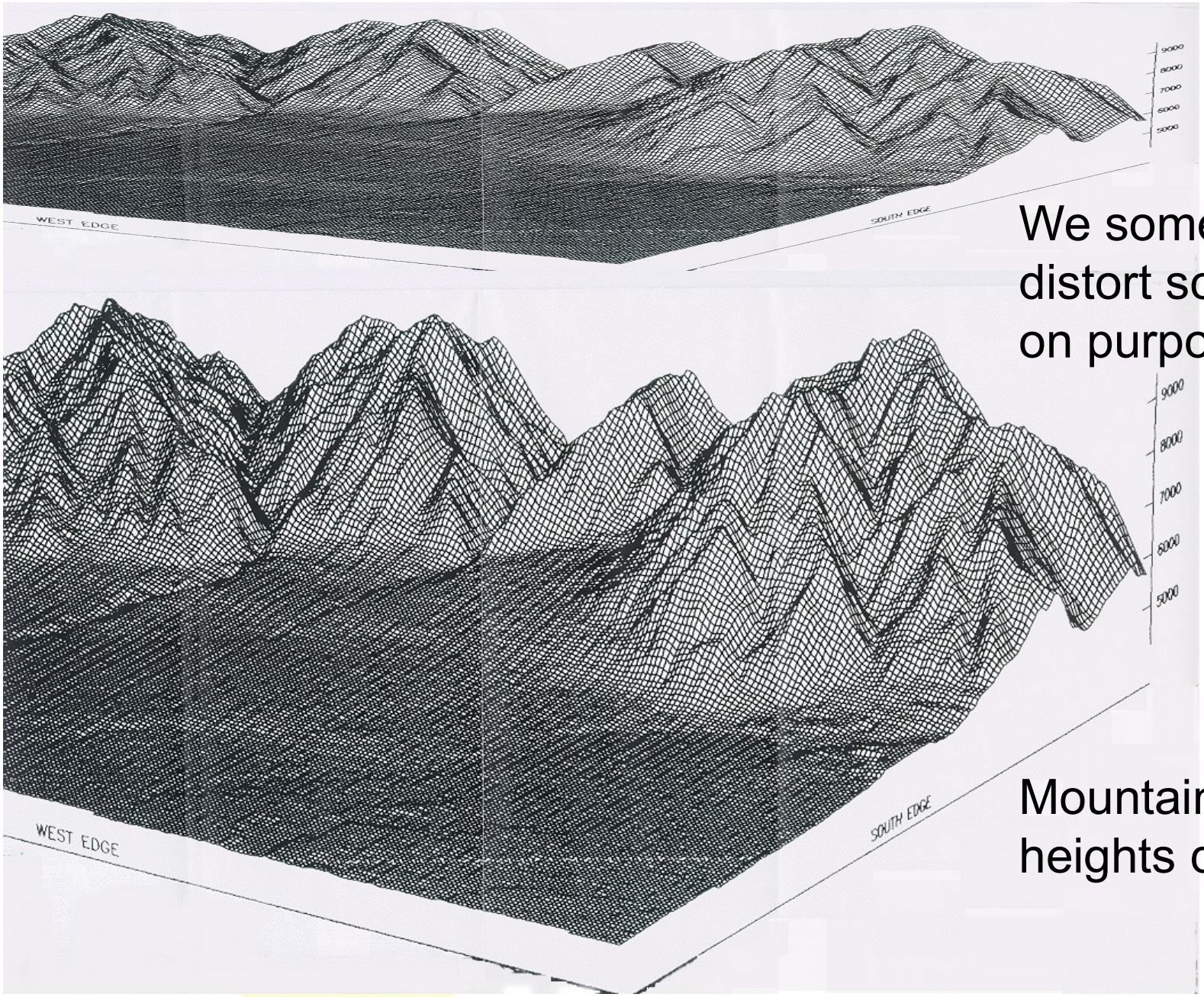


Relief Map of California



- Mountains
- Hills
- Hillsides
- Sea level
- Below sea level

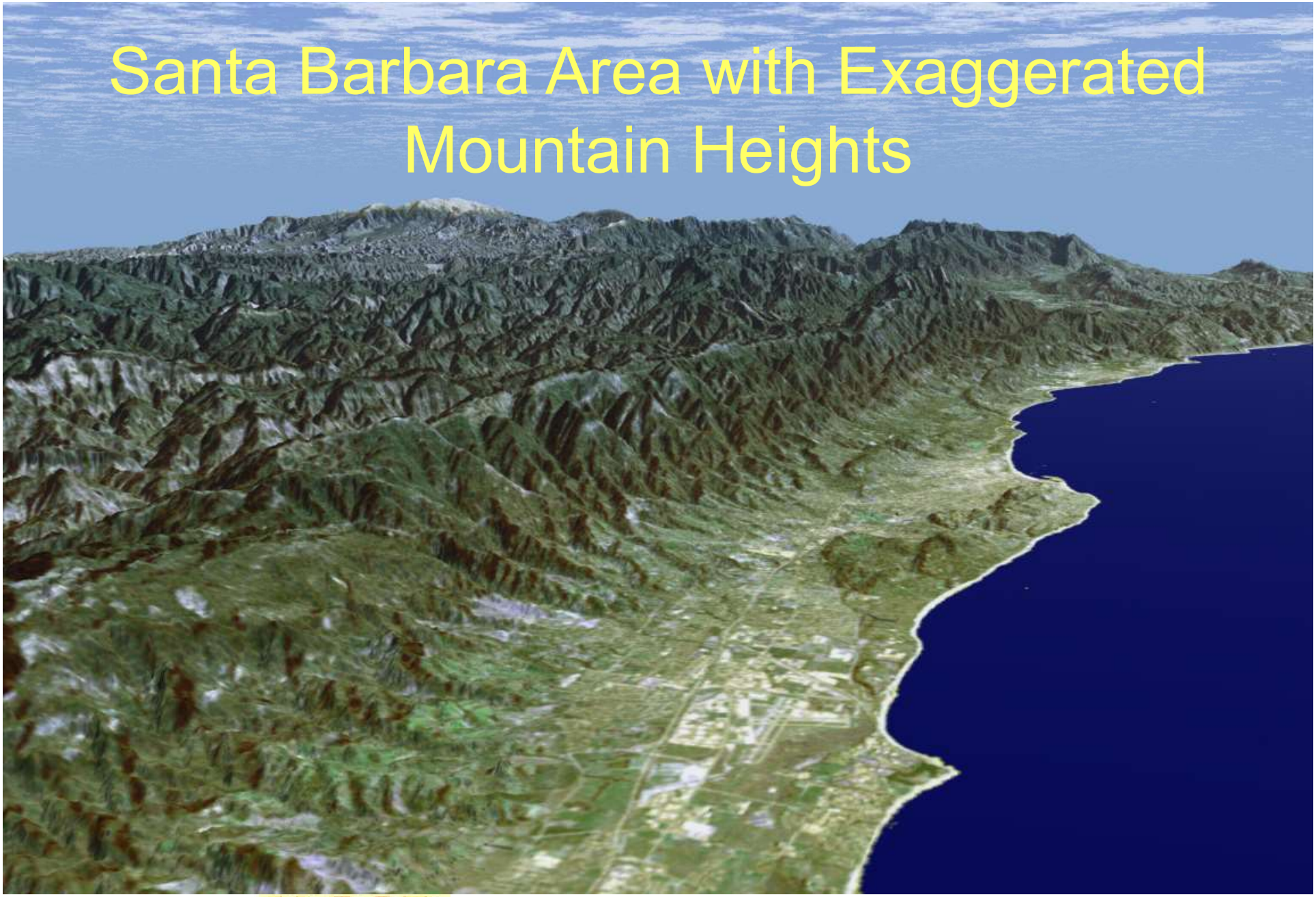

A Closer Look
at the Relief Map
for S. California
0
50
100 mi



We sometimes distort scale on purpose

Mountains, with heights doubled

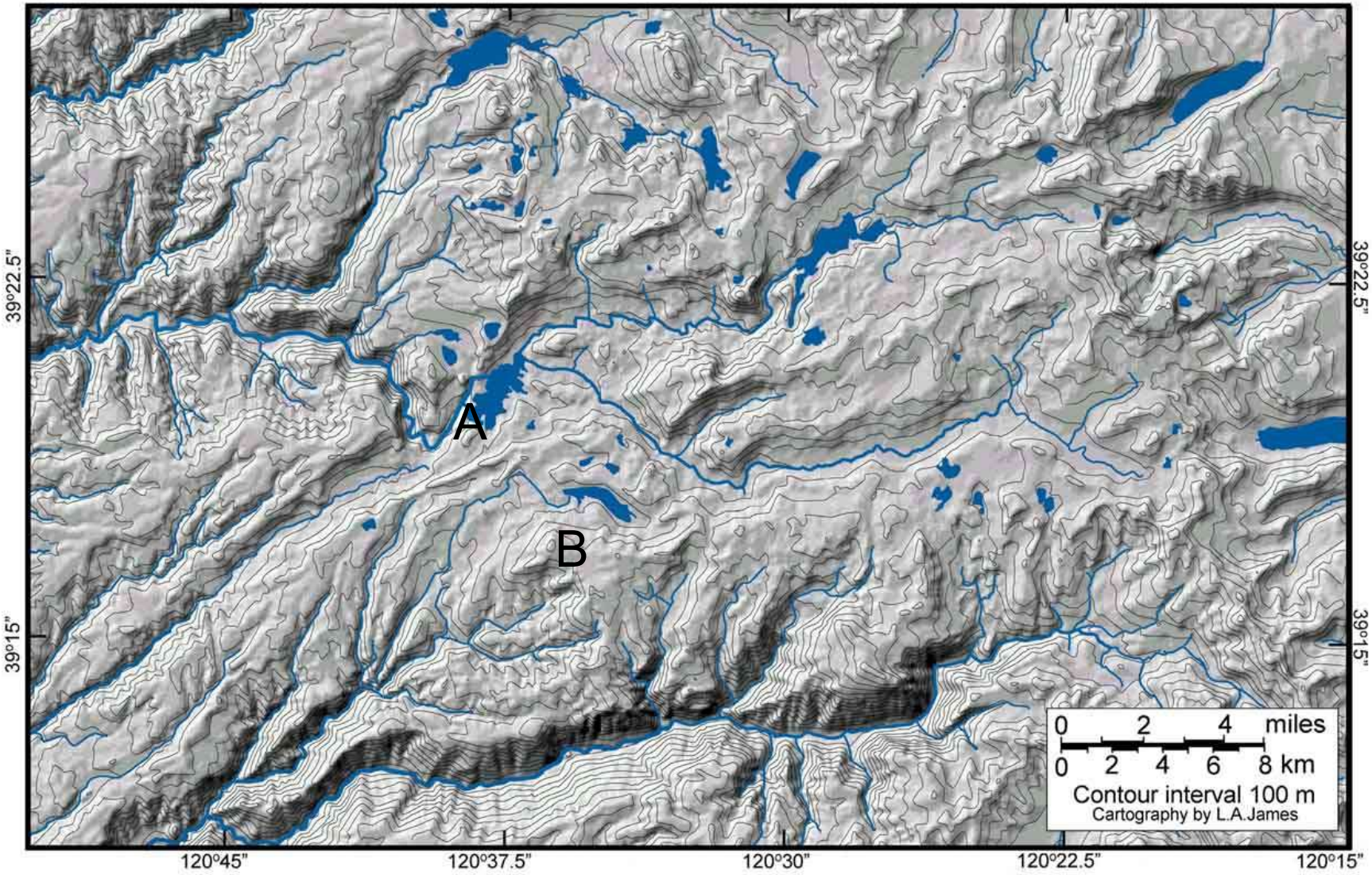
Santa Barbara Area with Exaggerated Mountain Heights



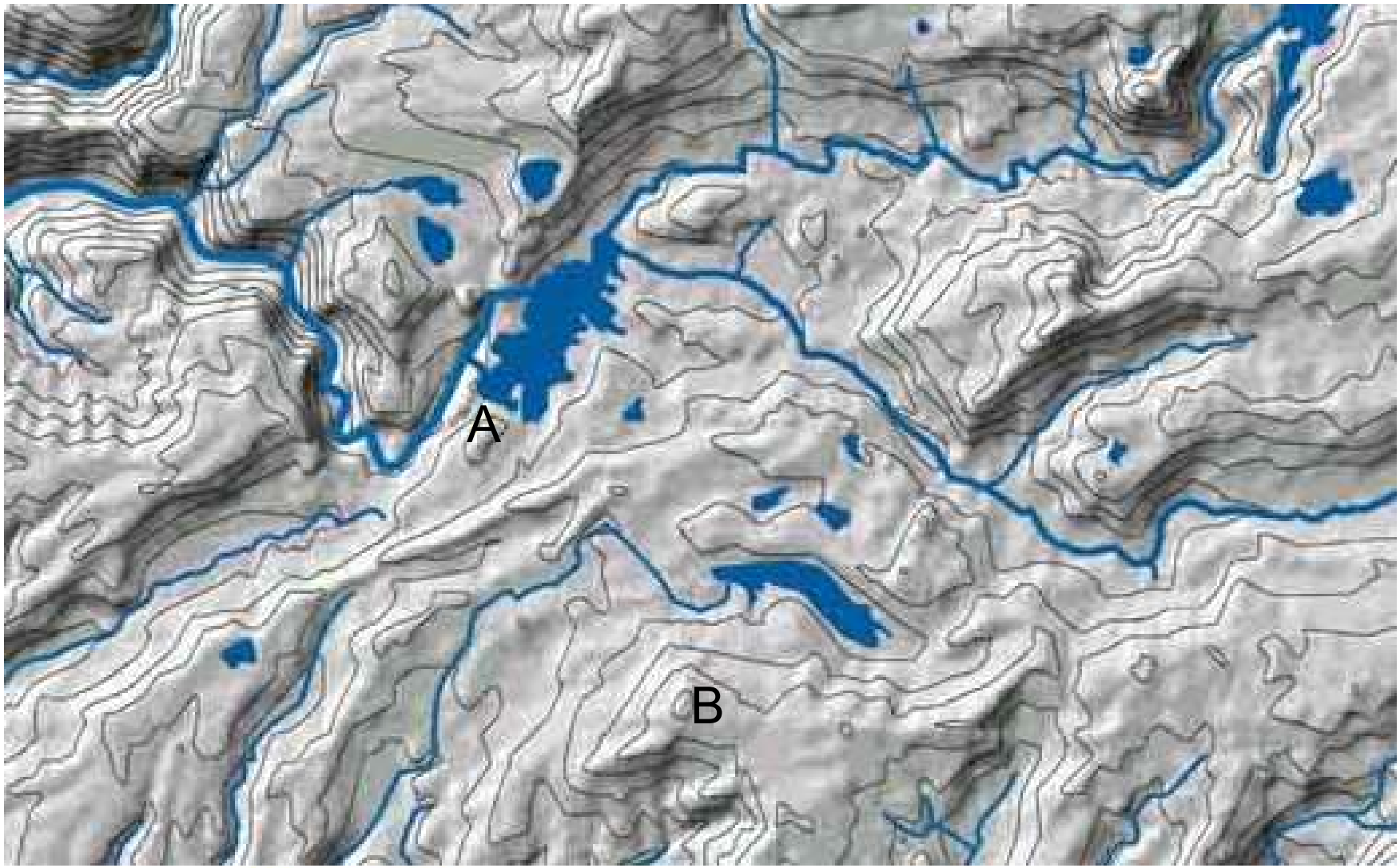
Topographic Maps

7-minute video: http://www.youtube.com/watch?v=SymUFSwEm3c

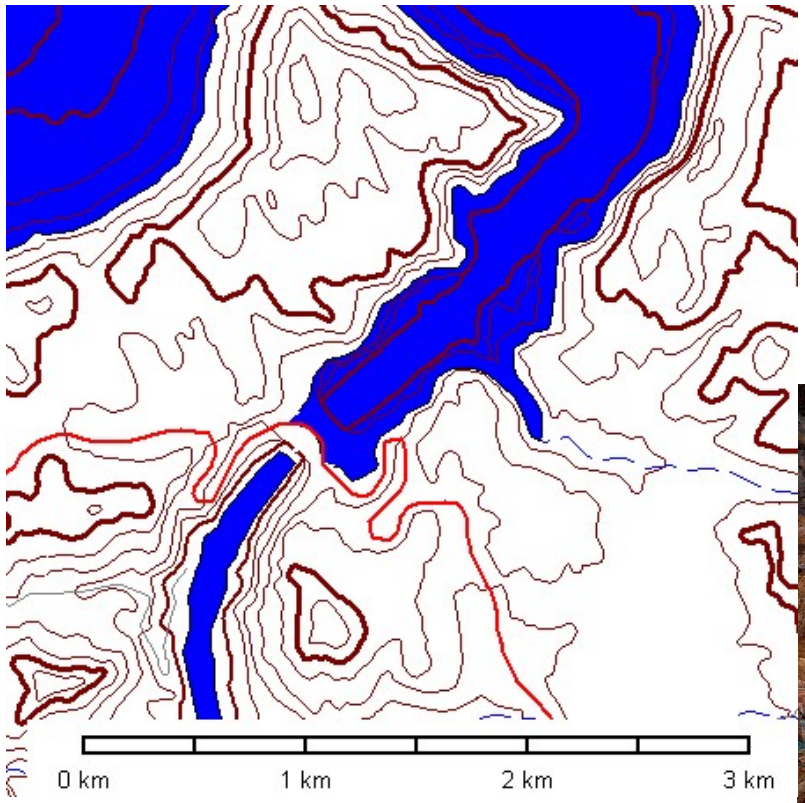
Contours Add a Third Dimension to Maps



Draw a Hiking Trail from A to B



Contour Maps and Road/Bridge Building

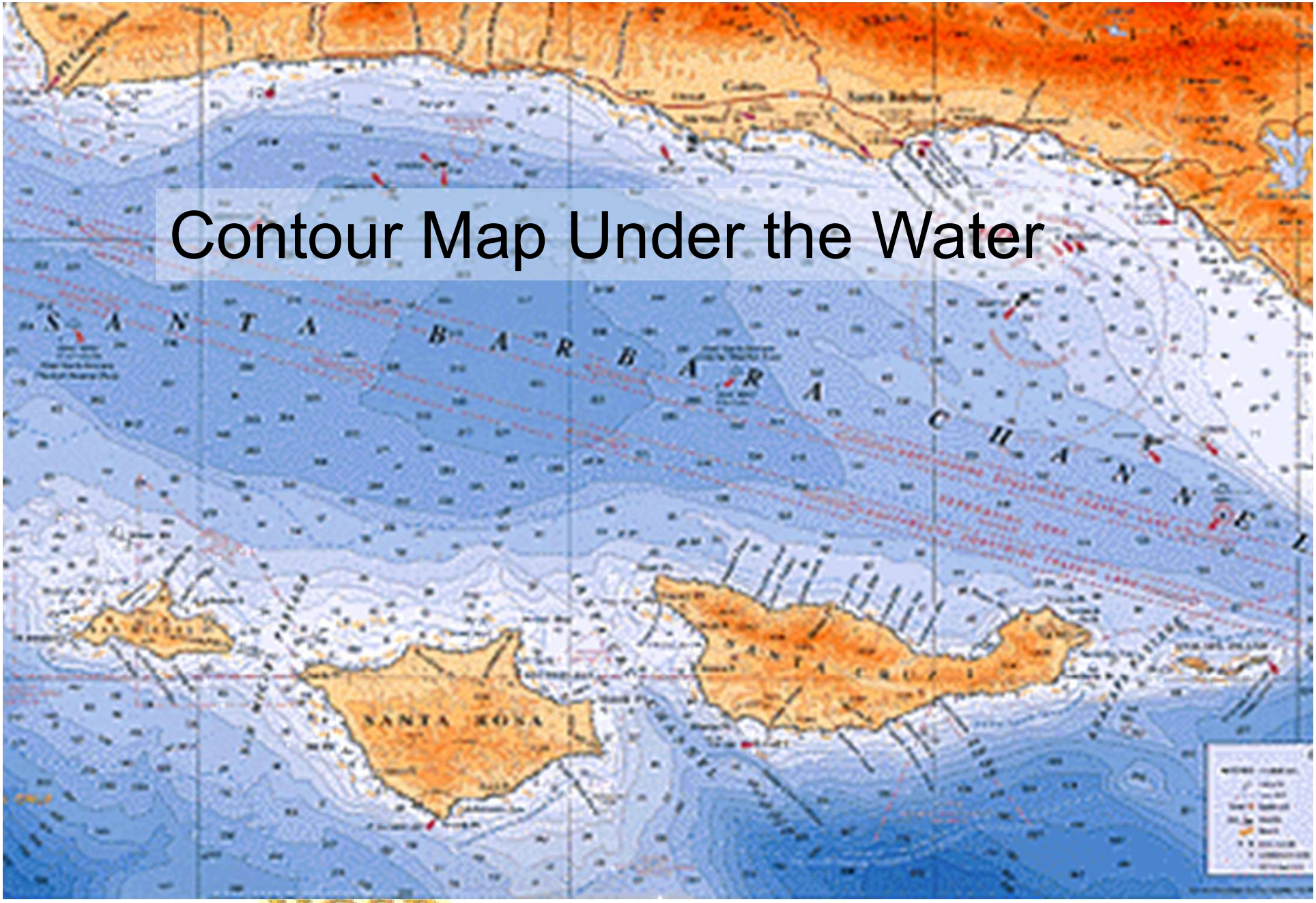


Contour map of Hoover Dam's surrounding area

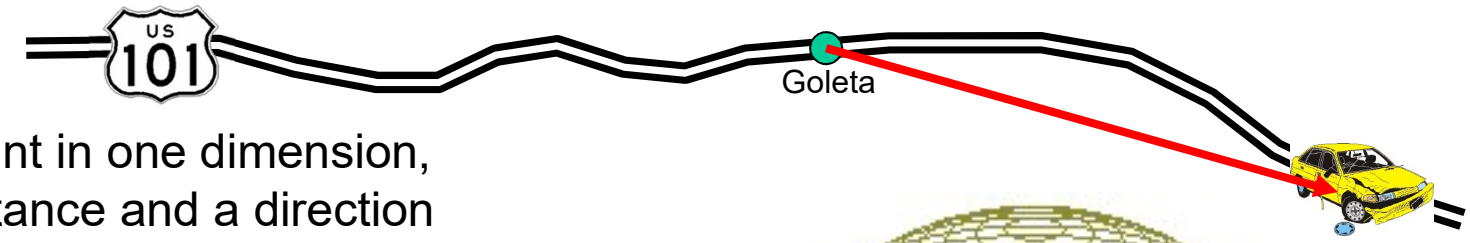
New bridge connecting the two sides of Hoover Dam (Nevada to Arizona)

Example of a Snaking Road

Contour Map Under the Water

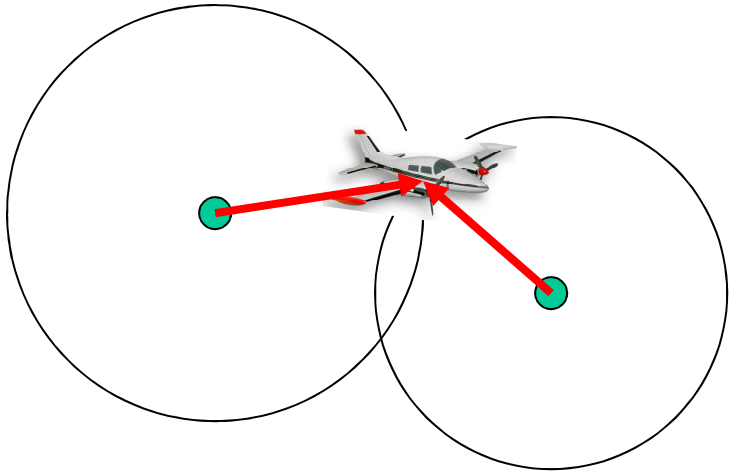


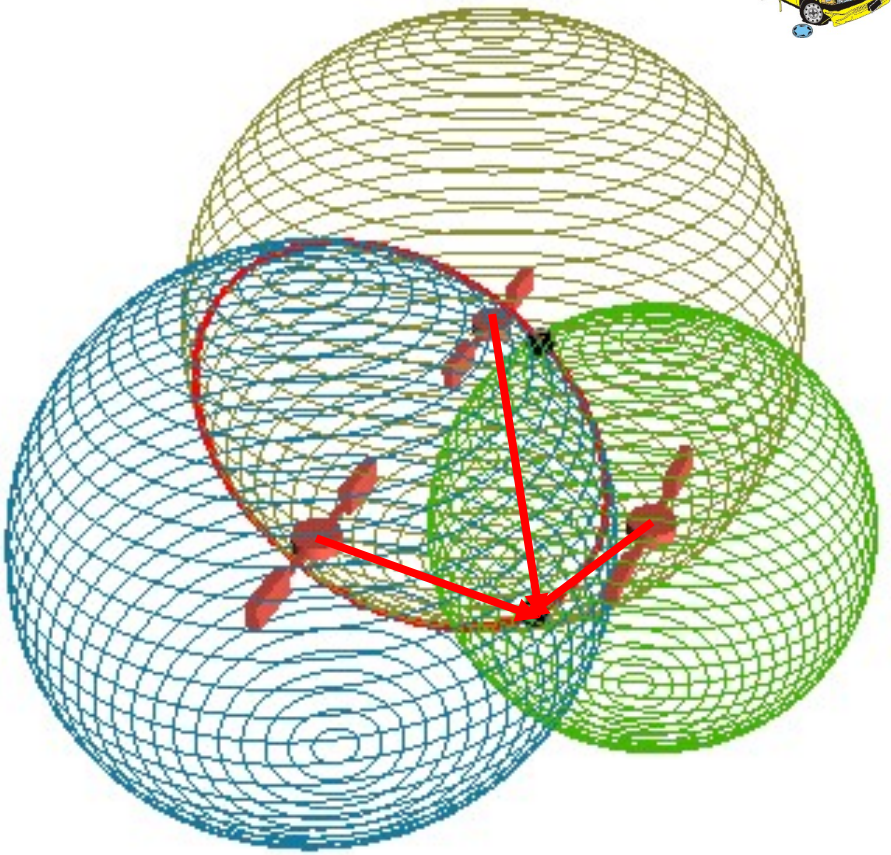
Where in the World Are You?



To locate a point in one dimension,
we need a distance and a direction

To locate a point in two dimensions,
we need two distances and a little bit
more information





To locate a point in three dimensions,
we need three distances and a little bit more information

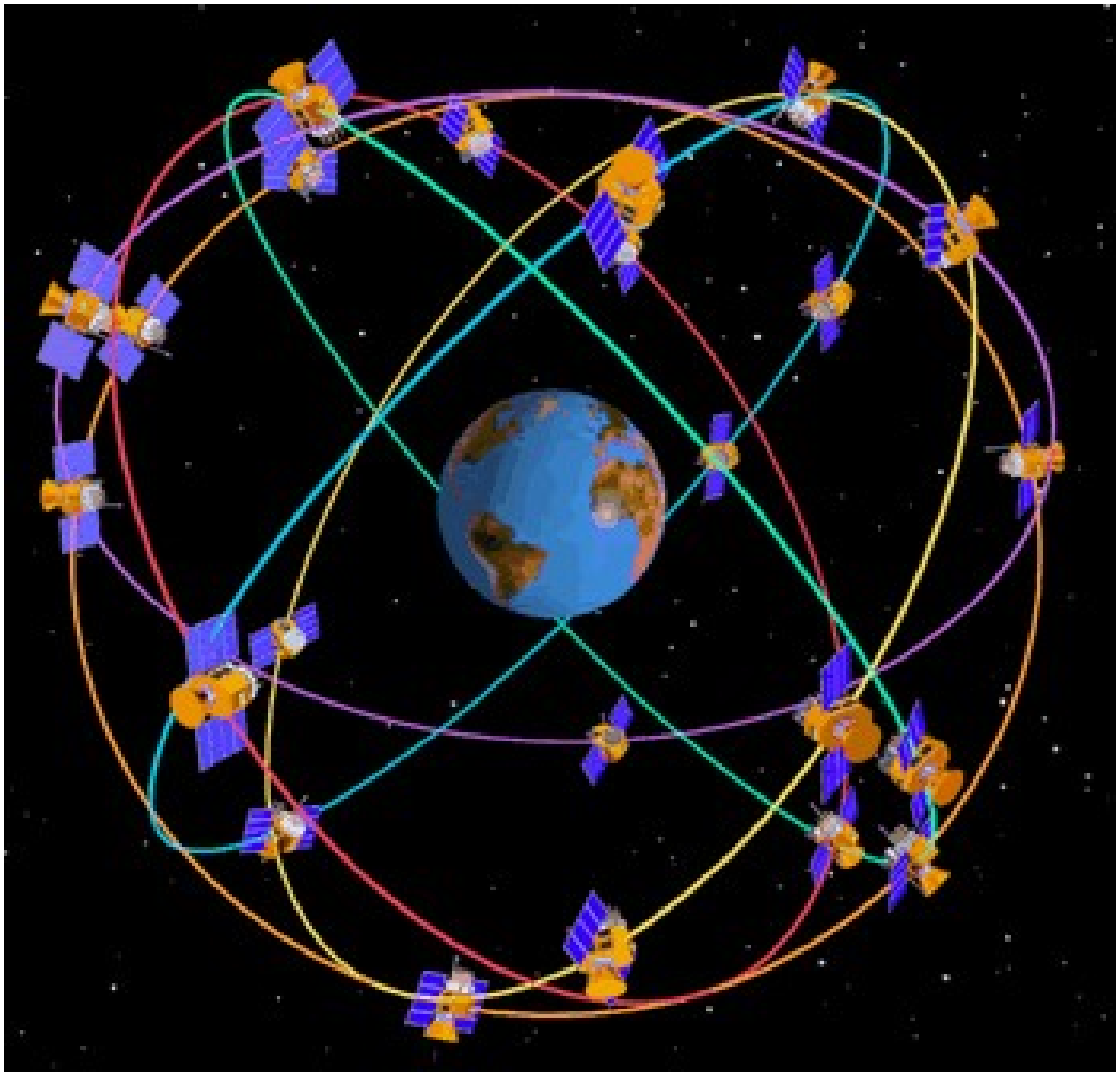
Global Positioning System (GPS)

How GPS Works: The Satellites

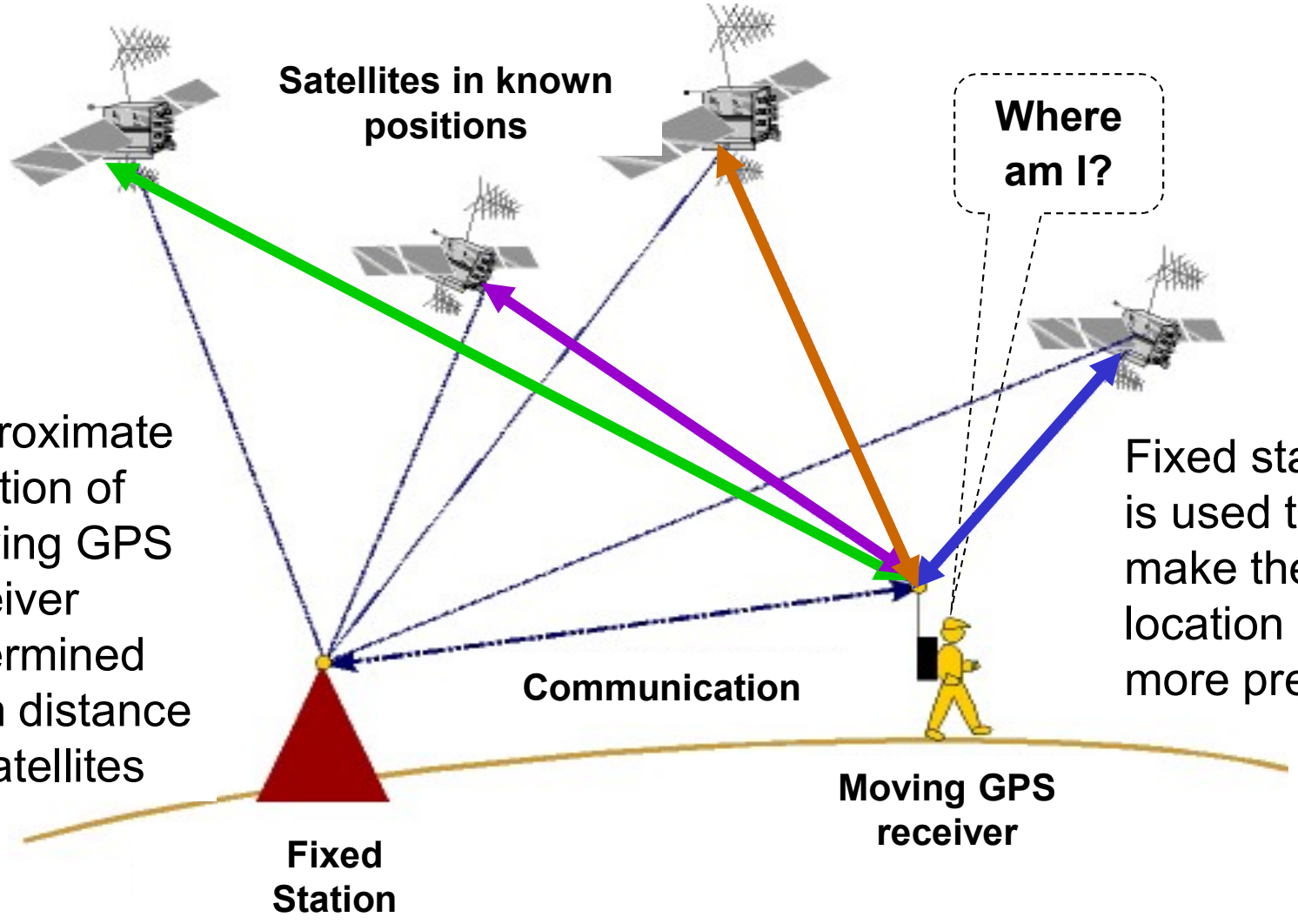
There are at least 24 satellites at any time; new ones are launched to replace older ones that need to be retired

The satellites are positioned so that from any point on earth, radio communication with at least four of them is possible

Each satellite follows a single orbit, passing over the same places on earth each time



How GPS Works: The Measurements



The GPS Receiver on Your Smartphone

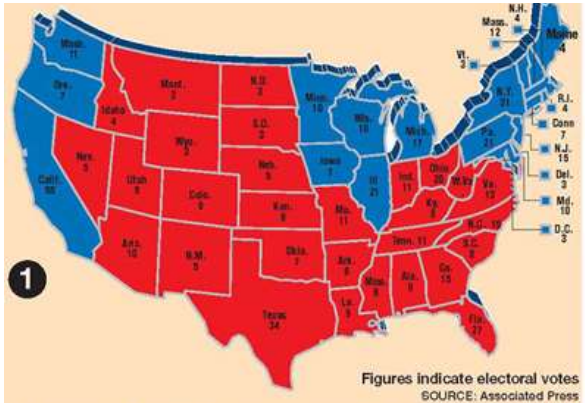
5-minute video: http://www.youtube.com/watch?v=70cDSUI4XKE

GPS-Guided Navigation

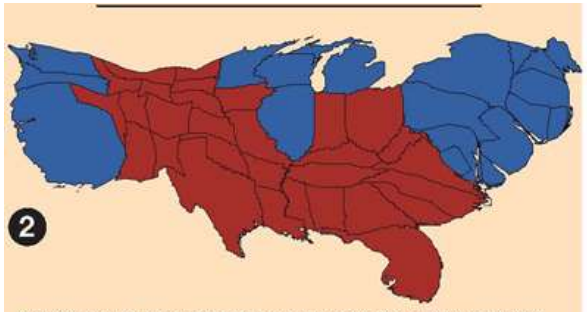
You know where you are (GPS) and where you want to go (address)
Roads form a known maze, stored on Google's servers
When there are multiple paths, you want the shortest/fastest path
To determine which path is faster, dynamic congestion data may be used

Cartograms Combine Geographic and Other Data

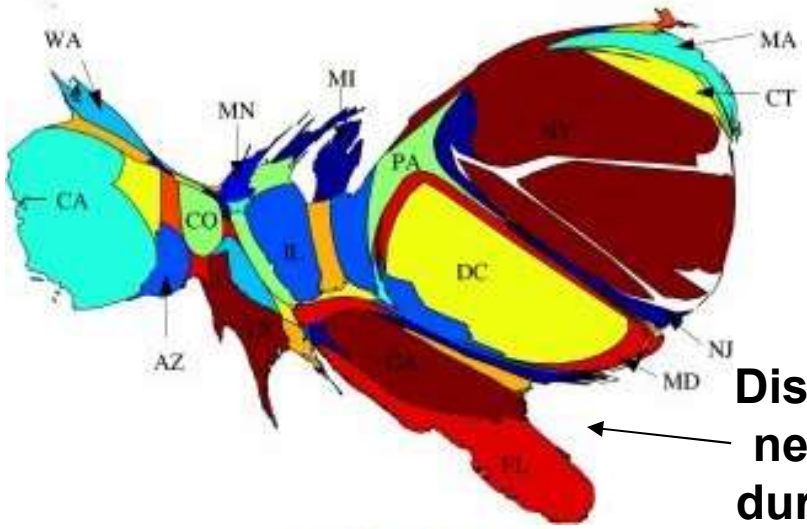
Outcome of the 2004 US presidential election (S. Fabrikant, UCSB Geog Dept)



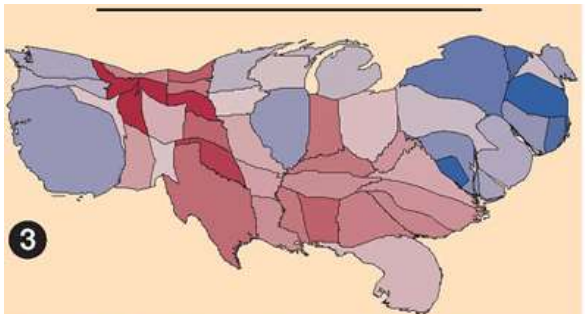
Standard US map, showing states won by Bush (red) and Kerry (blue)



Cartogram, with states distorted to have areas proportional to electoral votes



Distribution of news stories during 1994-98



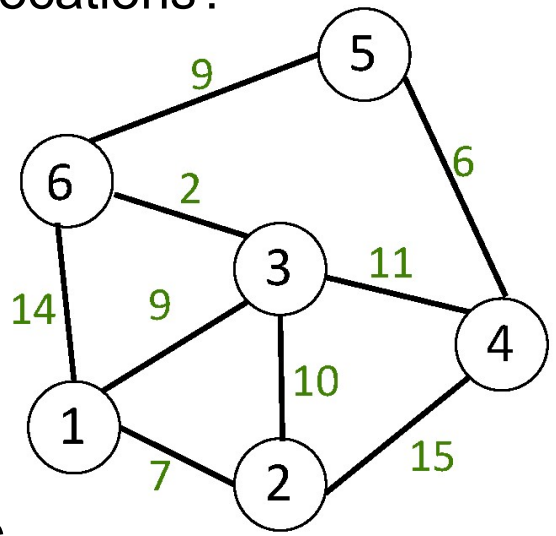
Cartogram, with relative support level for candidates indicated by shading

The Traveling Salesperson Problem

If the number on an edge represents travel distance, time, or cost, what is the shortest/fastest/cheapest tour of the six locations?

A salesperson may want to visit all the locations

A truck may have deliveries at all the locations



Other Optimization Problems

Assuming numbers shown are travel times for a fire truck in minutes, where is the best location for a single fire station?

Where should fire stations be placed for a response time of 8 minutes?