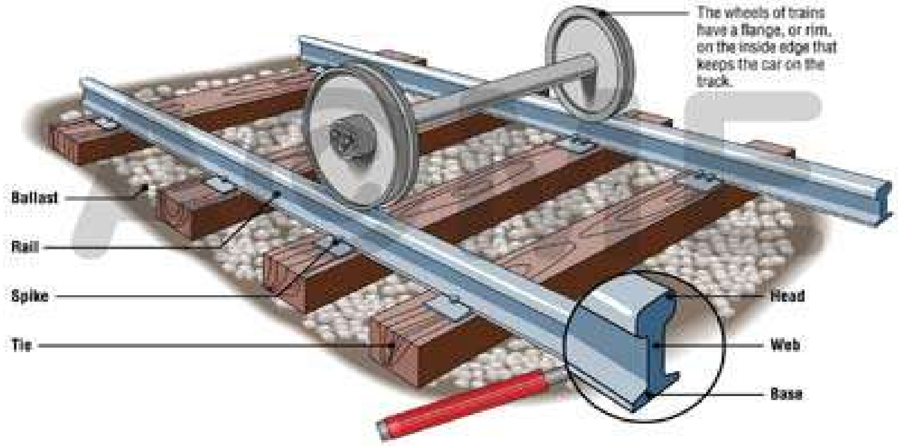
Sorting Networks

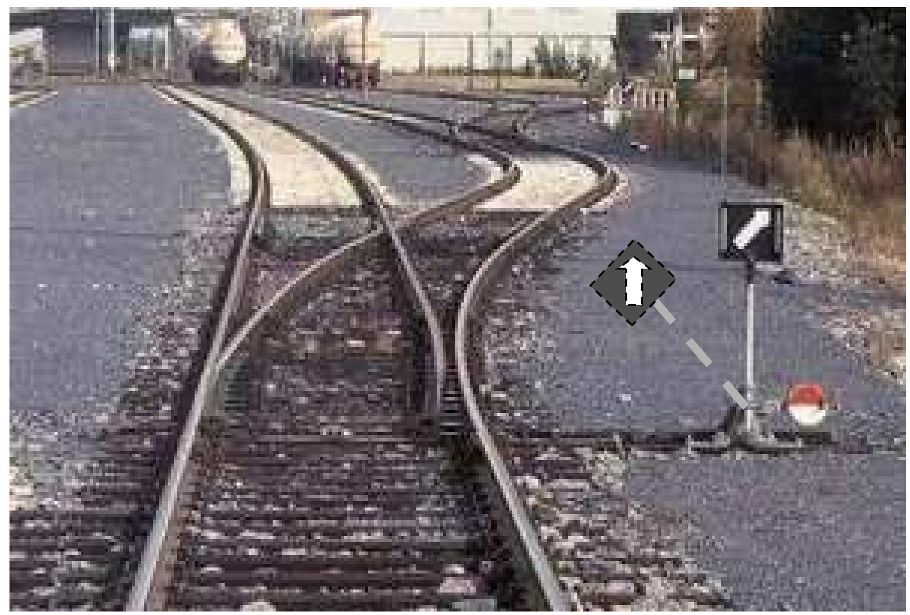
A Lecture in CE Freshman Seminar Series:
Ten Puzzling Problems in Computer Engineering

About This Presentation

Edition	Released	Revised	Revised	Revised	Revised
First	May 2007	May 2008	May 2009	May 2010	May 2011
		May 2012	May 2015	May 2016	May 2020

Railroad Tracks and Switches





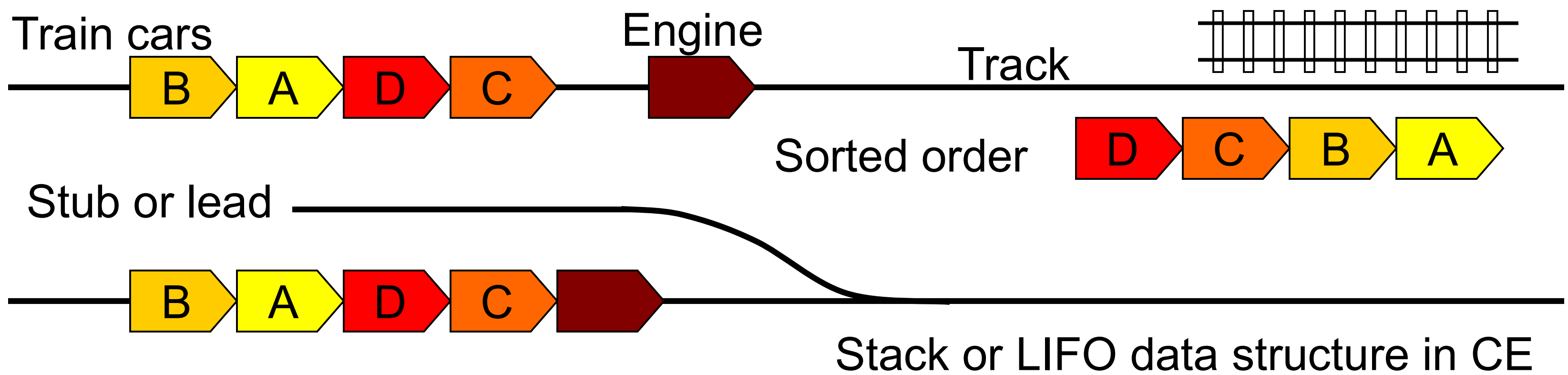
Coupling and Decoupling of Train Cars

Train cars and engines can be coupled and decoupled quickly

An engine can push a string of cars, or pull a desired subset by decoupling them from the rest

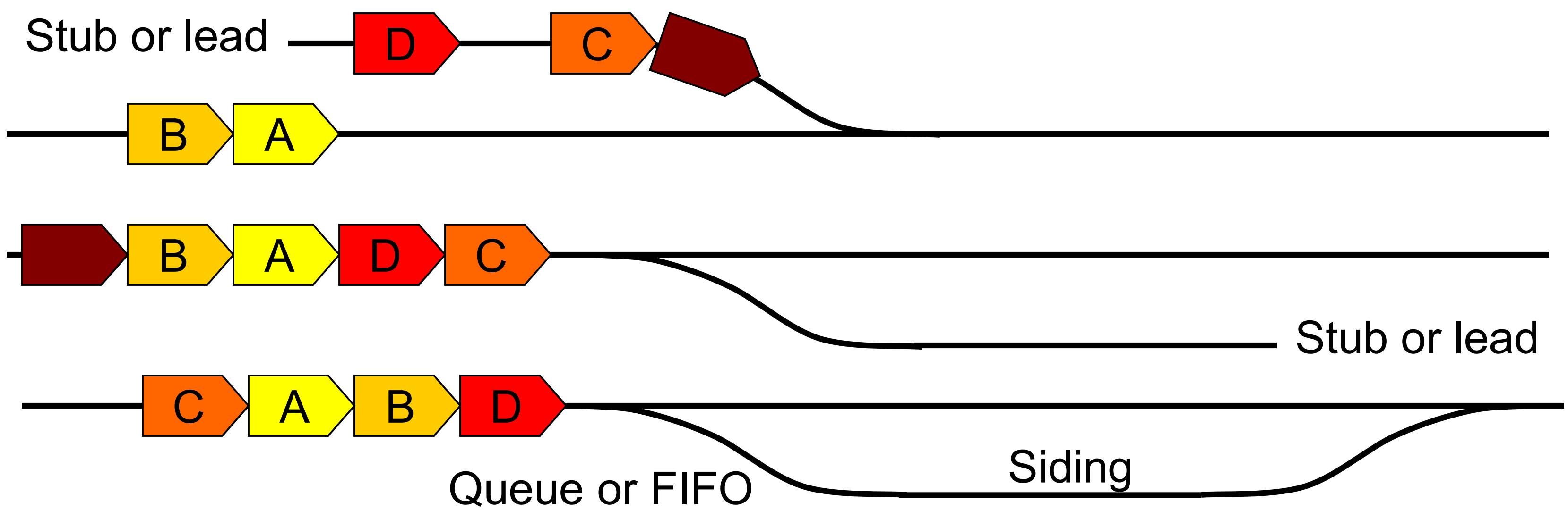
Railroad Yards Have Many Tracks and Switches

Rearranging Trains

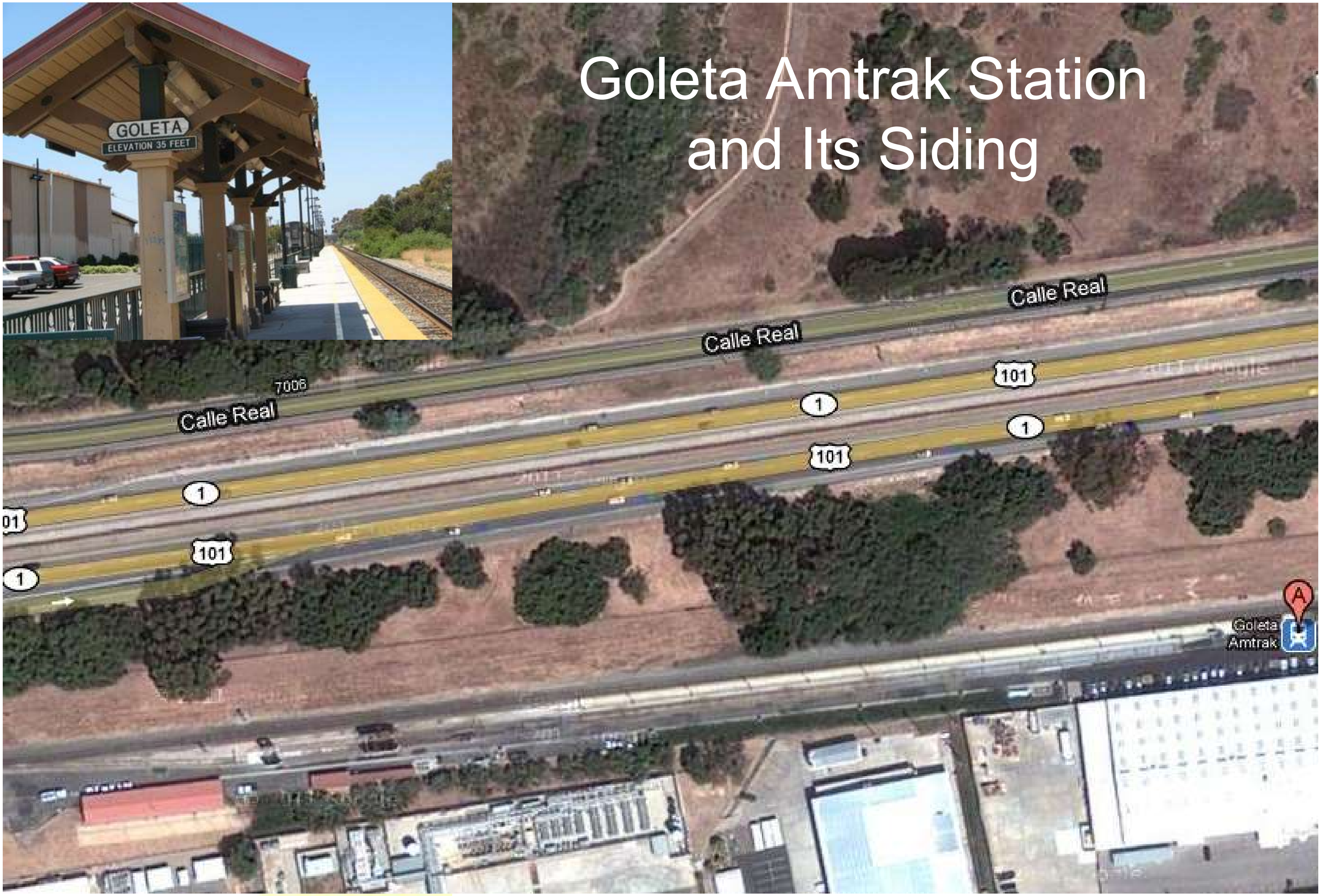


Sorting algorithm: Assemble train in stub, beginning with the last car

Repeat: If the next car is X, decouple train after X, push X into stub

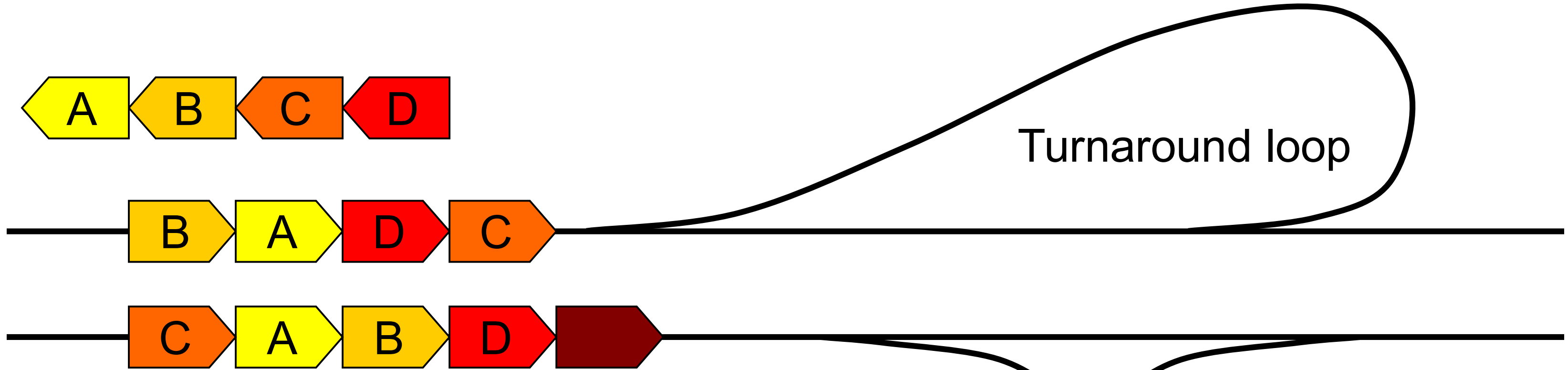


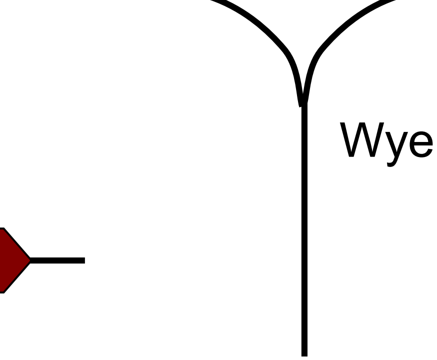
Goleta Amtrak Station and Its Siding



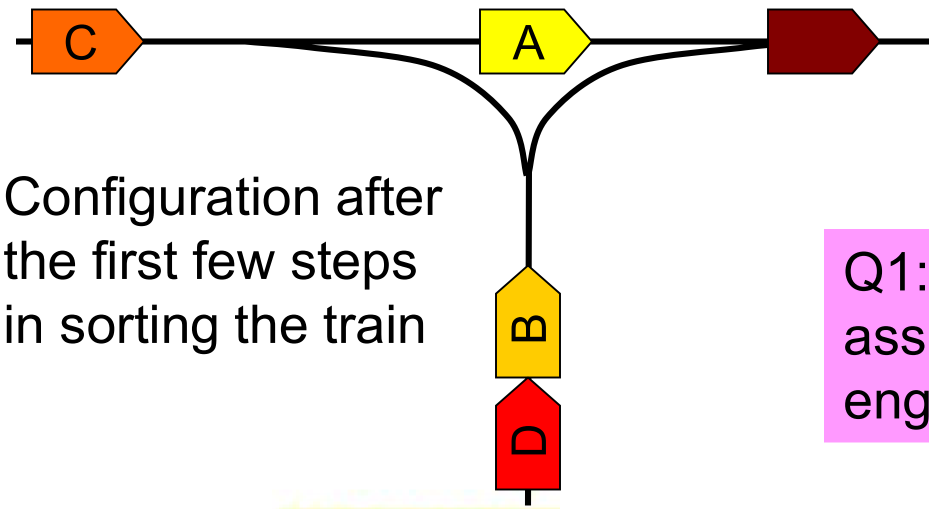
Model Railroad Yard

Rearrangement with Change of Direction





What types of sorting are possible with a turnaround loop? A wye?



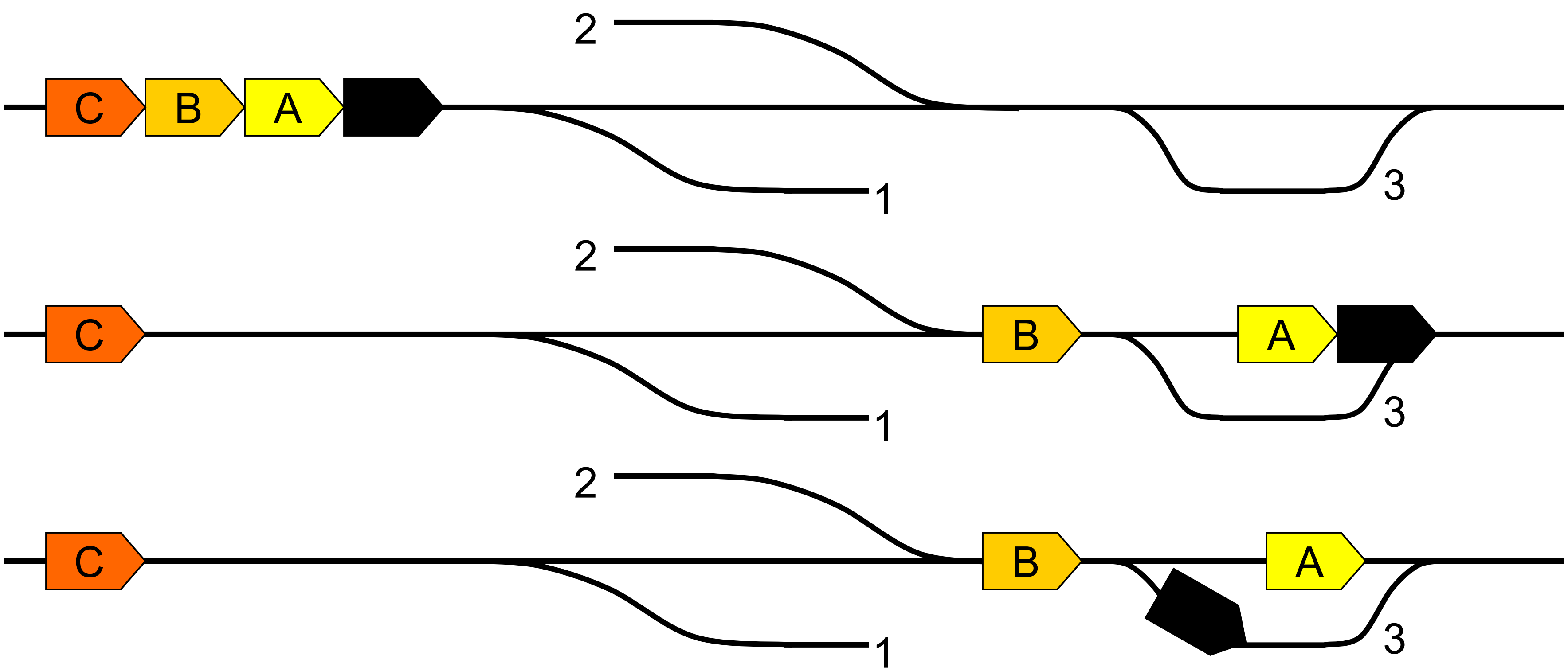
Configuration after the first few steps in sorting the train

Q1: Complete the sorting process, assuming that it is okay for the engine to end up behind the train


Model Train Turntable

Delivering Train Cars in a Specific Order

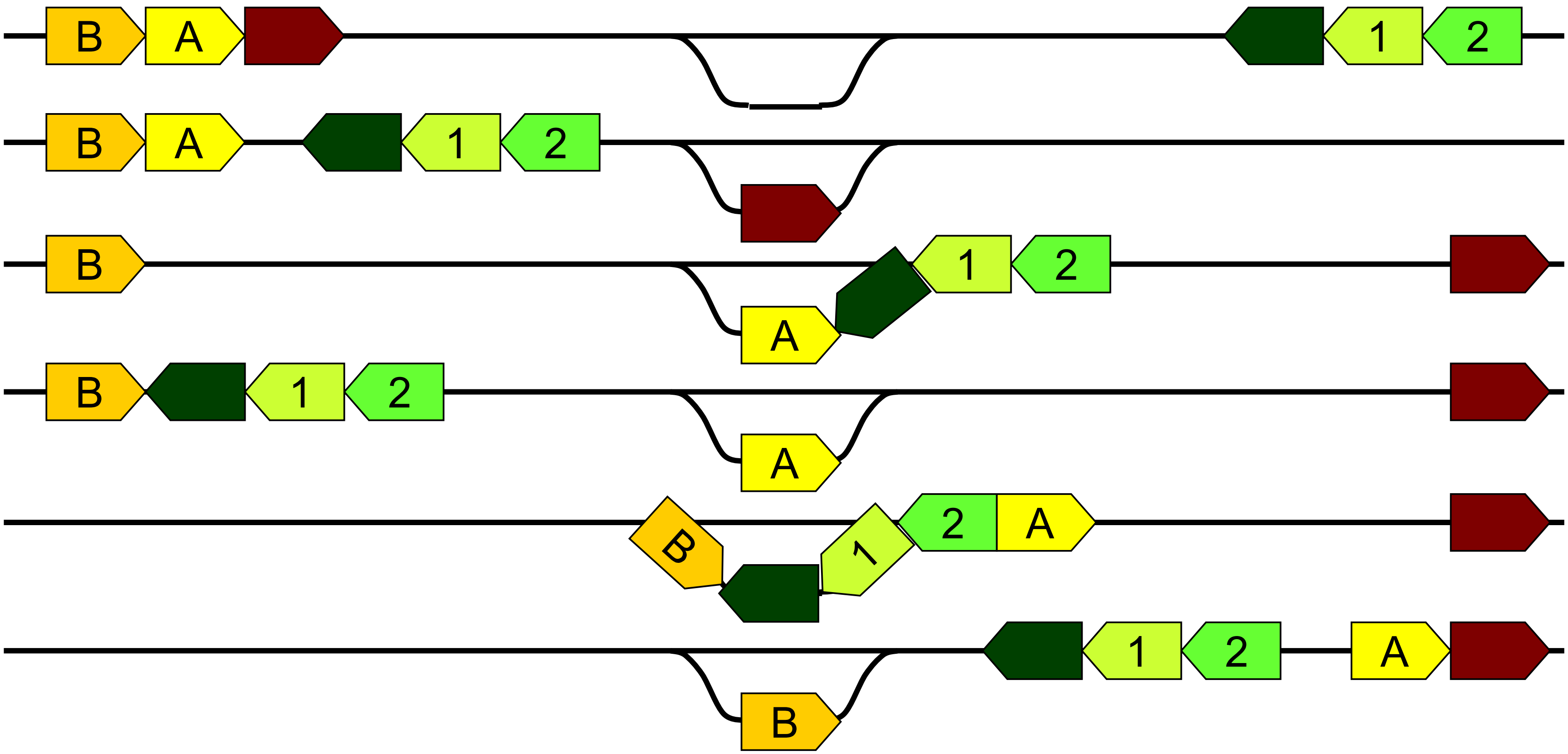
Cars in the train below have been sorted according to their delivery points. However, it is still nontrivial to deposit car A in stub 1, car B in stub 2, and car C in siding 3. Cars can be pulled or pushed by the engine.



Is there a better initial ordering of the cars for the deliveries in this puzzle?

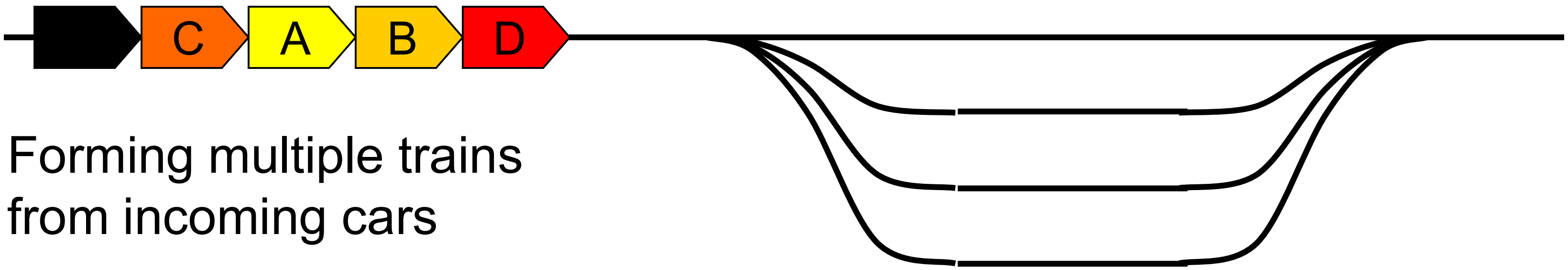
Train Passing Puzzle

The trains below must pass each other using a siding that can hold only one car or one engine. Show how this can be done.

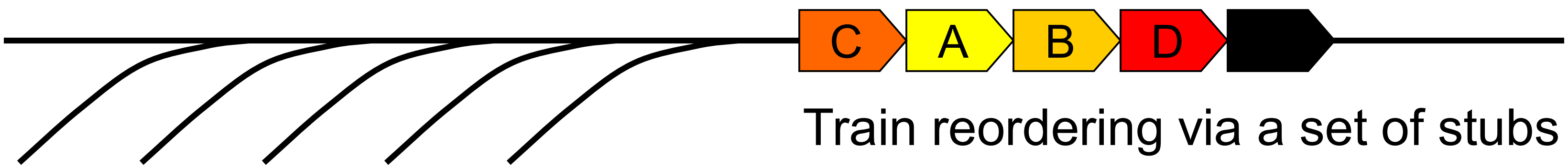


Q2: If the left and right trains have *L* and *R* cars, respectively, how many times will the siding be used for the trains to pass?

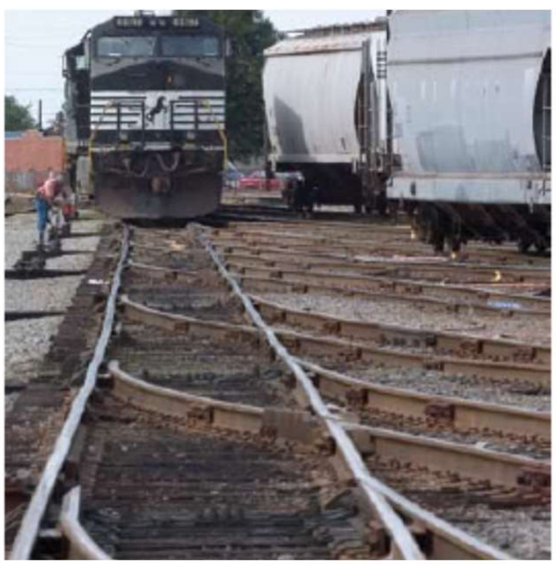
Fast Combining or Reordering of Train Cars



Forming multiple trains from incoming cars



Train reordering via a set of stubs



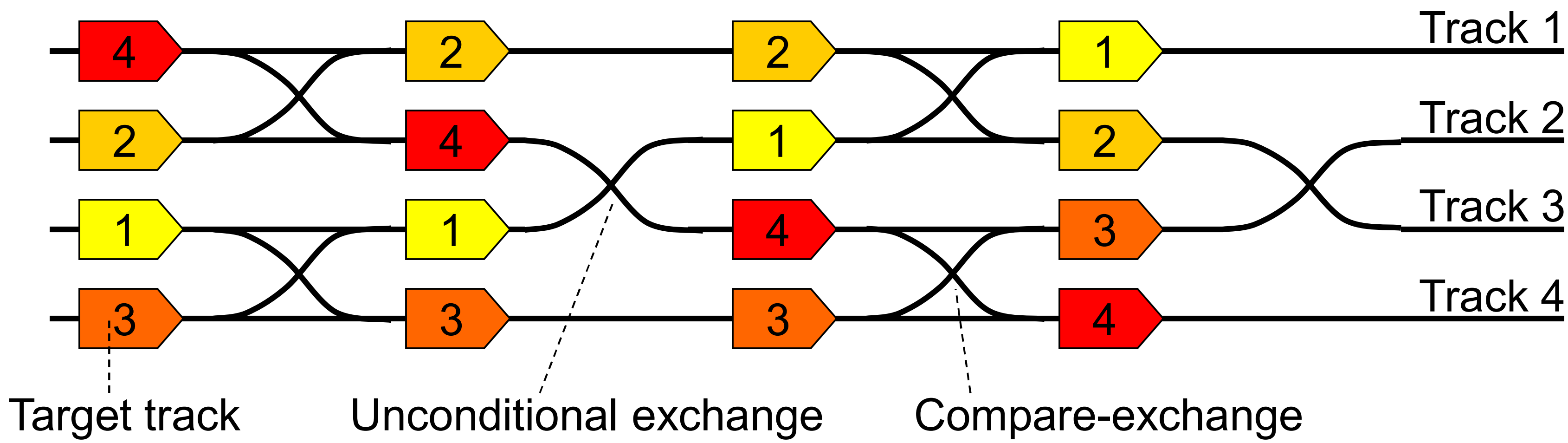
Cars are pushed or pulled by an engine

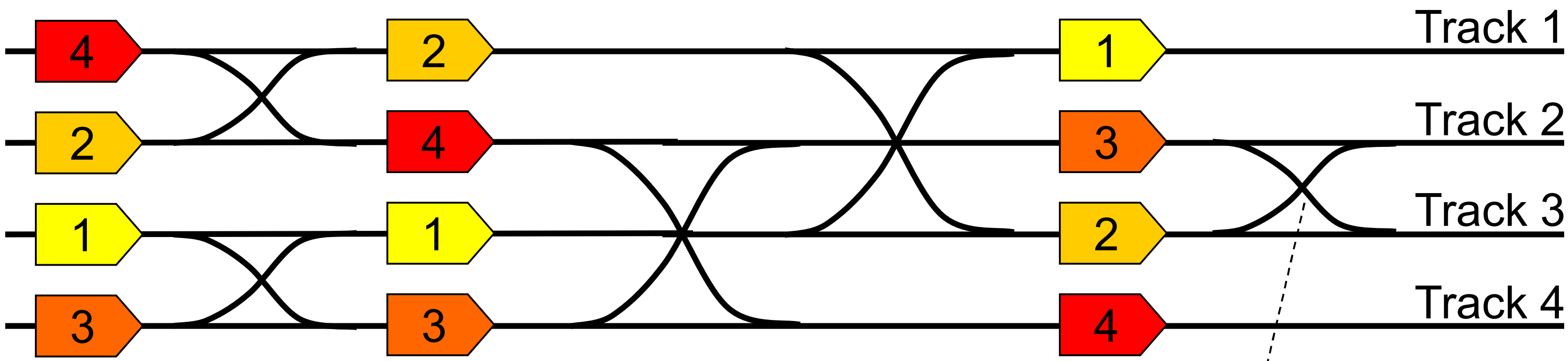
Alternatively, movement in one direction may be achieved via sloped rails

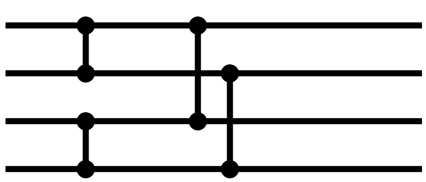
Switches used to be adjusted manually, but nowadays, electronic control is used

Q3: Sort the two trains shown above with the three sidings and five stubs

Sorting Train Cars in Parallel



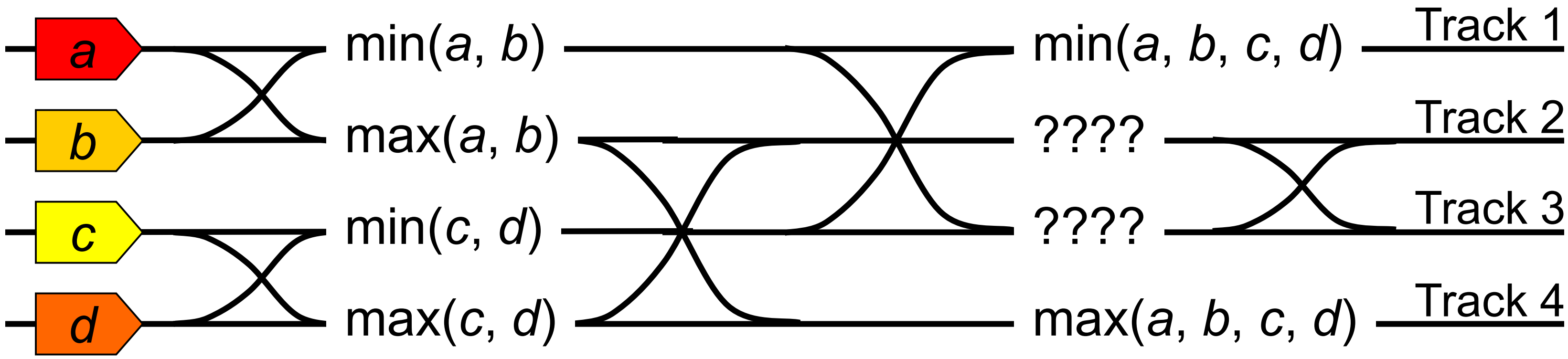




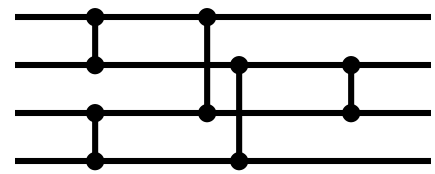
The net in stick diagram schematic

Is adding this compare-exchange element sufficient for producing a valid sorting network?

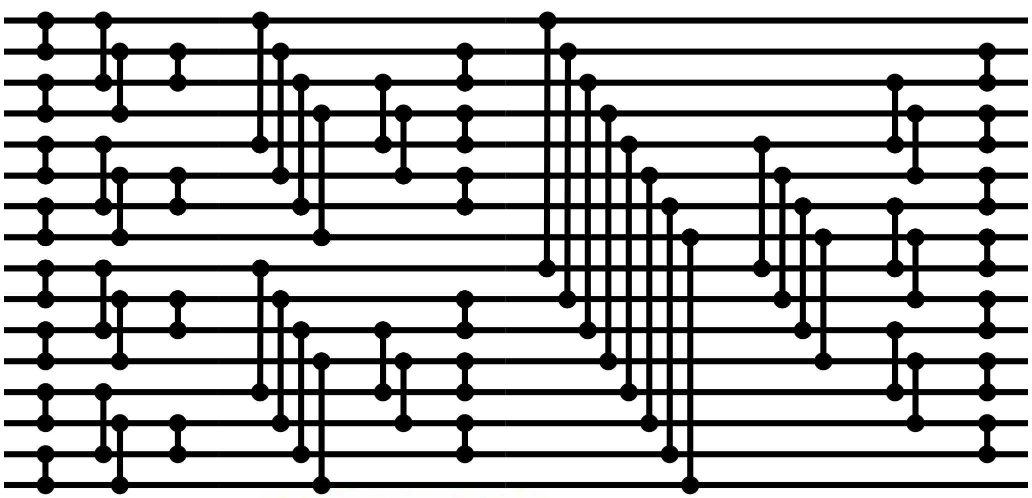
Validating a Sorting Network



In the example above, it was fairly easy to show the validity of the sorting network. Generally, it is much more difficult



Stick diagram schematic

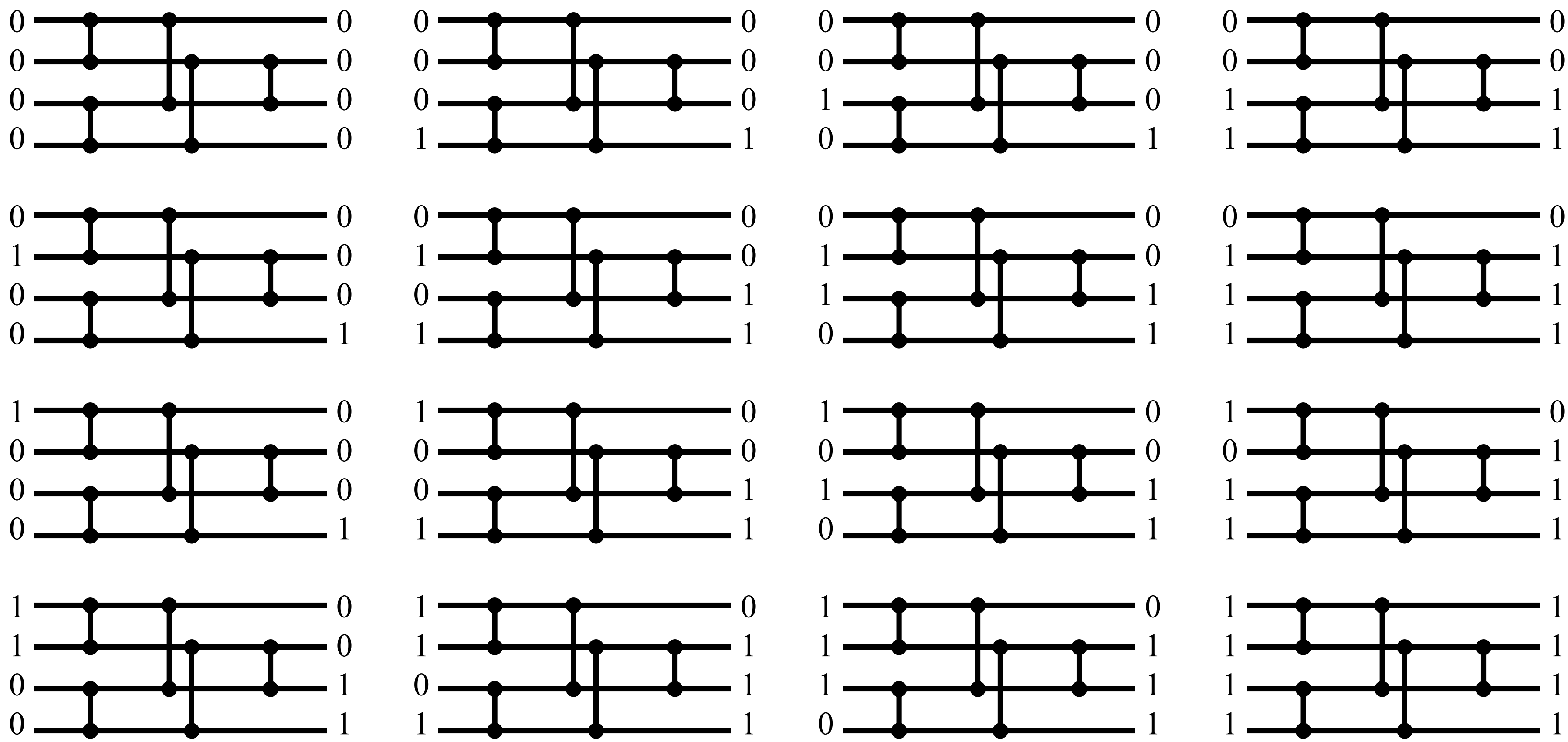


How would one establish the validity of this 16-input sorting network?

More importantly, how does one come up with this design in the first place?

The Zero-One Principle

A sorting net built of comparators is valid if it correctly sorts all 0-1 sequences

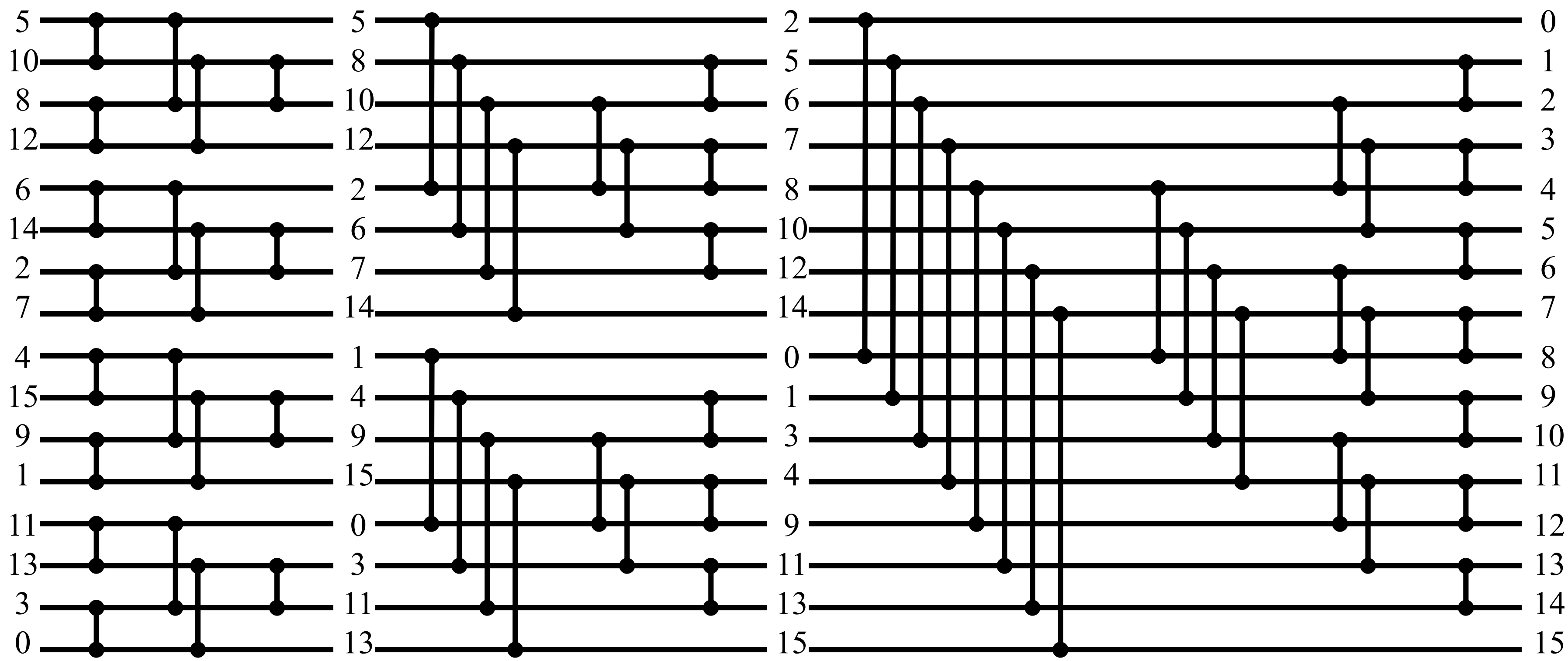


So, we can validate a sorting network using 2^n rather than $n!$ input patterns

$n = 12$: $2^n = 4096$, $n! = 479{,}001{,}600$ (thousands vs. half a billion)

A 16-Input Sorting Network

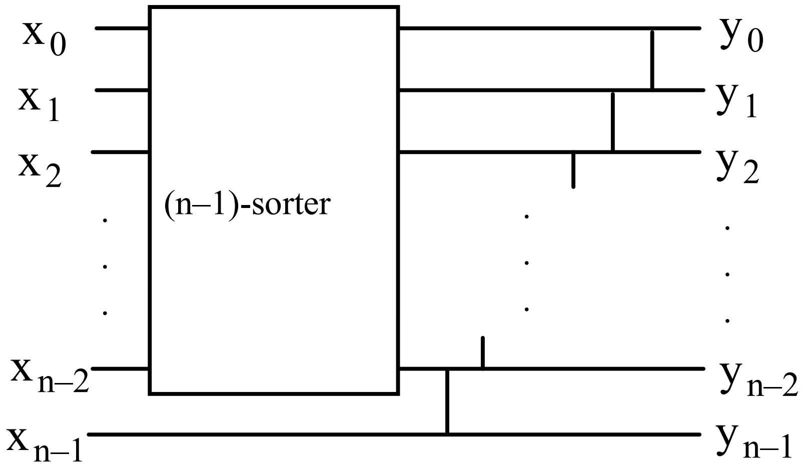
Use 4-input sorters, follow by (4, 4)-mergers, and end with an (8, 8)-merger



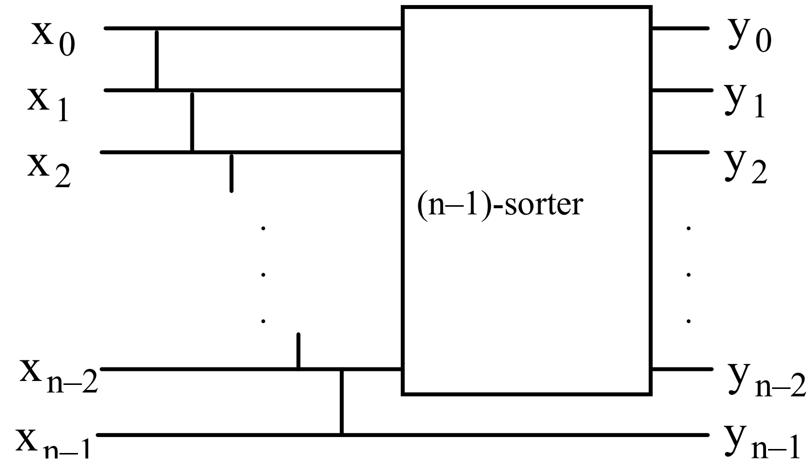
Using the 0-1 principle, we can validate this network via 16 + 25 + 81 tests

4-sorter tests (4, 4)-merger tests (8, 8)-merger tests

Insertion Sort and Selection Sort



Insertion sort



Selection sort

$C(n) = n(n-1)/2$
$D(n) = 2n-3$
Cost × Delay
$= \Theta(n^3)$

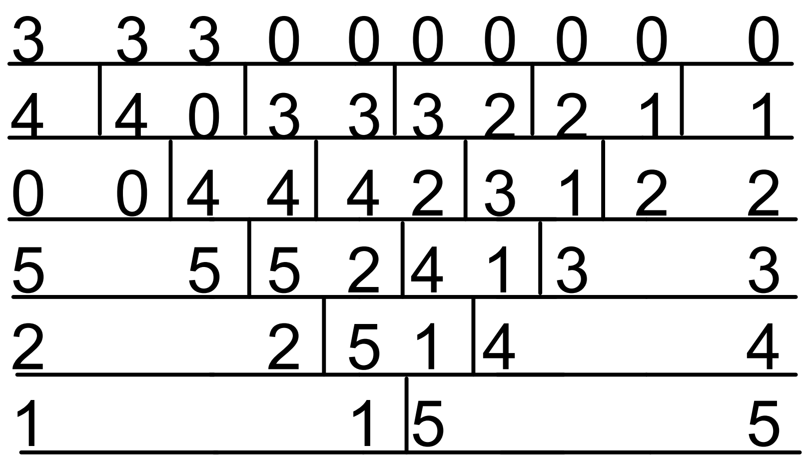
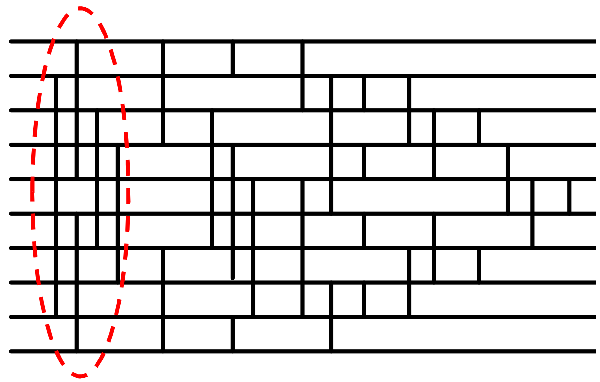


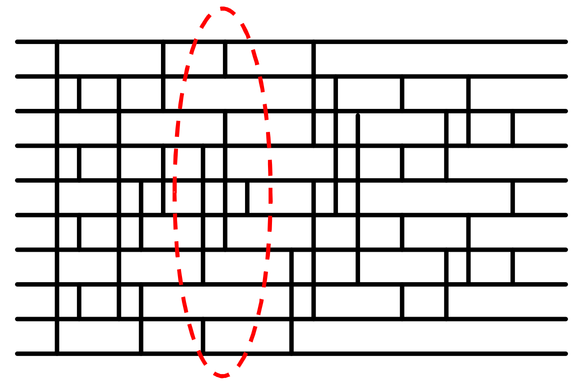
Fig. 7.8 Sorting network based on insertion sort or selection sort.

The Best Sorting Networks



Cost = 29 Delay = 9

Which 10-input sorting network is better?



Cost = 31 Delay = 7

Cost × Delay = 29 × 9 = 261

Cost × Delay = 31 × 7 = 217

Criterion 1: The number of sticks or compare-exchange blocks (cost)

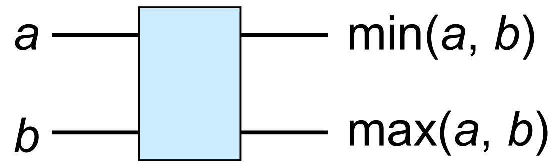
Criterion 2: The number of compare-exchanges in sequence (delay)

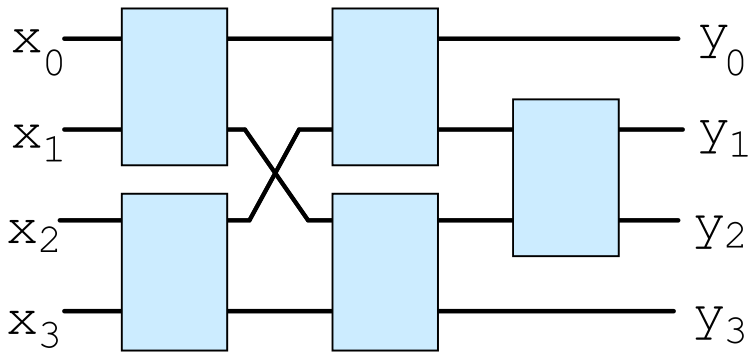
Criterion 3: The product of cost and delay (cost-effectiveness)

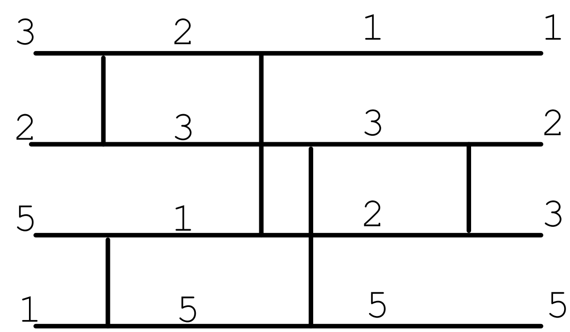
The most cost-effective *n*-input sorting network may be neither the fastest design, nor the lowest-cost design

Electronic Sorting Networks

Electronic sorting networks are built of 2-sorters building blocks







Applications of sorting networks:

Directing information packets to their destinations in a network router

Connecting *n* processors to *n* memory modules in a parallel computer

Q4: In the stick diagram of a 4-sorter on the top right, show that removing the top or bottom line and its comparators yields a valid 3-sorter but that removing one of the two middle lines does not