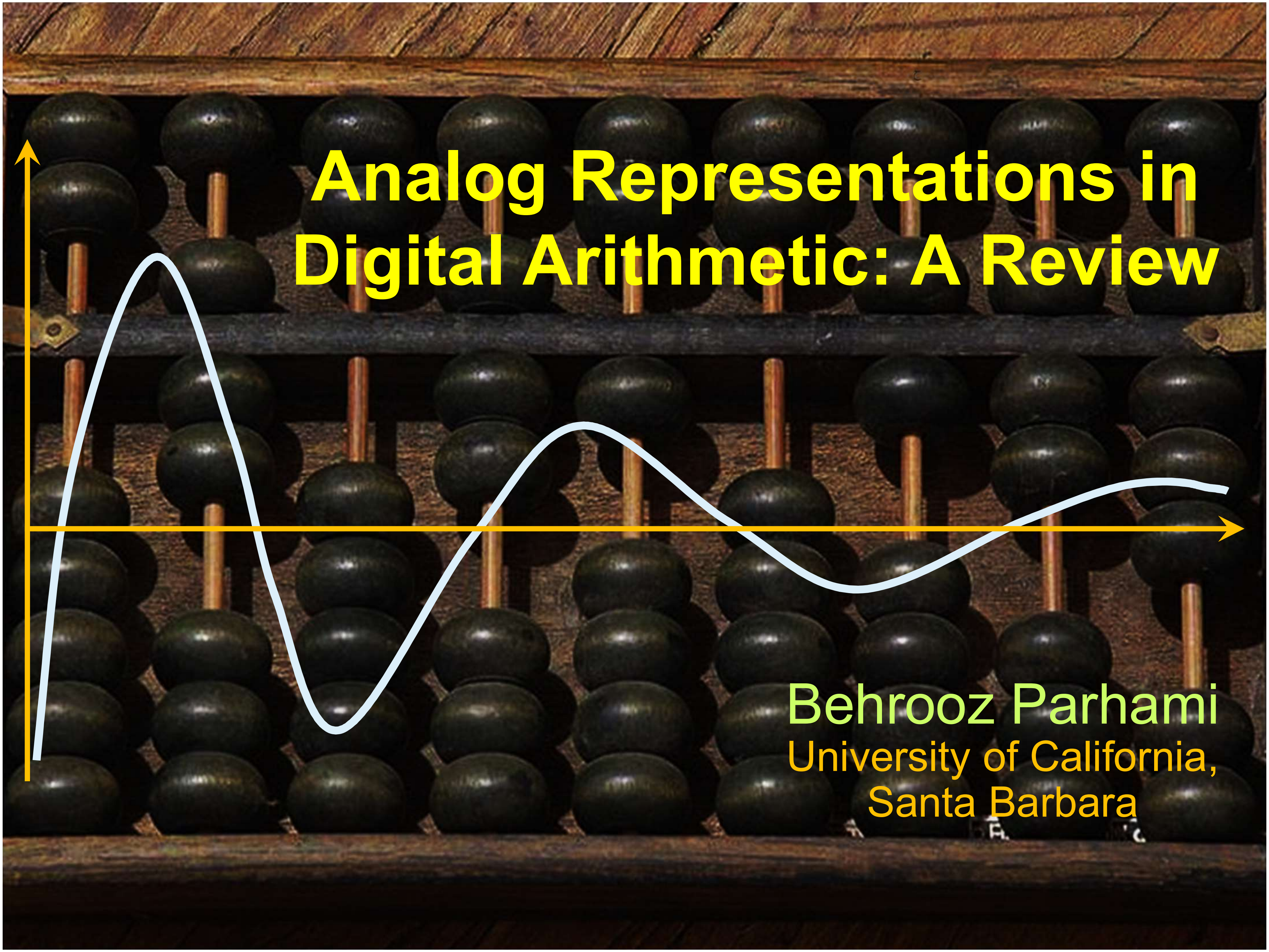


Analog Representations in Digital Arithmetic: A Review

Behrooz Parhami
University of California, Santa Barbara

About This Presentation

This slide show was first developed in fall of 2018 for an October 2018 talk at the 52nd Asilomar Conference on Signals, Systems, and Computers), Pacific Grove, CA, USA. All rights reserved for the author. ©2018 Behrooz Parhami

Edition	Released	Revised	Revised	Revised
First	Fall 2018			

Analog Computing is Back!

Digital data has been replacing analog data for years:
From 1986 to 2007, share of digital stored data went from near-zero to 90% [1]; a decade later, digital is fully dominant

We now struggle with complexity and energy limitations:
The solution seems to be approximate and analog computing

Hybrid digital/analog can give us the best of both worlds:
Digital's higher latency and power used only when needed

Hence, multi-resolution representations are desirable:
Combine fast, efficient low-precision computation with slower, energy-intensive high-precision computation

Hybrid Digital/Analog Representations

Continuous-valued number system (CVNS):
A positional number system with analog digits

Continuous-digit residue number system (CD-RNS):
Inspired by how positional info is represented in rat's brain

Here, we qualitatively compare the CVNS and CD-RNS representations

Race logic represents information as timing delays:
Results derived based on relative signal propagation delays

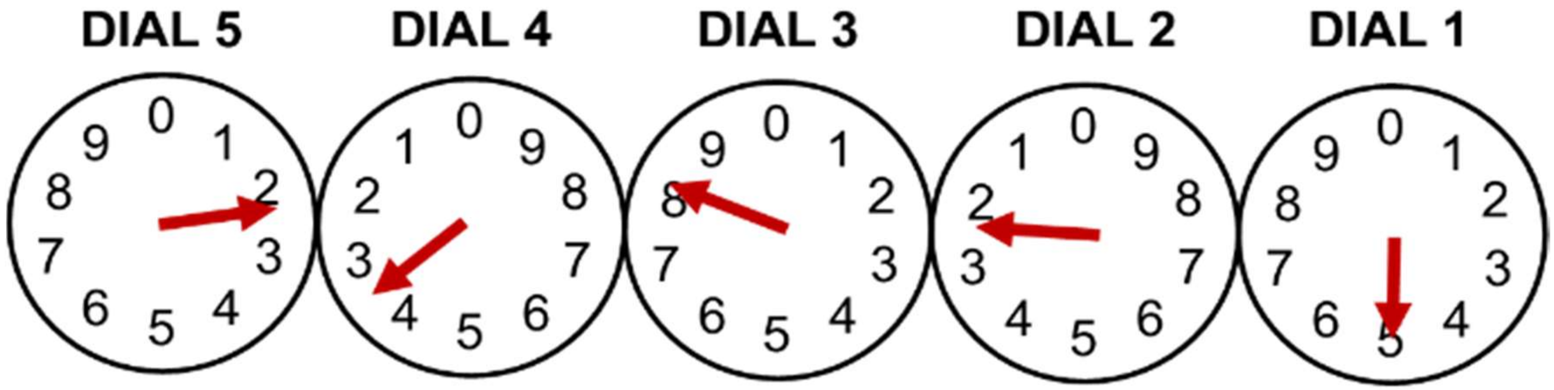
Space-time logic mimics spike-based neural computing:
Signal timing, as well signal magnitude, carries information

Continuous-Valued Number System

Positional representation with analog digits

Contains a form of natural redundancy (digit values overlap)
- Leftmost analog digit ~2.38 tells us something about the next digits
- Overlaps make the representation/arithmetic more immune to noise

Can be based on unsigned or signed digits; e.g., (–5, 5)



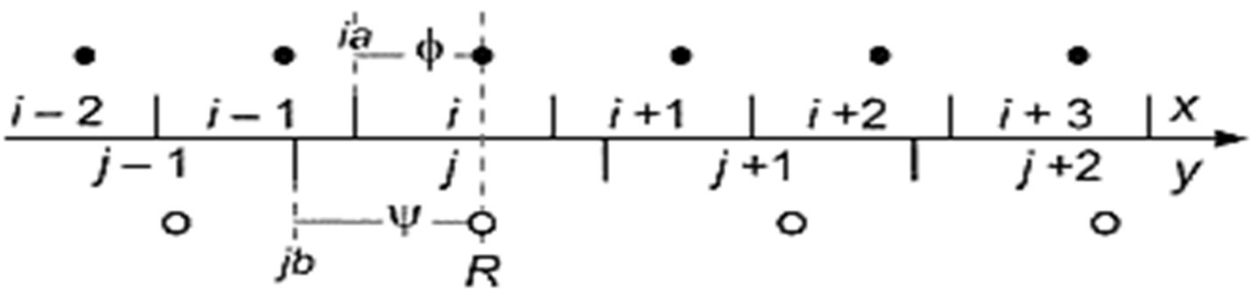
The electric-meter analogy for CVNS [4]

Continuous-Digit RNS Representation

Modular arithmetic with continuous residues

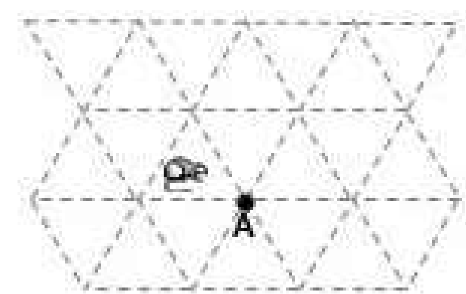
Extension of RNS to non-integer moduli and residues
- Offers precision-range-robustness trade-offs
- More accurate residues widen the range and increase robustness

Of interest to neuroscientists: Rat's navigation system
- Rat uses "space cells" (absolute) and "grid cells" (relative)
- Can return to home position in the dark, without any visual cues



Localization with 2 grids in 1D space [6]

Rat's hex grid



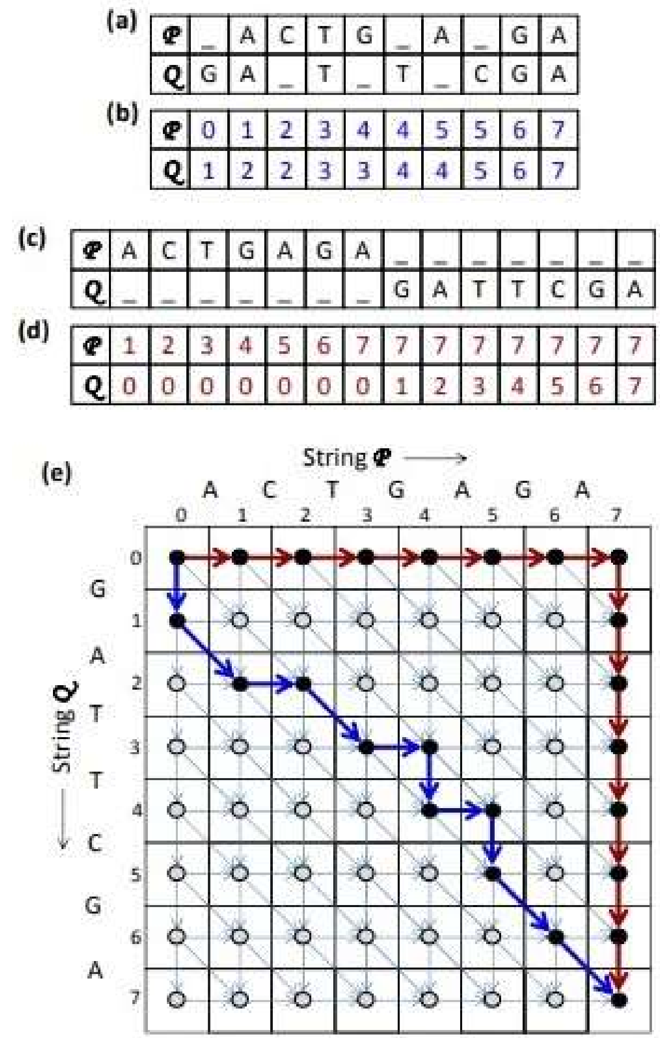
Time-Delay-Based Race Logic

Not useful for general use (yet)
Quite efficient in some domains

Example: String alignment [8]
(as in DNA/protein matching)
Closeness judged by "edit distance"

2D array of simple hardware cells
$O(m^2)$ hardware complexity

Paths represent alignments
Horizontal, vertical, diagonal
  moves have different latencies
Fastest path = Best alignment



Brain-Like Space-Time Computing

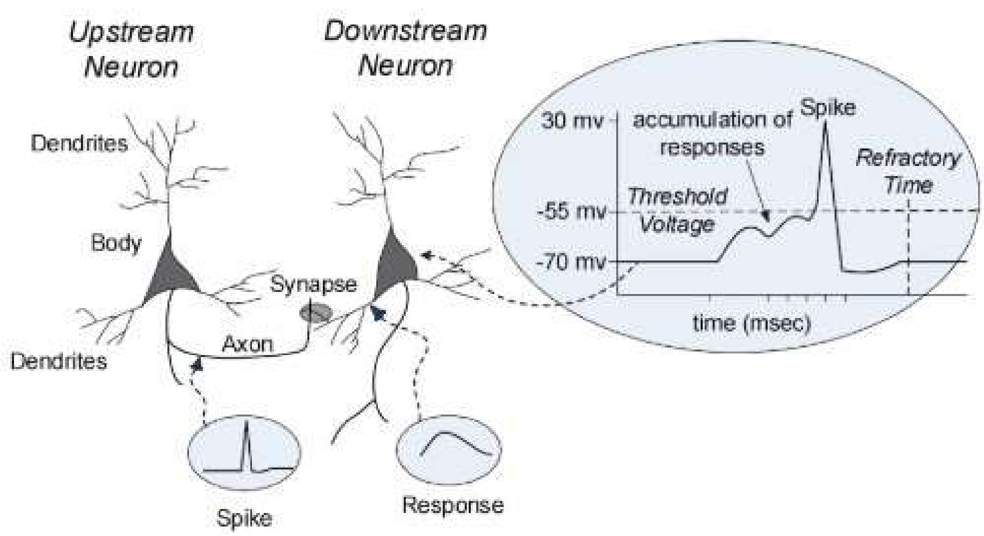
Example: Transmitting one 8-bit byte representing *n*

Binary: Delay = 9 slots; Average energy = 5 spikes

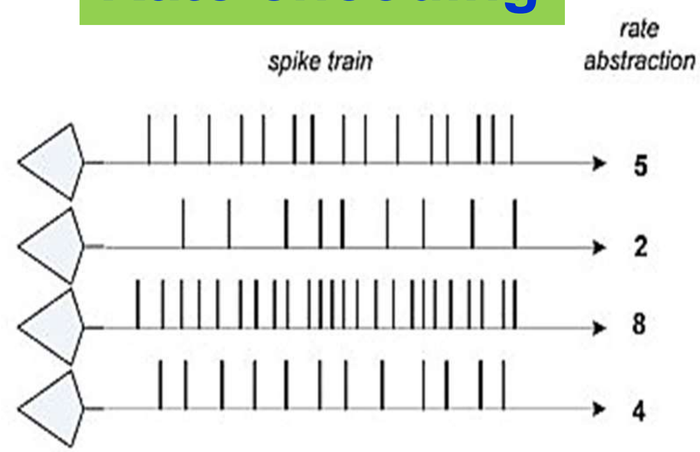
- Start/synch spike, followed by 8 spikes/no-spikes

Space-time: Average delay = 130 slots; Energy = 2 spikes

- Start/synch spike, followed by *n* no-spikes, then end spike



Rate encoding



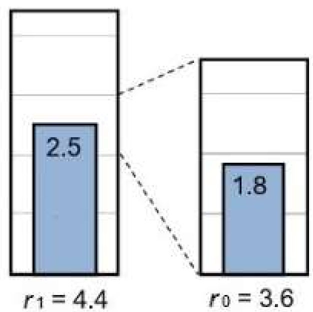
CVNS and CD-RNS Similarities

Two-level scheme: Analog representation at the low (digit) level and digital interpretation at the high (inter-digit) level

CVNS likely has performance edge in general applications

Mixed-radix format: CVNS is based on fixed integer radix, but extension to mixed and non-integer radices is possible

Example mixed-radix CVNS: Representation of
$9.0 = 2.5 \times 3.6 = 2 \times 3.6 + 1.8$



Approximate computing: Both CVNS and CD-RNS suited to low-precision, adaptive-precision, and lazy arithmetic

CVNS and CD-RNS Differences

Word-level parallelism: CD-RNS has greater affinity with parallel processing of digits in add/subtract/multiply

Input/output overheads: CVNS has simple/direct forward and reverse conversion processes (low-cost and low-energy)
CD-RNS conversions are even more complex than RNS

Noise immunity: Consider 2-digit CVNS and CD-RNS
CVNS range decreases quadratically with increased noise immunity
CD-RNS range decreases linearly with increased noise immunity
(cutting the radix *r* in half, versus using the equation $\mu\varepsilon \leq \mu_0\mu_1$)

Fault tolerance: CVNS can be protected through coding
CD-RNS has precision-robustness trade-off built in

Conclusions and Future Work

Analog computing is making a comeback and hybrid digital/analog computing is becoming more attractive

D/A computing can be combined at various levels:

- Representation level, as in CVNS and CD-RNS
- Analog approximation, digital refinement
- Neuromorphic computing paradigm
- Multi-level combination methods

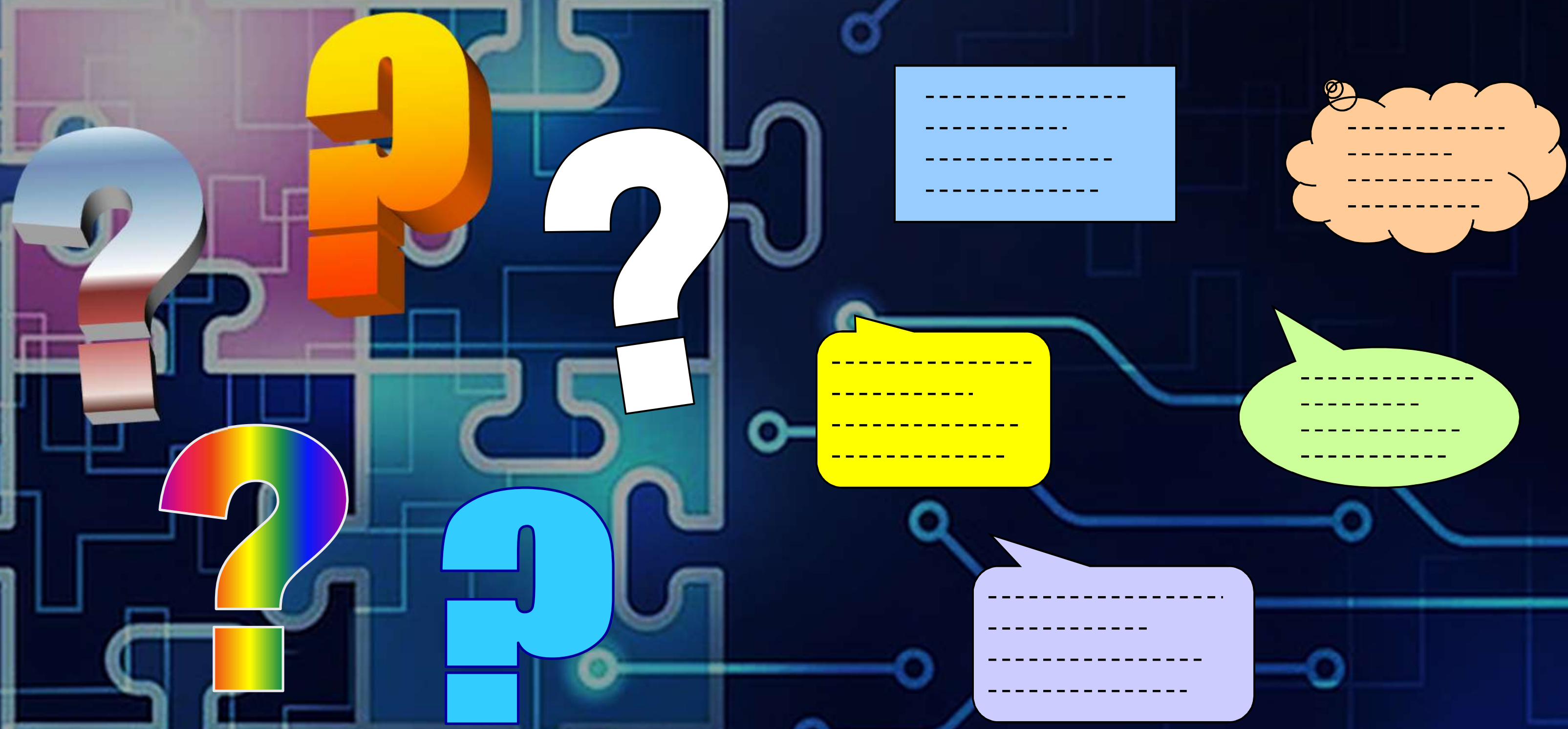
Future work and more detailed comparisons

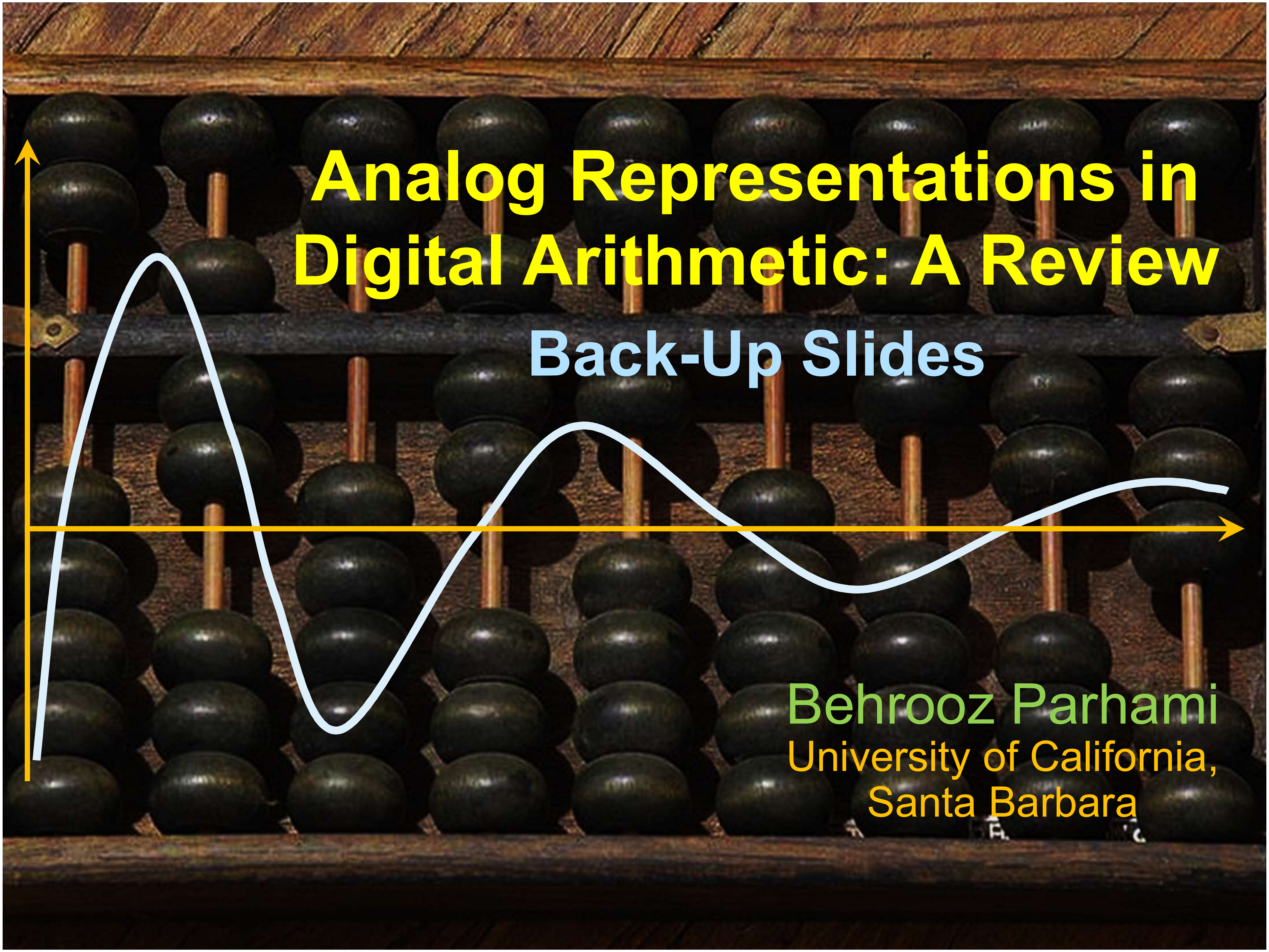
- Assessment of relative speeds in application contexts
- Quantifying cost and energy requirements
- Effects of radix and moduli selection
- Other D/A combination methods

Questions or Comments?

parhami@ece.ucsb.edu

http://www.ece.ucsb.edu/~parhami/

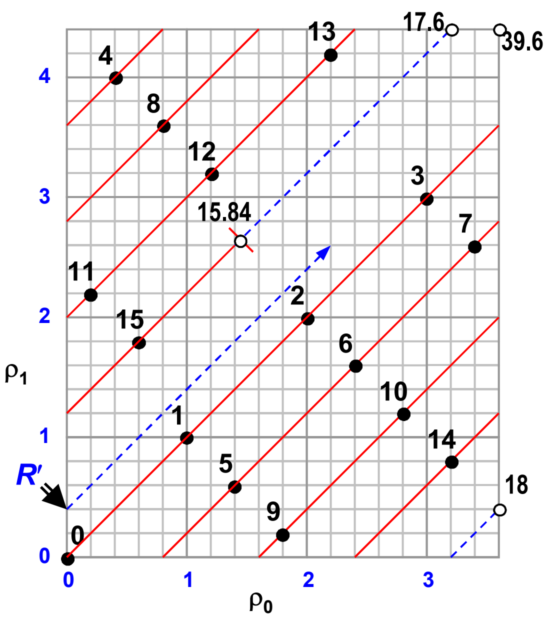


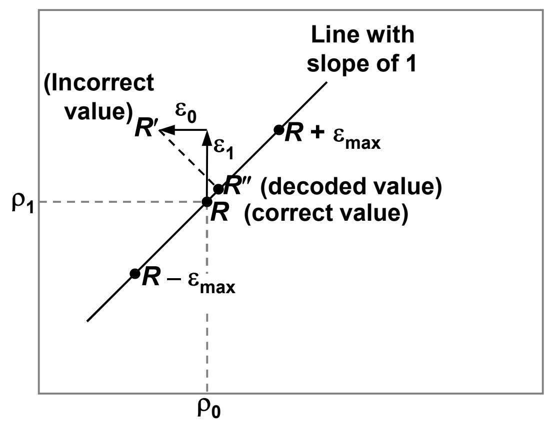

Analog Representations in Digital Arithmetic: A Review
Back-Up Slides
Behrooz Parhami
University of California, Santa Barbara

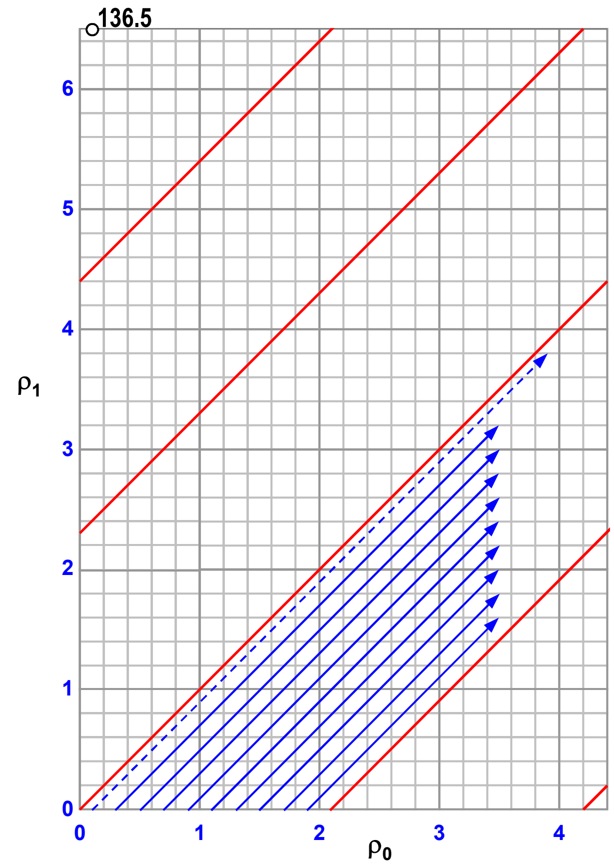
Range-Precision Trade-off in CD-RNS

Dynamic range is proportional to the product of moduli, divided by maximum error

Left diagram: Range extension beyond Πm_i
Middle diagram: Decoding error
Right diagram: CD-RNS with $m_1 = 6.5$, $m_0 = 4.4$





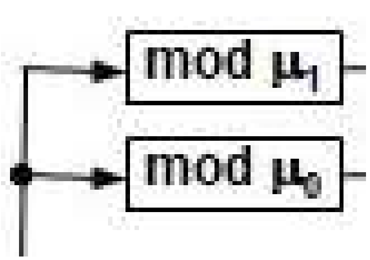


Forward/Reverse Conversion in CD-RNS

CRT-like methods of conventional RNS do not carry over

Small errors in residues may be amplified through the conversion process
A possible way out: View reverse conversion as nonlinear optimization
Single-layer feedback network (see diagram on the bottom-right)
Converges to the correct result within the *RC* time constant of the circuit

Forward conversion



Reverse conversion