Number Representation and Arithmetic in the Human Brain

Jennifer Volk
Prof. Behrooz Parhami

UC SANTA BARBARA

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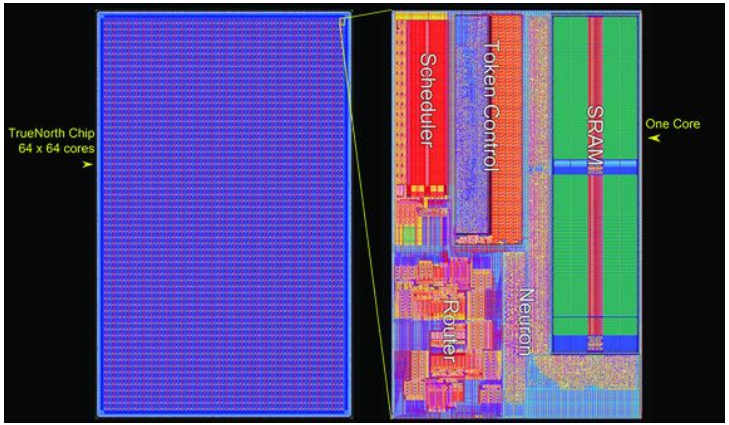
Topics to be Covered

1. Existing neuromorphic technologies
2. Number representation in the human brain
3. Generalizable qualities of arithmetic & parts of the brain responsible
4. Choose a model: analog, digital, or hybrid?
5. Error sources and sources of latency
6. Future work
7. Conclusion

Existing Neuromorphic Architectures

(TrueNorth, Neurogrid, BrainScaleS, SpiNNaker:)

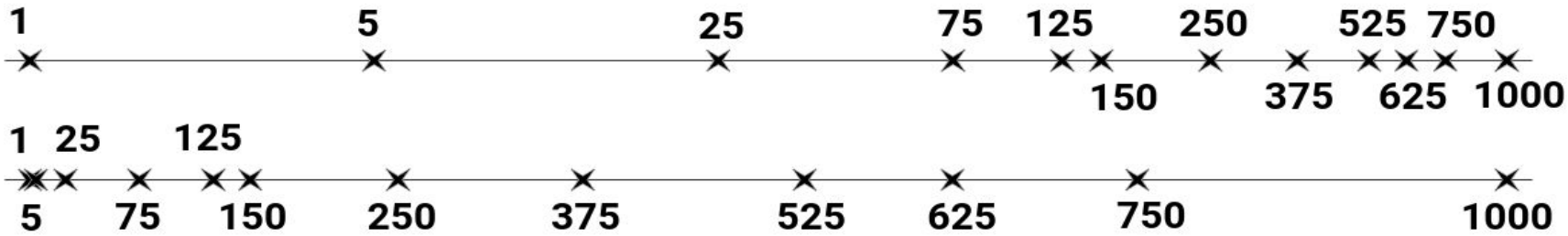
- Utilize an artificial version of a biological neuron
- Focus on high interconnectivity
- Physical connectivity limited to 2D
- Low efficiency



True North layout
Yale Engineering

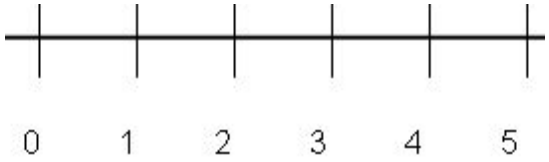
The Human Number Line: Logarithmic or Linear?

- **Mental visualization tends to be linear**



- **Mental encoding is hypothesized to be logarithmic**

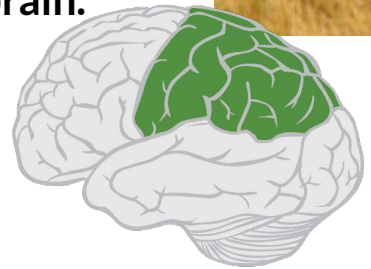
Number Sense



- **Applies to small numbers (usually zero to 5);**
- **May have been advantageous wrt evolution;**
- **Is encoded in a distinct region of the brain.**

  (This means faster cognition)

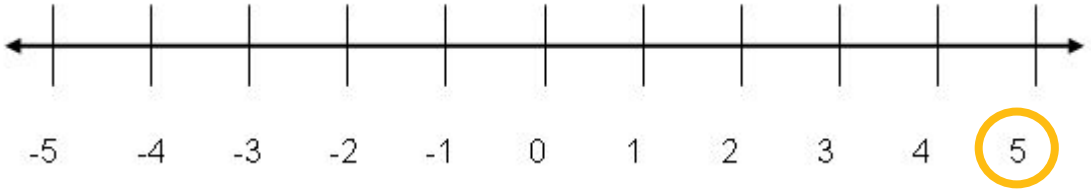
[4,5,6,7]



The Triple Code

Numbers are encoded in the brain in three forms:

- **Verbal** /'faɪv/
- **Numeral** 5
- **Quantitative**



[7]

Different Types of Arithmetic Considered

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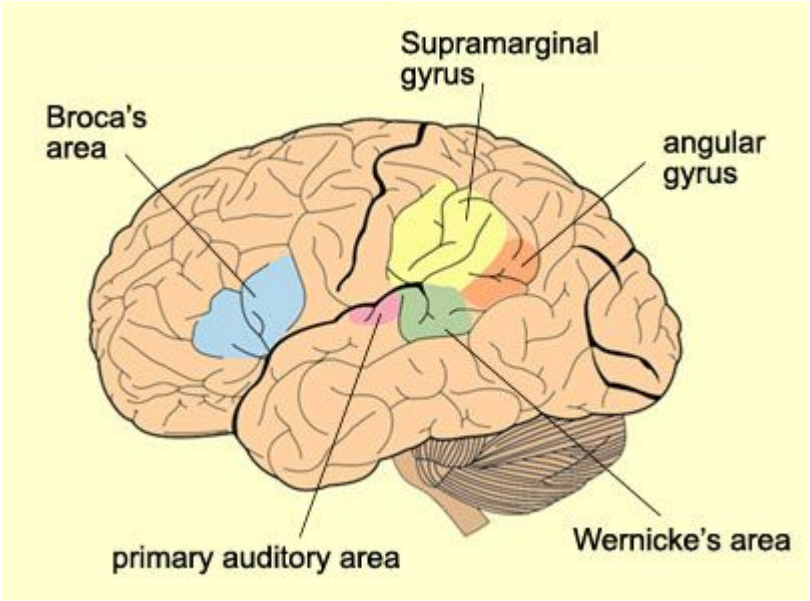
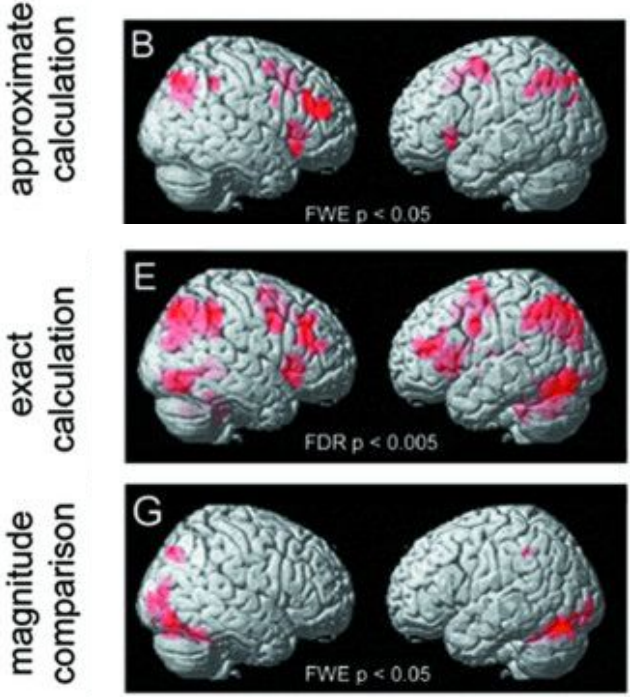
- Small operands vs Large (1 to 5 versus > 5)
- Calculations vs Comparisons (8 + 9 = 17 versus 100 <= 101)
- Approximate vs. Exact (10 + 21 ≅ 30 versus 25 + 24 = 49)

Approximate vs Exact Computations vs Comparisons

Approx vs Exact Calculations vs Comparison

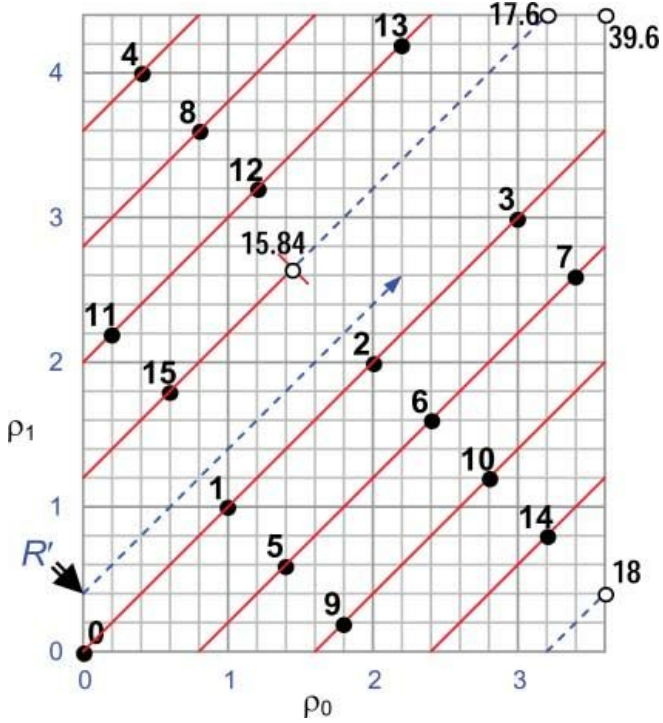




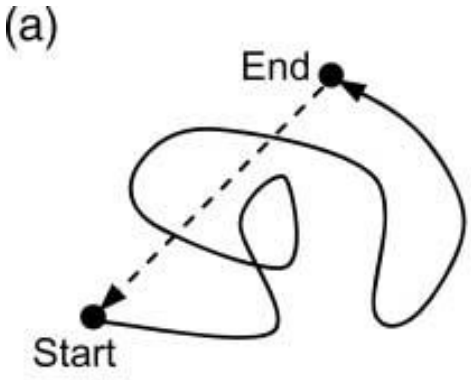
K. Kucian

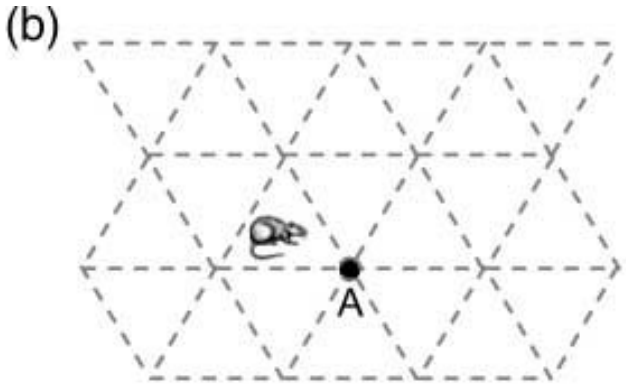
[8]

Is the Human Brain Analog, Digital, or Hybrid?



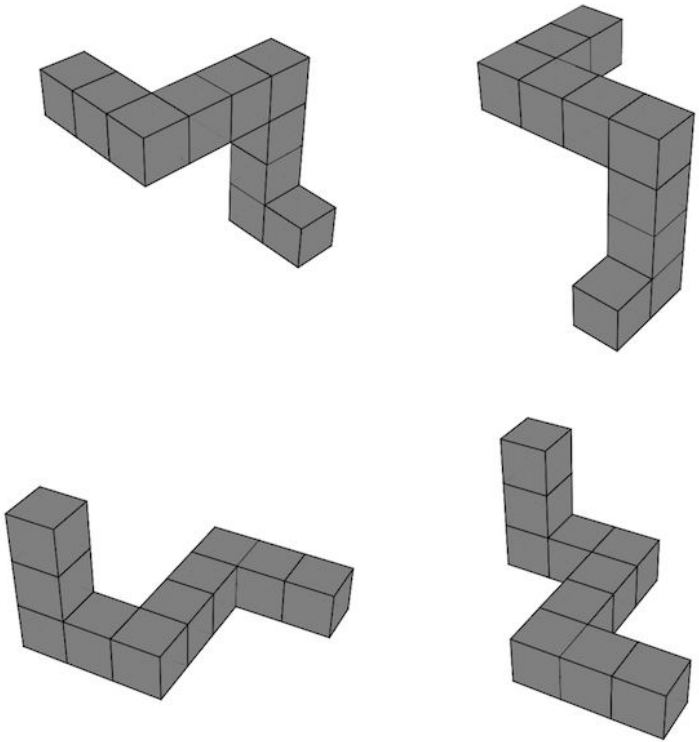
Analog computing by spatial reference





Parhami, 2015

Analog, Digital, or Hybrid?

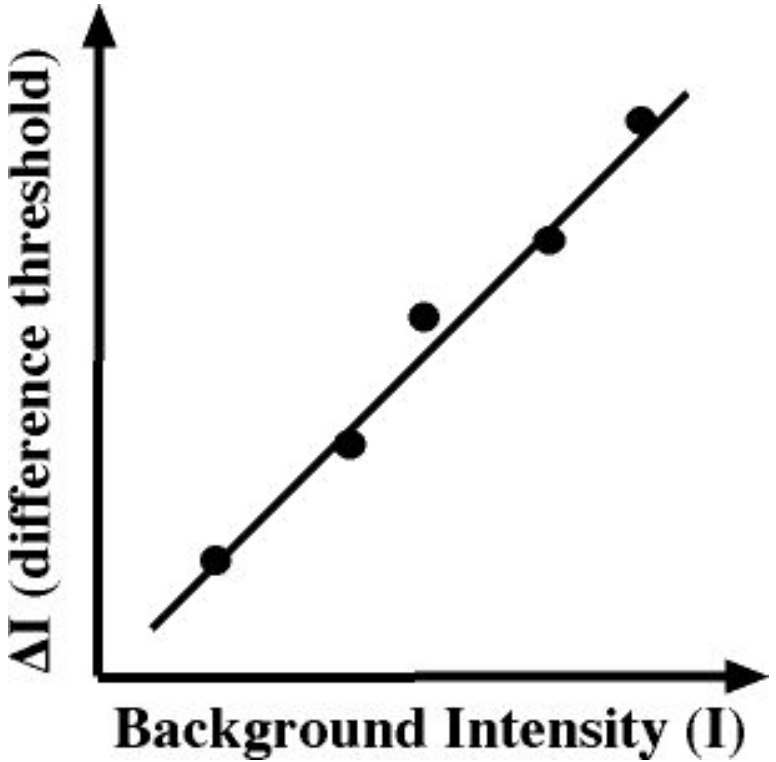


Analog computing example (spatial orientation)

(Top two are the same, bottom two different)

C. J. Maley, 2018

Analog, Digital, or Hybrid?

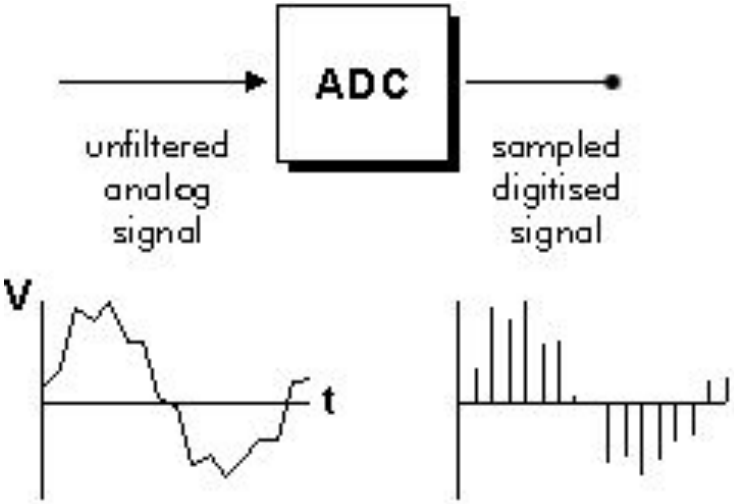


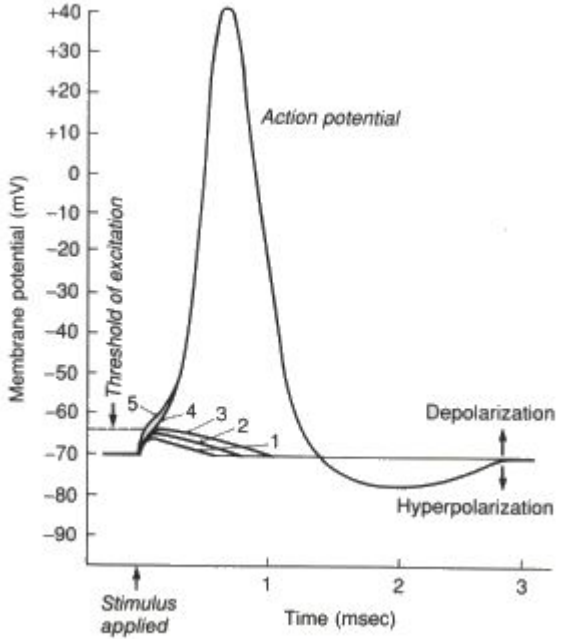
[16]

- **Digital (JND, Weber's law)**

Analog, Digital, or Hybrid?

- **Case for hybrid (take-aways, best of both worlds)**





[17,18]

Error Sources and Sources of Latency

- **Distance Effect**

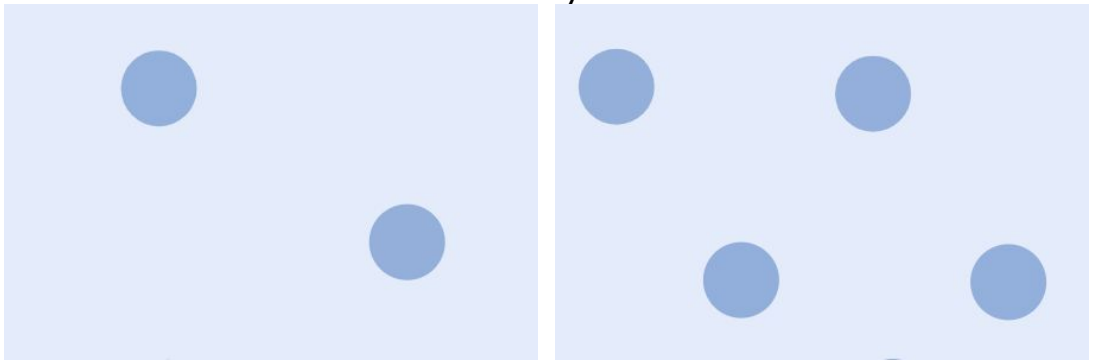
19 + 18 = 250?	**Fast to reject**
10 * 10 = 5000?	**Fast**
56 > 58?	**Slow**
56 > 100?	**Fast**

Error Sources and Sources of Latency

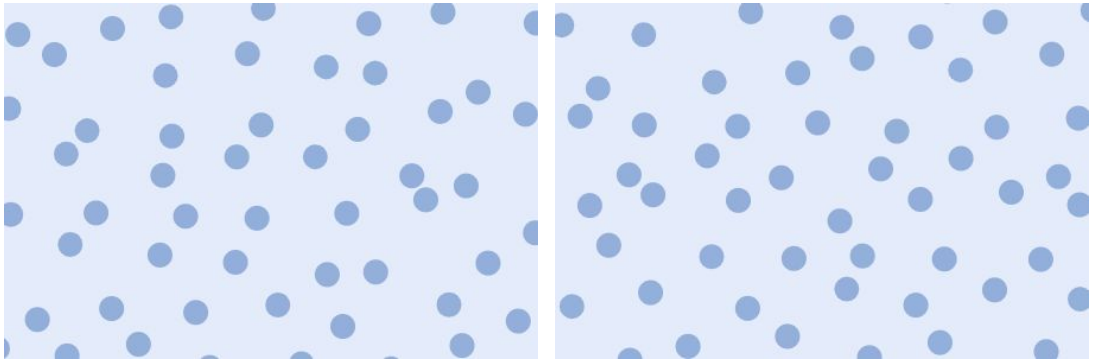
Size Effect:

Which has more dots?

Easy



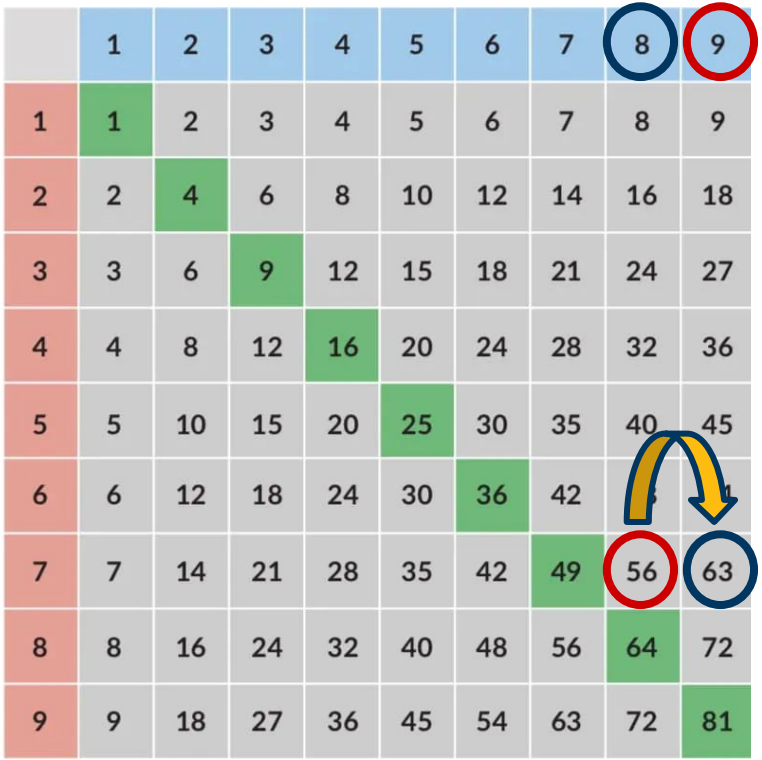
Hard



[12]

Error Sources and Sources of Latency

- Additive carries are slow
- Stored/memorized arithmetic facts can be noisy and error-prone
- Mathematical facts stored in the original language in which they were learned



	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42		
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

[13]

Neural Error-Tolerance and Robustness

- The brain is a noisy biological environment

- Error-checking features:
  - Odd-Even Rule — **19 + 16 ≠ 34**
  - Factor-Checking — **7 * 5 = 30?**
  - Distance Effect — **26 + 29 = 2?**

Future Work

- Signal processing (neuronal input/output) (human)
- Neuromorphic CMOS Scaling (artificial)
- Mathematical training (human)
- Artificial arithmetic (artificial)

Conclusion

- Human brains are bad at math, computers are good at math (and can perform it with less power) [19]
- But humans are better at other things
- Therefore we may say that a chip performing things like language synthesis, object recognition, etc. may benefit from architectural choices closer to the human brain

Thank you!

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