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Extended-Fault Diameter of Mesh Networks

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Abstract

The diameter of a network in the presence of faulty nodes is an important indicator of its resilience. Certain popular topologies have been shown to possess fault diameters that are only one unit greater than those of the corresponding fault-free networks, provided that the number of faults is strictly less than the network's connectivity. Examples of such robust networks include hypercubes, m-ary q-cubes, cube-connected cycles, and star graphs. We derive the extended-fault diameter of $m_0 \times m_1 \times \ldots \times m_{q-1}$ *mesh networks. Mesh networks, which play an important role in the implementation of parallel processors are not node-symmetric (even when all the* m_i *are equal) and thus do not easily lend themselves to theoretical analyses; hence, the fault diameter of meshes has not been previously dealt with. Furthermore, we do not require the number of faults to be less than the network's connectivity, as has been done in previous studies. Such a requirement would limit the number of faulty nodes to 1 in the case of 2D meshes, which is unduly restrictive. Hence, our motivation to define and study a network's extended-fault diameter.*

Keywords: Fault diameter, Faulty processor array, Incomplete mesh networks, Mesh-connected computer, Network fault tolerance, Robustness.

Notation

$c(G)$	Connectivity of a network G
$D(G)$	Diameter of a network G
$D_{EF}(G,f)$	Extended-fault diameter of network G with f worst-case faulty nodes
$D_F(G)$	Fault diameter of a network G with fewer than $c(G)$ faults
f	Number of faulty nodes
g	Number of good nodes that are cut off from the rest of the network by faulty nodes
m	Side of a mesh or torus with equal-length sides along all dimensions
m_i	Side length of a mesh or torus along dimension i, $0 \le i \le q-1$
p	Total number of nodes or processors
q	Number of dimensions in a hypercube, mesh, or torus

1. Introduction

The sheer volume of hardware components in a massively parallel computer makes the occurrence of faults inevitable. It is, therefore, imperative that mechanisms for fault detection, recovery, and tolerance be incorporated into such systems. Clearly, hardware fault detection and recovery is not sufficient in itself. Algorithm and software design must also take into consideration the fact the system resources and connectivity may vary as resources fail or are put back into operation following repair. Assuming that such robust algorithms can been implemented, the hardware designer's task is to ensure timely fault detection and recovery, as well as provisions for graceful degradation. The latter property requires, among other things, the relative insensitivity of key network parameters such as diameter, average distance, and bisection, to the occurrence of faults. While these latter properties are still difficult to ensure, they are significantly simpler that tackling hardware and software issues together.

Let the connectivity $c(G)$ of a network G be the minimum number of nodes whose removal would partition the network. We know, for example, that $c(q\text{-cube}) = q$ and $c(q\text{-star}) = q - 1$. Thus, if the number f of faults in a network G is less than $c(G)$, all remaining nodes will be connected. The diameter of the resulting network with a worst-case pattern of $c(G) - 1$ faults is known as the fault diameter $D_F(G)$ [Kris87].

Definition 1 (fault diameter): The fault diameter $D_F(G)$ of a network G with up to $c(G) - 1$ faulty nodes is the diameter of the surviving portion (guaranteed to be connected) when the faults occur in a worst-case configuration that maximizes the diameter. ▮

Certain popular topologies have been shown to possess fault diameters that are only one unit greater than those of the corresponding fault-free networks, provided that the number of faults is strictly less than the network's connectivity. Examples of such robust networks include hypercubes [Kris87], cube-connected cycles [Kris87], k-ary q-cubes [Day97], and star graphs [Lati93b]. Fault diameter with fault sets whose sizes equal or exceed the connectivity $c(G)$ of a network, but that are otherwise restricted to exclude certain "forbidden" patterns, has also been investigated [Esfa89], [Lati93a].

In practice, many parallel machines are designed with interconnection networks other than the ones named above. For example, meshes are quite popular due to their scalability and compact packaging/layout, as are torus networks with unequal sides (unlike k-ary n-cubes whose sides are all equal). Hence, numerous studies have dealt with properties of meshes and tori that pertain to efficient algorithm implementation and fault tolerance. Examples of mesh- and torus-based parallel machines include several recent multiteraflops computers developed as part of the DOE's ASCI program, Intel Paragon, SGI/Cray T3D/E, and Tera MTA [Parh99].

Definition 2 (mesh/torus network): A q-dimensional (qD) mesh, consists of $p = m_0 m_1 \ldots m_{q-1}$ processing degree-$2q$ elements arranged in an $m_0 \times m_1 \times \ldots \times m_{q-1}$ grid, where adjacent processors along the various dimensions are connected via bidirectional communication links. If end-around links are used along each dimension, a torus or, in the special case of $m_0 = m_1 = \ldots = m_{q-1} = m$, an m-ary q-cube results. Without loss of generality, we always assume $m_0 \le m_1 \le \ldots \le m_{q-1}$. ▮

The preceding variations from the highly regular, and theoretically more appealing, k-ary n-cubes are in part necessitated by packageability constraints. They may also result from the desire to make the architecture expandable in small increments (e.g., by increasing the length of just one of the three sides in a 3D mesh). Furthermore, the mesh and torus networks used are often low-dimensional with small connectivities, making the assumption that the number of faults is smaller than the network's connectivity (which is only 2 for a 2D mesh) quite restrictive. Despite these observations, studies of the fault diameter of torus networks have been limited to k-ary n-cubes, with no corresponding result available for meshes of any type.

We are thus motivated to define the notion of extended-fault diameter in this paper and to derive results on the extended-fault diameters of mesh networks that are applicable in the aforementioned cases of practical importance. Results for torus networks are currently being developed.

2. Why Extended Fault Diameter?

Like previous researchers dealing with the notions of fault diameter and computing with faulty ensembles of processing nodes, we assume the availability of a suitably complete and reliable mechanism for detecting faulty system elements. We note that if a pattern of faults in the system causes certain nonfaulty elements to become isolated from the rest of the system, the assumed fault detection mechanism will consider such elements to be faulty. This is true because isolated parts are indistinguishable from faulty elements based on their observed behavior (nonresponsiveness). As long as the number of isolated elements is relatively small, the connected parts of the system that contain the bulk of system resources can continue to function at a reduced performance level. We thus consider restricting the number of faults to $c(G) - 1$ unduly restrictive in the studies of fault diameter. Because we allow $f \ge c(G)$, we need to define what we mean by fault diameter under such a condition.

Definition 3 (extended-fault diameter): The extended-fault diameter $D_{\mathrm{EF}}(G,f)$ of a network with f worst-case faulty nodes is the diameter of a connected component of the surviving configuration that has the largest diameter among all such connected components. ▮

Defining the extended-fault diameter as above makes a lot of sense, especially for low-dimensional mesh and torus networks. A 2D mesh, for example, can become disconnected with as few as two faults. However, if this happens, only one nonfaulty node will become isolated from the remaining $p - 3$ good nodes. There is no reason why such a $(p - 3)$-node subnetwork cannot be put to work as one would do for the $(p - 1)$-node network resulting from a single fault in the mesh. Once the mesh loses its perfect connectivity, losing a small number of good nodes that become inaccessible is not all that important, particularly given that such occurrences have diminishingly small probabilities.

Theorem 1: In an $m_0 \times m_1$ mesh, the occurrence of f faults, $f < m_0$, will make at most $f(f - 1)/2$ good nodes unavailable, thus limiting the total number of unusable nodes to at most $f(f + 1)/2$.

Proof: Consider the smallest rectangular area enclosing the g isolated good nodes and their surrounding f faulty nodes. To maximize g, this rectangular area must be at one corner of the mesh. If it is not, we can shift it to one corner, thus causing at least the same number of good nodes to become isolated with fewer than f faults. We note that the postulated rectangular area located at one corner cannot touch the opposite sides of the mesh, since this would violate the condition $f < m_0$ (recall from Definition 2 that m_0 is the smaller of the two sides). So we have the situation depicted in Fig. 1, where the faulty nodes trace a path from one side to the neighboring side at a corner of the mesh. If g is to be maximal for the given number f of faults, the path cannot contain horizontal or vertical segments of the types shown in the upper left corner of Fig. 1. For example, the vertical segment at the upper right of the heavy line in Fig. 1 can be replaced by a diagonal segment that leads to an increase in g without affecting f. Thus, the path traced by the faulty nodes must be diagonal (see the heavy dotted line at the lower left corner of Fig. 1), leading to g being as large as $(f - 1) + (f - 2) + \ldots + 2 + 1 = f(f - 1)/2$. ▮

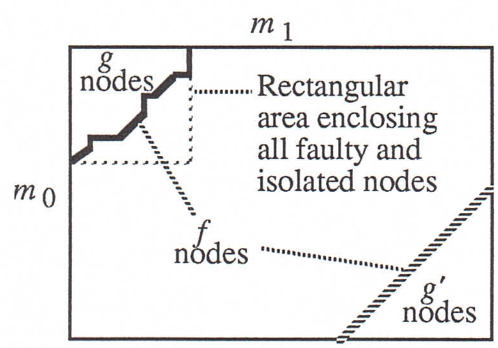

Fig. 1. Faulty nodes (heavy lines) isolating good nodes at corners of a 2D mesh.

Corollary 1: In an $m_0 \times m_1$ mesh with f faults, $f < m_0$, a connected subnetwork of size greater than $p/2$ is guaranteed to exist.

Proof: Immediate upon noting that $f \le m_0 - 1 \le \sqrt{p} - 1$ and that the number of faulty plus isolated good nodes satisfies $f(f+1)/2 \le (\sqrt{p} - 1)\sqrt{p} < p/2$. ∎

We thus see that it indeed takes a large number of faults to render the surviving connected portion of a 2D mesh too small to be of practical use; besides, such a worst-case pattern of faults is quite unlikely. To be more precise, the probability that $f(f-1)/2$ good nodes become unavailable due to f faults is no greater than $4(p-f)!/p!$ (approximated by $4/p^f$ for $f << p$).

As a practical matter, it may be advantageous to purposely disable certain good nodes of a mesh even though the faults do not completely isolate them. For example, if in Fig. 1, all but one of the processors on the heavy dotted line are faulty, the good nodes in the lower right corner will be connected to the rest of the mesh via a single node. The performance of any communication-intensive algorithm, such as all-to-all broadcast or total exchange will be severely degraded by the limited bandwidth into and out of the corner area. In such a case, it will likely be advantageous to let the diagnosis and reconfiguration facilities deactivate the weakly connected processors, thus sacrificing their processing power for the greater good of efficient systemwide communication. Such processors are thus effectively viewed as being faulty. This again points to the need for studying the effects of random and/or worst-case faults beyond the artificially imposed limit $f \le c(G) - 1$.

Note that 2D meshes represent the worst cases among the architectures of practical interest in terms of the number of nodes that can be isolated by f faulty nodes; the situation only improves once we go to torus networks and higher-dimensional meshes. This can be viewed as further evidence for the importance of extended-fault diameter in the case of mesh networks. As an example, the analog of Theorem 1 for 2D torus networks is given below.

Theorem 2: In an $m_0 \times m_1$ torus, the occurrence of f faults, $f < 2m_0$, will make at most $f^2/16$ good nodes unavailable, thus limiting the total number of unusable nodes to at most $f(f/16 + 1)$.

Proof: The proof is similar to that of Theorem 1, but there is no "corner". Consider a region of g good nodes isolated from the rest of the torus by f faulty nodes that surround them. It is easy to see that if g is to be maximized for a given f, the boundary defined by the faulty nodes cannot contain vertical or horizontal line segments involving three or more processors. This is true because it is always possible to replace such segments by diagonal segments that increase g without affecting f. The dotted segment in Fig. 2 represents an example. Thus, provided that f is a multiple of four, the isolated region will be in the shape of a diamond with $f/4 + 1$ faulty nodes on each of its four sides and $(f/4)^2$ nodes inside. Augmenting the above with the observation that for f not a multiple of four, g will be less than $(f/4)^2$ completes the proof. ∎

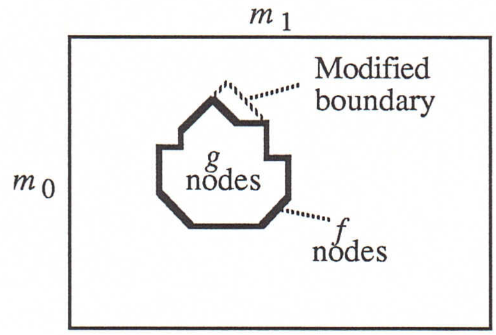

Fig. 2. Faulty nodes (heavy lines) isolating good nodes in a 2D torus.

Corollary 2: In an $m_0 \times m_1$ torus with f faults, $f < 2m_0$, a connected subnetwork of size greater than $3p/4 - 7\sqrt{p}/4$ is guaranteed to exist.

Proof: Immediate upon noting that $f \le 2m_0 - 1 \le 2\sqrt{p} - 1$ and that the number of faulty plus isolated good nodes satisfies $f(f/16 + 1) \le (2\sqrt{p} - 1)(\sqrt{p}/8 + 15/16) < p/4 + 7\sqrt{p}/4$. ∎

3. Results for Mesh Networks

In this section, we derive bounds on the extended fault diameter of mesh networks. We begin by considering the case of 2D meshes and then show how the results can be extended to higher-dimensional meshes.

Theorem 3: The extended fault diameter of an $m_0 \times m_1$ mesh with f faulty nodes ($f \ge 1$) is at most $m_0 + m_1 + 2f - 4$; i.e., no more than $2(f-1)$ hops over the diameter of the fault-free mesh.

Proof: We prove the desired upper bound by induction on m_0 and m_1. Clearly, the result holds for a mesh with $m_0 = 1$, $m_1 = 1$, or $m_0 = m_1 = 2$, since in these cases, the diameter does not increase with faults. Using the induction hypothesis that the diameter bound $m_0 + m_1 + 2\phi - 6$ holds for any $(m_0 - 1) \times (m_1 - 1)$ mesh having ϕ faults ($m_0 \ge 2$, $m_1 \ge 2$, $\phi \le f$), we show that the claimed upper bound holds for the $m_0 \times m_1$ mesh with f faults.

To establish the required diameter bound, we consider the length of the shortest path between an arbitrary pair of nodes A and Z and prove that it satisfies the given bound. Unless A and Z are on opposite sides on the boundary of the $m_0 \times m_1$ mesh, they will be within an $(m_0 - 1) \times (m_1 - 1)$ submesh. Without loss of generality, we assume that in this latter case, A and Z are within the submesh that excludes row $m_0 - 1$ and column $m_1 - 1$ from the original $m_0 \times m_1$ mesh. The proof then consists of two parts, corresponding to the aforementioned two cases.

Case (i): Assume that neither A nor Z is in row $m_0 - 1$ or column $m_1 - 1$. If A and Z are connected in the $(m_0 - 1) \times (m_1 - 1)$ submesh, then, by our induction hypothesis, the length of the shortest path between them is at most $m_0 + m_1 + 2\phi - 6$, which is clearly less than $m_0 + m_1 + 2f - 4$. Now, suppose that A and Z are not connected in the $(m_0 - 1) \times (m_1 - 1)$

submesh. So, the shortest path between A and Z must contain $h \geq 1$ "dips" into row $m_0 - 1$ and/or column $m_1 - 1$, as shown in Fig. 3a, where the zigzag lines represent arbitrary rectilinear paths. If there are several such shortest paths, we take the one for which the number h of dips is minimal. Each group of nodes in a shaded segment within row $m_0 - 2$ or column $m_1 - 2$ contains at least one fault, for a total of at least h faults in row $m_0 - 2$ and column $m_1 - 2$. Similarly, each group of nodes in a hatched segment within row $m_0 - 1$ or column $m_1 - 1$ contains at least one fault, for a total of $h - 1$ faults or more in row $m_0 - 1$ and column $m_1 - 1$. Note that the existence of a fault in the L-shape hatched segment in the lower right corner of Fig. 3a is due to the fact that we have assumed a minimal number of dips in the shortest path shown.

If we replace all faulty nodes in the shaded parts of row $m_0 - 2$ (column $m_1 - 2$) with fault-free nodes, and at the same time replace any fault-free north (west) neighbor of such nodes with faulty nodes, the total number of faults in the $(m_0 - 1) \times (m_1 - 1)$ submesh will not increase. This transformation establishes a path between A and Z that does not visit row $m_0 - 1$ or column $m_1 - 1$ (instead it goes through the now fault-free shaded areas) and is $2h$ hops shorter than the one shown. This new path must be a shortest path in the transformed $(m_0 - 1) \times (m_1 - 1)$ submesh which contains no more than ϕ faults, since due to replacing the north (west) neighbors of the transformed faulty nodes by new faulty nodes, no new shorter path can be created. By the induction hypothesis, the length of this path is at most $m_0 + m_1 + 2\phi - 6$. Thus, the length of the path shown in Fig. 3a is no more than $m_0 + m_1 + 2\phi - 6 + 2h = m_0 + m_1 + 2(\phi + h - 1) - 4$ which is in turn no more than $m_0 + m_1 + 2f - 4$.

To conclude the proof for case (i), we must deal with a complication that arises from the fact that the fault-free node to the north (west) of the faulty node F in the transformation discussed above may be one of the nodes on the shortest path (see node G in Fig. 3b). In this case of "nested" dips, we can apply the transformation to each of the dips in turn, beginning with the innermost dip, to create the required shortest path of length $m_0 + m_1 - 6 + 2\phi$ or less.

Case (ii): If A and Z cannot be enclosed in an $(m_0 - 1) \times (m_1 - 1)$ submesh, then they must be on opposite sides on the boundary of the mesh (perhaps one or both being in a corner). Without loss of generality, let A be in row $m_0 - 1$ and Z in row 0. Furthermore, assume that the path from A to Z goes through node B, the neighbor of A in Row $m_0 - 2$ and that Z is not in column $m_1 - 1$ (proofs for other cases are similar). As in case (i), we consider two subcases. If B and Z are connected in the $(m_0 - 1) \times (m_1 - 1)$ submesh that excludes row $m_0 - 1$ and column $m_1 - 1$, then, by our induction hypothesis, the length of the shortest path between them is at most $m_0 + m_1 - 6 + 2\phi$. The shortest path from A to Z, which is at most one hop longer than that from B to Z, thus has a length of at most $m_0 + m_1 - 5 + 2\phi$, which is less than $m_0 + m_1 - 4 + 2f$. Now, suppose that B and Z are not connected in the $(m_0 - 1) \times (m_1 - 1)$ submesh. So, the shortest path between them must go through row $m_0 - 1$ and/or column $m_1 - 1$, as depicted in Fig. 3c. Based on the proof of case (i), the shortest path from B to Z has a length of at most $m_0 + m_1 - 4 + 2(\phi + h - 1)$. The shortest path from A to Z, therefore, is no longer than $m_0 + m_1 - 5 + 2(\phi + h)$ hops. The proof is complete upon noting that each of the hatched areas in row $m_0 - 1$ and column $m_1 - 1$ must contain at least one fault, for a total of at least h faults in row $m_0 - 1$ and column $m_1 - 1$. ∎

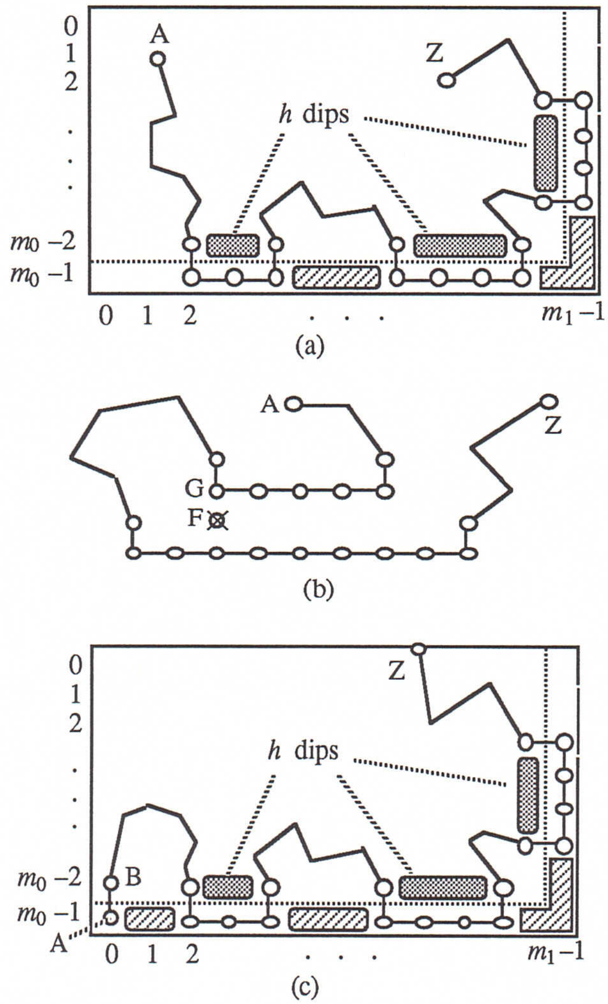

Fig. 3. Examples for the two cases in the proof of Theorem 2.

Theorem 4: The bound $m_0 + m_1 + 2f - 4$ of Theorem 3 is tight in that there exist faulty meshes whose diameters exactly match the given bound for suitably chosen fault patterns.

Proof: To show that the bound $m_0 + m_1 - 4 + 2f$ is tight, consider the example in Fig. 4a which corresponds to an $m \times m$ mesh with $f = m - 1$ faults on one of its diagonals. The diametral path between A and Z is of length $4m - 6$; i.e., $2m - 4 = 2(f - 1)$ hops more than the diameter of the fault-free mesh. The bound is also tight for $f = 1$, but for other small values of f, the bound is never reached. However, it is easy to construct examples where the diameter is increased by $f - 1$ as opposed to $2(f - 1)$. One class of examples include fault patterns that diagonally cut a corner of the mesh, leaving just one edge processor on that cut fault-free. Other degenerate fault patterns exist that raise the diameter to become asymptotically close to the given bound. The example depicted in Fig. 4b is a $(6l + 3) \times (6l + 3)$ mesh with diagonally aligned faults and a snakelike path. The length of the snakelike path, which is clearly the unique shortest path between A and Z, is $24l^2 + 16l + 6$; i.e. $24l^2 + 4l + 2$ hops more than the fault-free diameter $12l + 4$. The number of faults is $f = 12l^2 + 8l + 2$. So, in this case, the increase in diameter is $2f - O(\sqrt{f})$. ∎

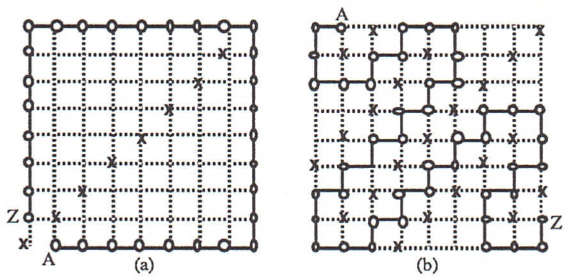

Fig. 4. Examples demonstrating that the bound of Theorem 3 is tight.

For a qD mesh, $q > 2$, the situation improves. The following theorem provides a bound on the extended fault diameter of a qD mesh for $q \geq 2$. Even though the bound is not tight for $q > 2$, it does show that the increase in diameter due to faults becomes less of a problem as q increases.

Theorem 5: The extended fault diameter of an $m_0 \times m_1 \times ... \times m_{q-1}$ mesh with f faulty nodes ($q \geq 2$, $f \geq q - 1$) is upper bounded by $m_0 + m_1 + ... + m_{q-1} + 2f/(q - 1) - q - 2$; i.e., no more than $2f/(q - 1) - 2$ hops larger than the diameter of the fault-free qD mesh.

Proof outline: The proof technique of Theorem 3 can be applied with suitable generalizations. The key observations are: (1) The analogs of row $m_0 - 1$ and column $m_1 - 1$ of Fig. 3 in the generalized construction are $(q - 1)$D meshes resulting from fixing one of the coordinates at its maximum value, and (2) The shaded and hatched regions, again $(q - 1)$D meshes, must each contain at least $q - 1$ faults, where $q - 1$ is the connectivity of a $(q - 1)$D mesh. ∎

4. Conclusion

We introduced the notion of extended-fault diameter for interconnection networks and established exact values and upper bounds for it in the case of mesh networks. We argued that extended-fault diameter is a natural and useful extension of the notion of fault diameter because it eliminates the latter's restriction to an unrealistically small set of faults, especially for the practically important case of low-dimensional processor arrays (e.g., to at most one fault for 2D mesh and 3 faults for 2D torus). Clearly, the possibility of multiple faults in a system with many thousands of processors cannot be totally dismissed.

To summarize our results for meshes, we now know the following about the extended-fault diameter of a 2D mesh network with sides m_0 and m_1, $m_0 \leq m_1$:

1. The diameter is unaffected for $f = 1$.
2. It can increase by $f - 1$ (or more?) for $2 \leq f < m_1$.
3. It can increase by $2(f-1) - (m_1 - m_0)$ (or more?) for $m_0 - 1 \leq f < m_1$. This is based on the fault pattern in Fig. 4a, folded downward and extended at its right.
4. It can increase by at most $2(f - 1)$ for $m_0 - 1 \leq f$.

We also know that the impact of faults on diameter becomes less severe as the number q of dimensions increases. In particular, the diameter increase is upper-bounded by $2f/(q - 1) - 2$ for a qD mesh. For 2D meshes, there are gaps in our knowledge of how the extended-fault diameter varies as the number f of faults increases. There is room for improvement in tightening some of the bounds.

Further research is required to apply the new notion of extended-fault diameter to other parallel processing networks. We are now working on the extended-fault diameter of torus networks, as well as pruned torus networks [Kwai97] derived by systematically removing certain links from complete tori.

References

[Day97] Day, K. and A.E. Al-Ayyoub, "Fault Diameter of k-ary n-cube networks", *IEEE Trans. Parallel and Distributed Systems*, Vol. 8, pp. 903-907, Sep. 1997.

[Esfa89] Esfahanian, A.H., "Generalized Measures of Fault Tolerance with Applications to N-Cube Networks", *IEEE Trans. Computers*, Vol. 38, pp. 1586-1591, Nov. 1989.

[Kris87] Krishnamoorthy, M.S. and B. Krishnamurthy, "Fault Diameter of Interconnection Networks", *Computers & Mathematics with Applications*, Vol. 13, pp. 577-582, 1987.

[Kwai97] Kwai, D.-M. and B. Parhami, "A Class of Fixed-Degree Cayley-Graph Interconnection Networks Derived by Pruning k-ary n-cubes", *Proc. Int'l Conf. Parallel Processing*, Aug. 1997, pp. 92-95.

[Lati93a] Latifi, S., "Combinatorial Analysis of the Fault Diameter of the n-Cube", *IEEE Trans. Computers*, Vol. 42, pp. 27-33, Jan. 1993.

[Lati93b] Latifi, S., "On Fault Diameter of the Star Graph", *Information Processing Letters*, Vol. 46, pp. 143-150, 1993.

[Parh93] Parhami, B., "Fault Tolerance Properties of Mesh-Connected Parallel Computers with Separable Row/Column Buses", *Proc. 36th Midwest Symp. Circuits and Systems*, Aug. 1993, pp. 1128-1131.

[Parh95] Parhami, B. and C.Y. Hung, "Robust Shearsort on Incomplete Bypass Meshes", *Proc. 9th Int'l Parallel Processing Symp.*, Apr. 1995, pp. 304-311.

[Parh98] Parhami, B. and C.-H. Yeh, "The Robust Algorithm Approach to Fault Tolerance on Processor Arrays: Fault Models, Fault Diameter, and Basic Algorithms", *Proc. Joint Parallel Processing Symp. and the Symp. on Parallel and Distributed Processing*, 1998, pp. 742-746.

[Parh99] Parhami, B., *Introduction to Parallel Processing: Algorithms and Architectures*, Plenum Press, 1999.

[Yeh97] Yeh, C.-H. and B. Parhami, "Optimal Sorting Algorithms on Incomplete Meshes with Arbitrary Fault Patterns", *Proc. Int'l Conf. Parallel Processing*, Aug. 1997, pp. 4-11.