Correction to "Periodically Regular Chordal Rings"

Behrooz Parhami, *Fellow*, *IEEE*, and Ding-Ming Kwai

THE following are corrections to this paper which appeared in the June 1999 issue. We clarify them here since the errors make statements which may appear invalid or difficult to understand.

In the statement of Theorem 2, $g = 2$ should read $g \geq 2$:

Theorem 2. *The diameter D of an N-node PRC ring with group size* $g \geq 2$ *and skip distances* $s_1, s_2, ..., s_g$, *such that each* s_{h+1} *is divisible by* s_h, $1 \leq h \leq g$, *is exactly equal to* $\sum_{h=0}^{g} s_{h+1}/s_h - 3$.

In the second paragraph of the proof, $g \leq 2$ should read $g \geq 2$:

Proof. The bound for D_{Alg3} given above clearly shows that, when each s_{h+1} is divisible by s_h, any node can be reached in no more than $\sum_{h=0}^{g} s_{h+1}/s_h - 2$ steps. Hence;

$$D \leq \sum_{h=0}^{g} s_{h+1}/s_h - 2.$$

However, for $g \geq 2$, a special situation arises for skips s_1 and s_0. Consider the final part of the route starting with the transition from s_2-type steps to s_1-type steps:

$$\begin{array}{ccc} s_0 & \underbrace{s_1\ s_1 \ldots s_1}_{x_1 \text{ steps}} & s_0 & \underbrace{s_0\ s_0 \ldots s_0}_{x_0 \text{ steps}} \end{array}.$$

If $x_1 = s_2/s_1 - 1$ and $x_0 = s_1 - 2$, as discussed in our earlier worst-case analysis, the steps shown above add up to:

$$1 + s_1(s_2/s_1 - 1) + 1 + 1(s_1 - 2) = s_2.$$

Thus, the worst-case values for x_1 and x_0 cannot occur simultaneously and we can write:

$$D \leq \sum_{h=0}^{g} s_{h+1}/s_h - 3.$$

The proof is complete upon noting that the distance from node 0 to node $N - s_1 + g - 2$ is exactly $\sum_{h=0}^{g} s_{h+1}/s_h - 3$. □

In the leftmost column of Table 1, the number 1024 was misaligned. It is corrected here.

In the proof of Theorem 3, the appearance of uppercase "delta" in the second paragraph should be lowercase:

Proof. Given the group length g, there are $n - \log_2 g$ permissible powers of 2 from which the g skip distances s_h, $1 \leq h \leq g$, can be selected. These are $2^{n-1}, 2^{n-2}, \ldots, 2^{\log_2 g} = g$. Thus, we need to have:

$$n - \log_2 g \geq g.$$

It is then easy to see that the group length g satisfies:

$$g + \log_2 g = n - \varepsilon \text{ where } 0 \leq \varepsilon \leq g.$$

Let $2^{m_h} = s_{h+1}/s_h$, $0 \leq h \leq g$, be the hth skip ratio. Recall that $s_{g+1} = 2^n$ and $s_0 = 1$. We then have:

$$m_0 = \log_2 g + \delta \quad \text{where } \delta \geq 0$$

$$\sum_{h=0}^{g} m_h = n$$

$$D = \sum_{h=0}^{g} 2^{m_h} - 3 \quad \text{(by Theorem 2)}.$$

Since $m_0 \leq n - g = \log_2 g + \varepsilon$, we have $0 \leq \delta \leq \varepsilon$. Select the remaining m_h values, $1 \leq h \leq g$, such that $m_h = 2$ in $\varepsilon - \delta$ cases and $m_h = 1$ in the remaining $g - \varepsilon + \delta$ cases. It is easily verified that:

$$n = \sum_{h=0}^{g} m_h = g + \log_2 g + \varepsilon$$

$$D = \sum_{h=0}^{g} 2^{m_h} - 3 = (2^{\delta} g - 2\delta) + 2g + 2\varepsilon - 3.$$

The diameter is minimized for $\delta = 0$, leading to:

$$D = 3g + 2\varepsilon - 3 = 2n + g - 2\log_2 g - 3.$$

The proof is complete upon showing that reduction of the group length to $g/2$ does not improve the diameter relative to the above. As shown earlier, a $2^{n-g/2-1}$-node PRC ring with group length $g/2$ has the minimized diameter:

$$2(n - g/2 - 1) + g/2 - 2(\log_2 g - 1) - 3.$$

The factor of $2^{g/2+1}$ increase in the number of nodes leads to all $g/2 + 1$ skip ratios 2^{m_h} being multiplied by 2 (the m_h values being incremented by 1). The diameter then becomes

$$\begin{aligned} D' &= 2[2(n - g/2 - 1) + g/2 - 2(\log_2 g - 1)] - 3 \\ &= 4n - g - 4\log_2 g - 3 = D + 2\varepsilon, \end{aligned}$$

which is no less than that obtained for group length g. □

The equation in the second to last line of the proof of the corollary to Theorem 4 should read:

Proof. From $s_g \geq 2s_{g-1} \geq \ldots \geq 2^{g-1}s_1$, we conclude that $s_h \leq s_g/2^{g-h}$. Substituting this upper bound for s_h in the upper bound for B given by Theorem 4, we get:

$$B \leq 2 + 2(2 - 1/2^{g-1})s_g/g.$$

The preceding inequality, combined with $s_g/g \geq 2^{g-1}$, yields $B \leq 4s_g/g$. □

• *The authors are with the Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106-9560. E-mail: parhami@ece.ucsb.edu.*

For information on obtaining reprints of this article, please send e-mail to: tpds@computer.org, and reference IEEECS Log Number 100341.

TABLE 1
Minimum Diameter D_{opt} and the Associated Average Internode Distance $\Delta_{\mathrm{opt}D}$ for PRC Rings with Different Power-of-2 Sizes *N* and Group Lengths *g*

N	g	D_{opt}	$\Delta_{\mathrm{opt}\,D}$	s_h $(1 \le h \le g)$
8	2	3	2.0	2, 4
16	2	4	2.7	4, 6
32	2	6	3.6	6, 14
	4	6	3.6	4, 8, 12, 16
64	2	8	4.9	6, 20
	4	7	4.4	8, 20, 24, 28
128	2	10	6.4	40, 60
	4	8	5.4	16, 28, 36, 60
	8	10	6.1	8, 16, 24, 32, 40, 48, 56, 64
256	2	14	8.2	106, 116
	4	10	6.5	32, 68, 76, 116
	8	11	6.9	24, 32, 56, 72, 80, 104, 112, 120
512	4	12	7.7	36, 76, 168, 200
	8	12	7.8	8, 56, 88, 152, 160, 184, 200, 224
1024	2	22	13.5	252, 458
	4	14	9.0	212, 320, 344, 436
	8	13	8.7	208, 216, 264, 344, 376, 400, 464, 504