Compressive Cooperative Obstacle Mapping in Mobile Networks

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***Abstract*—In this paper we consider a mobile cooperative network that is tasked with building an obstacle map in an environment. We propose a framework that allows the robots to build the obstacle map non-invasively and with a small number of wireless channel measurements. By extending our previous work on sparse obstacle mapping, we show how the nodes can exploit the sparse representation of the map in order to build it with minimal sensing. The proposed work allows the robots to efficiently map an area before entering it. We propose two approaches based on random and coordinated wireless measurements. Our simulation and experimental results show the superior performance of the proposed framework.**

I. Introduction

Mobile intelligent networks can play a key role in emergency response, surveillance and security, and battlefield operations. The vision of a multi-agent robotic network cooperatively learning and adapting in harsh unknown environments to achieve a common goal is closer than ever. An important issue key to the robust operation of a mobile robotic network is the accurate mapping of the obstacles. In most related work, however, only areas that are directly sensed are mapped. The rich literature on Simultaneous Localization and Mapping (SLAM) fall into this category [1], [2]. Similarly, approaches based on generating an occupancy map also address reducing sensing uncertainty [3]. However, areas that are not sensed directly are not mapped. In general, there is currently no framework for cooperative obstacle mapping, based on a very small number of measurements.

Consider an obstacle map. By an obstacle map, we refer to a 2D (or 3D) map of the environment where we have zeros at locations where there is no obstacle and non-zero values at obstacle locations. Each non-zero value could be one to indicate the presence of an obstacle or could be the decay rate of the wireless signal within the object at that location, as we will see in the next section. By obstacle mapping, we then refer to a number of robots generating a spatial map of the obstacles. In this paper, we are interested in **non-invasive obstacle mapping**. By non-invasive mapping, we refer to the mapping of the obstacles without sensing them directly. Consider the case where a number of robots want to build a map of the obstacles inside a building before entering it. Non-invasive mapping allows the nodes to assess the situation before entering the building and can be of particular interest in several applications such as an emergency response or battlefield operation. In general, devising non-invasive mapping strategies can be considerably challenging.

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In our previous work [4], we proposed a framework for compressive obstacle mapping based on wireless channel measurements. In this paper, we extend our previous work to consider cooperative mapping based on both random and coordinated measurements. We furthermore show the performance of our proposed framework with an experimental setup, in which robots map a real obstacle on the campus of the University of New Mexico. A survey of the existing work in the literature shows very few works with a similar flavor. There are a number of radar-based approaches for imaging [5], [6], which are completely different from the proposed method. In [7] and [8], the authors build a network of several hundred fixed sensors in order to detect the presence of a person. The framework is based on making several measurements between pairs of sensors. Then the goal is to roughly track a person as opposed to building a map of obstacles. There is a need for prior learning in the area of interest as well. As such, those papers are more on detecting an obstruction to a wireless signal as opposed to obstacle mapping with minimal measurements.

To the best of authors' knowledge, aside from authors' work, there is currently no framework for cooperative non-invasive mapping of real obstacles, with minimal measurements. Our proposed theory and design tools are inspired by the recent breakthroughs in non-uniform sampling theory [9]. The new theory of *compressive sampling* (also known by other terms such as compressed sensing, compressive sensing or sparse sensing) shows that under certain conditions, it is possible to reconstruct a signal from a considerably incomplete set of observations, i.e. with a number of measurements much less than predicted by the Nyquist-Shannon theorem [9]. This opens new and fundamentally different possibilities in terms of information gathering and processing in mobile networks.

In this paper, we develop the fundamentals of cooperative non-invasive obstacle mapping in mobile networks from a compressive sampling perspective and based on only wireless channel measurements. By utilizing the sparse representation of the map in space or in spatial variations, we show how the vehicles can solve for the map cooperatively, based on minimal measurements, and more importantly in a non-invasive manner. More specifically, we propose two cases of 1) wireless random measurements and 2) wireless coordinated measurements, where the latter can result in the sparse sampling of the frequency transformation of the obstacle map. We show the superior performance of the proposed framework through both simulation and experiment. Overall, our framework is a new multidisciplinary approach that integrates robotics and wireless communications for efficient non-invasive obstacle mapping, based on wireless channel measurements.

II. An Overview of Compressive sampling theory [9]–[11]

The new theory of sampling is based on the fact that real-world signals typically have a sparse representation in a certain transformed domain. Consider a scenario where we are interested in recovering a vector $x \in \mathbb{R}^N$. For 2D signals, vector x can represent the columns of the matrix of interest stacked up to form a vector. Let $y \in \mathbb{R}^K$ where $K \ll N$ represent the incomplete linear measurement of vector x obtained by the sensors. We will have

$$y = \Phi x, \tag{1}$$

where we refer to Φ as the observation matrix. Clearly, solving for x based on the observation set y is an ill-posed problem as the system is severely under-determined ($K \ll N$). However, suppose that x has a sparse representation in another domain, i.e. it can be represented as a linear combination of a small set of vectors:

$$x = \Gamma X, \tag{2}$$

where Γ is an invertible matrix and X is S-sparse, i.e. $|\text{supp}(X)| = S \ll N$ where $\text{supp}(X)$ refers to the set of indices of the non-zero elements of X and $|\cdot|$ denotes its cardinality. This means that the number of non-zero elements in X is considerably smaller than N. Then we will have

$$y = \Psi X, \tag{3}$$

where $\Psi = \Phi \times \Gamma$. If $S \leq K$ and we knew the positions of the non-zero coefficients of X, we could solve this problem with traditional techniques like least-squares. In general, however, we do not know anything about the structure of X except for the fact that it is sparse (which we can validate by analyzing similar data). The new theory of compressive sensing allows us to solve this problem.

Theorem 1 (see [9] for details and the proof): If $K \geq 2S$ and under specific conditions, the desired X is the solution to the following optimization problem:

$$\min ||X||_0, \text{ such that } y = \Psi X, \tag{4}$$

where $||X||_0 = |\text{supp}(X)|$ represents the zero norm of vector X.

Theorem 1 states that we only need $2 \times S$ measurements to recover X and therefore x fully. This theorem, however, requires solving a non-convex combinatorial problem, which is not practical. For over a decade, mathematicians have worked towards developing an almost perfect approximation to the ℓ_0 optimization problem of Theorem 1 [12], [13]. Recently, such efforts resulted in several breakthroughs.

More specifically, consider the following ℓ_1 relaxation of the aforementioned ℓ_0 optimization problem:

$$\min ||X||_1, \text{ subject to } y = \Psi X. \tag{5}$$

Theorem 2: (see [9], [10]) Assume that X is S-sparse. The ℓ_1 relaxation can exactly recover X from measurement y if matrix Ψ satisfies the Restricted Isometry Condition for $(2S, \sqrt{2} - 1)$, as described below.

Restricted Isometry Condition (RIC) [11]: Matrix Ψ satisfies the RIC with parameters (Z, ϵ) for $\epsilon \in (0, 1)$ if

$$(1 - \epsilon)||c||_2 \leq ||\Psi c||_2 \leq (1 + \epsilon)||c||_2 \tag{6}$$

for all Z-sparse vector c. Other conditions and extensions of Theorem 2 have also been developed [14], [15]. While it is not possible to define all the classes of matrices Ψ that satisfy RIC, it is shown that random partial Fourier matrices [16] as well as random Gaussian [17]- [18] or Bernoulli matrices [19] satisfy RIC (a stronger version) with the probability $1 - O(N^{-M})$ if

$$K \geq B_M S \times \log^{O(1)} N, \tag{7}$$

where B_M is a constant, M is an accuracy parameter and O($\cdot$) is Big-O notation [9]. Eq. 7 shows that the number of required measurements could be considerably less than N.

While the recovery of sparse signals is important, in practice signals may rarely be sparse. Most signals, however, will be compressible, i.e. most of signal's energy is in very few coefficients. In practice, the observation vector y will also be corrupted by noise. The ℓ_1 relaxation and the corresponding required RIC condition can be easily extended to the case of noisy observations with compressible signals [20].

A. Basis Pursuit (BP): Reconstruction Using ℓ_1 Relaxation

The ℓ_1 optimization problem of Eq. 5 can be posed as a linear programming problem [21]. The compressive sensing algorithms that reconstruct the signal based on ℓ_1 optimization are typically referred to as "Basis Pursuit" [10]. Reconstruction through ℓ_1 optimization has the strongest known recovery guarantees [11]. The ℓ_1 magic toolbox [22] provides several optimization tools for solving the aforementioned ℓ_1 relaxation and its variations. The computational complexity, however, can be high, especially when dealing with real data. SPARSA [23], GPSR [24] and AC [25] are a few examples of the continuing attempts to reduce the computational complexity of the convex relaxation approach. Overall, we found SPARSA to be more computationally efficient yet effective in solving this problem, especially when dealing with real data.

B. Matching Pursuit (MP): Reconstruction using Successive Interference Cancellation

While the ℓ_1 relaxation of the previous part can solve the compressive sampling problem with performance guarantees, its computational complexity can be high, as mentioned above. Alternatively, there are greedy approaches that can solve the compressive sampling problem more efficiently, at the cost of a (possibly slight) loss of performance. Next, we summarize such approaches.

The Restricted Isometry Condition implies that the columns of matrix Ψ should have a certain near-orthogonality property. Let $\Psi = [\Psi_1 \Psi_2 \dots \Psi_N]$, where Ψ_i represents the i^{th} column of matrix Ψ. We will have $y = \sum_{j=1}^{N} \Psi_j X_j$, where X_j is the j^{th} component of vector X. Consider recovering X_i:

$$\frac{\Psi_i^H y}{\Psi_i^H \Psi_i} = \underbrace{X_i}_{\text{desired term}} + \underbrace{\sum_{j=1, j\neq i}^{N} \frac{\Psi_i^H \Psi_j}{\Psi_i^H \Psi_i} X_j}_{\text{interference}} . \tag{8}$$

If the columns of Ψ were orthogonal, then Eq. 8 would have resulted in the recovery of X_i. For an under-determined system, however, this will not be the case. Then there are two factors affecting recovery quality based on Eq. 8. First, how orthogonal is the i^{th} column to the rest of the columns and second how strong are the other components of X. In other words, it is desirable to first recover the strongest component of X, subtract its effect from y, recover the second strongest component and continue the process. Adopting the terminology of CDMA (Code Division Multiple Access), we refer to such approaches as *Successive Interference Cancellation* [4]. In fact, if $X_i \neq 0$, one can think of Ψ_i coding X_i. If the i^{th} code is used as in Eq. 8, then ideally X_j for $j \neq i$ can not be decoded properly and only X_i can be recovered.

Recently, Tropp et al. independently proposed using a version of successive interference cancellation in the context of compressive sampling and derived the conditions under which it can result in almost perfect recovery [26]. They refer to it as Orthogonal Matching Pursuit (OMP). Similar to Successive Interference Cancellation, the basic idea of OMP is to iteratively multiply the measurement vector, y, by Ψ^H, recover the strongest component, subtract its effect and continue again. A variation of OMP, Regularized Orthogonal Matching Pursuit (ROMP), was later introduced by Needell et al. [11]. The main difference in ROMP as compared to OMP is that in each iterative step, a set of indices (locations of vector X with non-negligible components) are recovered at the same time instead of only one at a time, resulting in a faster recovery [11]. Other variations of this work have also appeared.

C. Reconstruction Using Total Variation (TV) Minimization

The spatial variations of the map (gradient) are also considerably sparse. Thus, another related sparsity-based reconstruction approach is to use the sparsity in the gradient [9], [22], [27]. Let $f = [f_{i,j}]$ denote an $m \times m$ matrix that represents the spatial function of interest. Define the following operators: $D_{h,i,j}(f) = \begin{cases} f_{i+1,j} - f_{i,j} & i < m \\ 0 & i = 0 \end{cases}$, $D_{v,i,j}(f) = \begin{cases} f_{i,j+1} - f_{i,j} & j < m \\ 0 & j = 0 \end{cases}$ and $D_{i,j}(f) = \begin{pmatrix} D_{h,i,j}(f) \\ D_{v,i,j}(f) \end{pmatrix}$. Then, the Total Variation function is defined as follows: $\text{TV}(f) = \sum_{ij} \sqrt{(D_{h,i,j}(f))^2 + (D_{v,i,j}(f))^2} = \sum_{ij} \|D_{i,j}(f)\|_2$. TV minimization approaches solve the following problem or a variation of it:

$$\min \text{TV}(f), \text{ subject to } y = \Psi_f \times v_f, \tag{9}$$

where v_f is a column vector that results from stacking up the columns of matrix f, and y is the observation vector, which is linearly related to v_f through matrix Ψ_f. The ℓ_1 magic toolbox provides a solver for this TV minimization problem. More recently, TVAL [27] is proposed for solving this problem more efficiently and robustly, which we will use in the next section.

III. Compressive Cooperative Mapping of Obstacles

In this section we show how a group of mobile nodes can build a map of the obstacles non-invasively. We start by summarizing our original proposed framework of [4]. We then extend that work and propose two different strategies for compressive obstacle mapping: 1) random wireless measurements and 2) coordinated wireless measurements, where the second approach can result in sampling in the frequency or space domain. Finally, and most importantly, we show the reconstruction of a real obstacle on UNM campus, using our proposed framework and two robotic units.

We consider building a 2D map of the obstacles in this paper. For instance, for real 3D structures, we reconstruct a horizontal cut of them, as shown in Section IV. It should, however, be noted that our proposed approach can also be easily extended to 3D maps. Figure 1 (both left and right) shows a sample 2D obstacle map where a number of vehicles want to map the space before entering it. Let $g_n(u, v)$ represent the binary map of the obstacles at position (u, v) for $u, v \in \mathbb{R}$. We have

$$g_n(u, v) = \begin{cases} 1 & \text{if } (u, v) \text{ is an obstacle} \\ 0 & \text{else} \end{cases} \tag{10}$$

Consider communication from Transmitter 1 to Receiver 1, as marked in Fig. 1 (right). A fundamental parameter that characterizes the performance of a communication channel is the received signal strength. There are three time-scales associated with the spatio-temporal changes of the channel quality and therefore received signal strength [28], as indicated in Fig. 2. The slowest dynamic is associated with the signal attenuation due to the distance-dependent power fall-off (path loss). Then there is a faster variation referred to as shadow fading (shadowing), which is due to the impact of the blocking objects. This means that each obstacle along the transmission path leaves its mark on the received signal by attenuating it to a certain degree characterized by its properties. Finally, depending on the receiver antenna angle, multiple replicas of the transmitted signal can arrive at the receiver due to the reflection from the surrounding objects, resulting in multipath fading, a faster variation in the received signal power [29].

A communication from Transmitter 1 to Receiver 1 in Fig. 1 (right), therefore, contains implicit information of the obstacles along the communication path. Consider the dashed ray (line) that corresponds to distance t and angle θ in Fig. 1 (right). This line is at distance t from the origin and is perpendicular to the

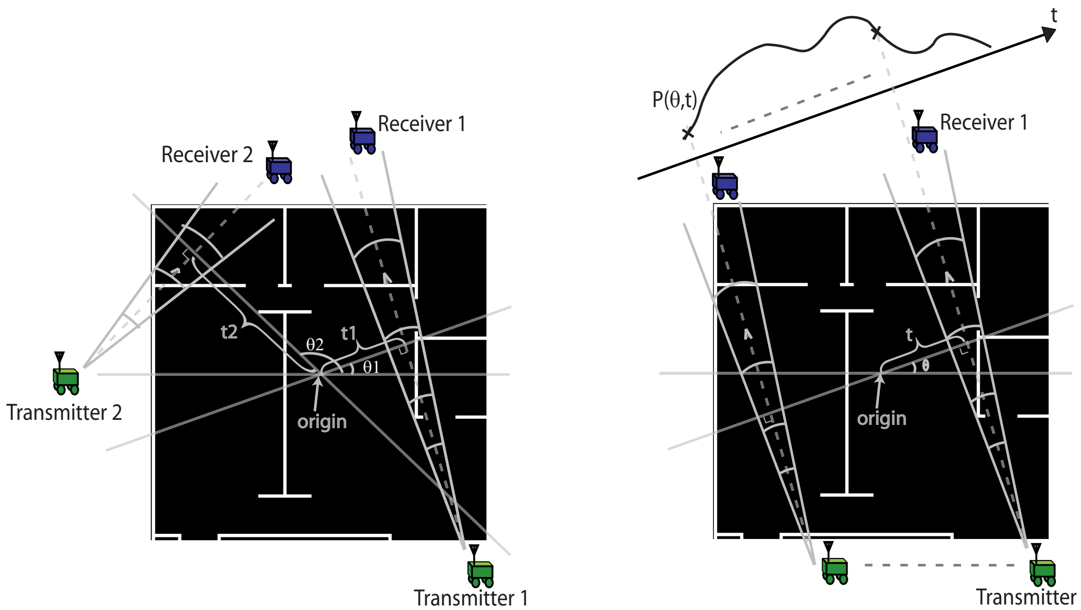

Fig. 1. An indoor obstacle map with the obstacles marked in white and the illustration of the proposed compressive cooperative mapping using (left) random and (right) coordinated wireless measurements.

line that is at angle θ with the x-axis. Let $P(\theta, t)$ represent the received signal power in the transmission along the ray that corresponds to distance t and angle θ, as shown in Fig. 1 (right). We will have [28], [29],

$$P(\theta, t) = P_s(\theta, t) w(\theta, t), \tag{11}$$

where

$$P_s(\theta, t) = \underbrace{\frac{\beta P_T}{\left(d(\theta, t)\right)^{\alpha}}}_{\text{path loss}} \times \underbrace{e^{\sum_i r_i(\theta,t) n_i(\theta,t)}}_{\text{shadowing due to obstacles}} \tag{12}$$

represents the contribution of distance-dependent path loss and shadowing. For the path loss term, P_T represents the transmitted power, $d(\theta, t)$ is the distance between the transmitter and receiver across that ray, α is the degradation exponent of the wireless signal and β is a constant that is a function of system parameters. For the shadowing (or shadow fading) term, r_i is the distance travelled across the i^{th} object along the (θ, t) ray and $n_i < 0$ is the decay rate of the wireless signal within the i^{th} object. Furthermore, the summation is over the objects across that line. As can be seen, shadowing characterizes wireless signal attenuation as it goes through the obstacles along the transmission path and therefore contains information about the objects along that line.

$w(\theta, t)$ of Eq. 11, on the other hand, is a positive random variable with unit average which models the impact of multipath fading. For more mathematical details on wireless channel modeling, readers are referred to [28]–[30]. We can then model $\ln P(\theta, t)$ as follows

$$\begin{aligned} \ln P(\theta, t) &= \underbrace{\ln P_T}_{\text{transmitted power in dB}} + \underbrace{\beta_{\text{dB}} - \alpha \ln d(\theta, t)}_{\text{path loss } (\leq 0)} \\ &+ \underbrace{\sum_i r_i(\theta, t) n_i(\theta, t)}_{\substack{\text{shadowing effect due to} \\ \text{blocking objects } (\leq 0)}} + \underbrace{w_{\text{dB}}(\theta, t),}_{\text{multipath fading}} \end{aligned} \tag{13}$$

where $\beta_{\text{dB}} = \ln \beta$ and $w_{\text{dB}} = \ln w(\theta, t)$. Then we have

$$\begin{aligned} h(\theta, t) &\triangleq \ln P(\theta, t) - \ln P_T - \left(\beta_{\text{dB}} - \alpha \ln d(\theta, t)\right) \\ &= \underbrace{\sum_i r_i(\theta, t) n_i(\theta, t)}_{\text{shadow fading effect}} + \underbrace{w_{\text{dB}}(\theta, t).}_{\text{multipath fading}} \end{aligned} \tag{14}$$

Path loss and shadowing represent the signal degradation due to the distance travelled and obstacles respectively and $w_{\text{dB}}(\theta, t)$ represents the impact of multipath fading. By using an integration over the line that corresponds to θ and t, we can express Eq. 14 as follows:

$$h(\theta, t) = \int \int_{\text{line } (\theta, t)} g(u, v) du dv + w_{\text{dB}}(\theta, t), \tag{15}$$

where

$$g(u, v) = \begin{cases} n(u, v) & \text{if } g_n(u, v) = 1 \\ 0 & \text{else} \end{cases} \tag{16}$$

with $g_n(u, v)$ representing the binary map of the obstacles (indicated by Eq. 10) and $n(u, v)$ denoting the decay rate of the signal inside the object at position (u, v) (see $n_i(\theta, t)$ in Eq. 12). $g(u, v)$ then denotes the true map of the obstacles including wireless decay rate information.

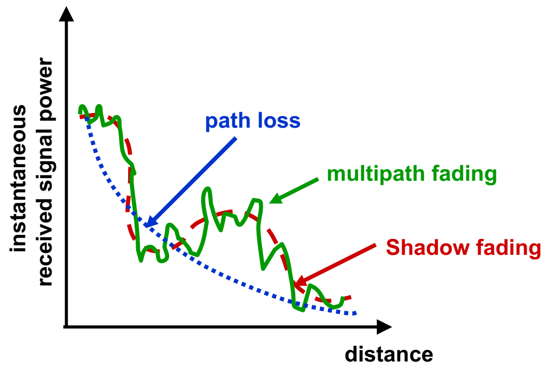

Fig. 2. A multi-scale representation of the received signal power in a wireless transmission.

A. Random Wireless Measurements

Consider Fig. 1 (left), where pairs of robots are making wireless measurements. In this case, we do not assume that the robots are attempting to have a specific pattern, i.e. the θ and t can be chosen randomly. In practice, the parameters of the path loss component of the received signal in Eq. 13 can be estimated by using a few Line Of Sight (LOS) transmissions in the same environment, as we have shown in [29]. Therefore, the impact of path loss can be removed and the receiving robot can calculate $h(\theta, t)$. Thus, for each θ and t pair, a wireless transmission and reception is made, which results in measuring a line integral of Eq. 14. Let X of Eq. 3 be the vector representation of g, where the columns are stacked up. Then vector y is the vector of the gathered samples of $h(\theta, t)$ of Eq. 14. In each row of Ψ, the non-zero elements correspond to the obstacle map pixels that the corresponding ray visited, with each non-zero value indicating the distance travelled in the corresponding pixel. We will have $y = \Psi X + e$, where e models the impact of multipath fading and measurement noise. Then, the sparsity in the spatial variations (TV) can be used for reconstruction, as we shall see later in this section.

B. Coordinated Wireless Measurements

In this section, we consider an obstacle mapping approach that is motivated by computed tomography approaches in medical imaging [31], geology, and computer graphics. For most cases, as we shall see in the next part, this approach has a better performance than the previous one. However, due to the environmental constraints, it may not always be possible to make coordinated measurements. Consider Fig. 1 (right) and the illustrated line at angle θ that passes through the origin. Two vehicles can move in a coordinated fashion such that at a given angle θ, a number of wireless channel measurements at different ts are formed. By changing t at a specific θ, a projection is formed, i.e. a set of ray integrals, as is shown in Fig. 1 (right). Clearly, this results in the immediate sampling in the space domain (similar to the random case). Then sparsity in spatial variations (TV) can be used for reconstructing the map. Alternatively, coordinated measurements can be used to acquire frequency samples, as we discuss next.

Frequency Sampling: Let $G(\theta_f, f)$ represent the 2D Fourier transform of g expressed in the polar coordinates. Let $H_t(\theta, f)$ denote the 1D Fourier transform of $h(\theta, t)$ with respect to t: $H_t(\theta, f) = \int h(\theta, t) e^{-j2\pi f t} dt$. We have the following theorem.

Fourier Slice Theorem [31]: Consider the case where there is no multipath fading in Eq. 15, i.e. $w_{\text{dB}} = 0$. Then $H_t(\theta, f)$, the Fourier transformation of $h(\theta, t)$ with respect to t, is equal to the samples of $G(\theta_f, f)$ across angle $\theta_f = \theta$.

By making a number of measurements at different ts for a given θ, the Fourier Slice Theorem allows us to measure the samples of the Fourier transform of the map g at angle θ. By changing θ, we can sample the Fourier transform of the map of the obstacles at different angles. We can then pose the problem in a compressive sampling framework. By measuring the received signal power across the rays, the vehicles can indirectly sample the Fourier transformation of the obstacle map. Then the sparsity in the spatial domain or TV can be used for reconstruction.

Let x of Eq. 1 denote the vector representation of G (2D Fourier transform of the obstacle map), where the columns of G are stacked up to form a vector. Then y represents the very few samples of G acquired using the proposed framework, i.e. wireless channel measurements across a number of rays and applying the Fourier Slice Theorem. For reconstruction based on sparsity in the spatial domain or TV, X will be the vector representation of g and Γ is the Fourier transform matrix. Φ represents a matrix with only one 1 in every row. If there are redundant measurements, there may be more than one 1 in every column. Otherwise, there will be at most one 1 in every column. Then the Ψ matrix that results from point sampling in the frequency domain and reconstruction using sparsity in the spatial domain will meet the RIC condition [16].

IV. Performance of the Proposed Obstacle Mapping Approaches

So far we proposed two approaches for obstacle mapping using random and coordinated sparse wireless measurements. We furthermore discussed different reconstruction possibilities using BP, MP and TV approaches. In this section, we show the performance of our proposed sparse obstacle mapping framework, using wireless channel measurements. We show the underlying tradeoffs between different methods. We also show the reconstruction of a real obstacle using robotic units and wireless measurements.

We start by showing the performance in a simulation environment. Fig. 3 compares the performance of the proposed random and coordinated approaches as a function of the percentage of the measurements taken in the reconstruction of a T-shaped obstacle. In this result, for the coordinated case, frequency samples are acquired using the Fourier Slice Theorem, as described earlier. The original T-shape is shown in Fig. 7 (left). Then Fig. 3 shows the normalized MSE of the reconstruction. As can be seen, the coordinated approach outperforms the random one considerably, as expected. However, at an extremely low sampling rate, it can be seen that the random approach outperforms the coordinated one. To see this more clearly, Fig. 4 shows the reconstruction of the aforementioned T-shaped obstacle map, for two different sampling rates. The top row shows the reconstruction for the case where only 0.77% measurements are taken whereas the bottom row shows the reconstruction quality for the case with 4.6% measurements. It can be seen that for the top row, the random projection can provide a recognizable reconstruction while the coordinated one can not provide any useful information. This makes sense as the coordinated approach makes measurements at only a few θs (see Fig. 1 (right)) but extensively and at different ts for each θ. As such, for an extremely small sampling rate, it only measures the obstacle map from a very small number of angles. The random approach, on the other hand, samples the map from possibly different views even at a considerably small sampling rate. If the sampling rate is not extremely small, however, the coordinated approach will outperform the random one considerably. This can be seen from Fig. 3. An example of it can also be seen from Fig. 4 (bottom row), where the coordinated approach can provide an almost perfect reconstruction with only 4.6% measurements. For this result, no multipath fading is considered.

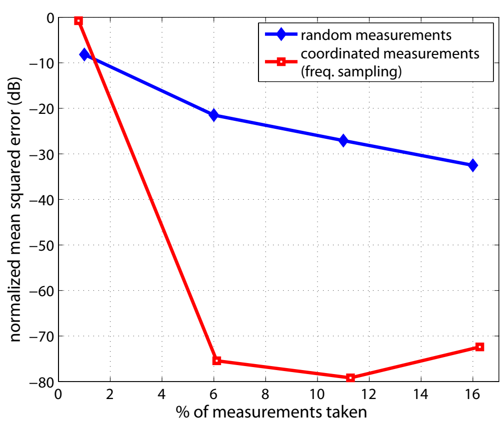

Fig. 3. Performance of the proposed sparse obstacle mapping framework using 1) random measurements and space sampling and 2) coordinated measurements and frequency sampling.

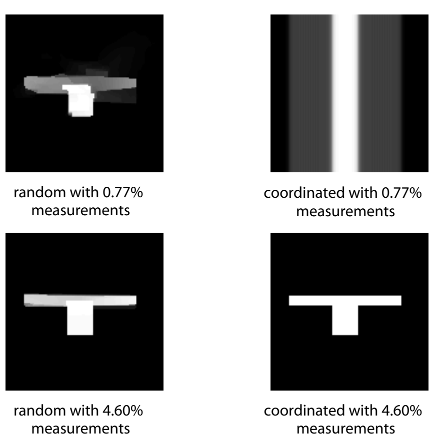

Fig. 4. Comparison of the proposed random and coordinated approaches at an extremely small (top row) and small (bottom row) sampling rates in the reconstruction of a T-shaped obstacle.

A. Compressive Reconstruction of a Real Obstacle

In the previous parts, we showed the performance of our proposed framework in a simulation environment. The aforementioned modeling of a wireless channel, however, can not possibly embrace all the propagation phenomena. As such, we do not expect perfect recovery with a very small number of measurements in a real environment. However, as long as the reconstruction is informative for the cooperative operation of the robots, it would be valuable. For instance, if it allows the vehicles to build a rough map of inside a room before entering it, it could be considerably valuable. In order to test our framework, we also built an experimental setup for cooperative obstacle mapping. In our setup, a number of robots that are equipped with transceivers, make a small number of wireless channel measurements, as proposed in this paper. Fig. 5 shows one of our robotic platforms. We tested our setup on the campus of the University of New Mexico, where two robots made a small number of wireless measurements in order to build a 2D map of the column of Fig. 6 (left). A horizontal cut (2D map) of the column has a T-shape as shown in Fig. 6 (right). Our robots then aim at reconstructing this structure based on only wireless channel measurements. We consider reconstruction in a horizontal plane, i.e. the goal is for the robots to reconstruct the horizontal cut of Fig. 6. Fig. 7 (middle) shows the reconstruction of the T-shape, based on only 9.09% coordinated wireless measurements. Earlier in this paper, we showed the performance of coordinated measurements and frequency sampling. Thus in this case, we show the reconstruction with coordinated measurements and space sampling. We note that the frequency sampling of the previous section results in a very similar reconstruction. The reconstruction is noisy as expected, due to several propagation phenomena that our modeling did not include. However, the T-shape structure can still be easily seen. To the best of authors' knowledge, this is the first time that robots have mapped a real obstacle cooperatively, based on a small number of wireless channel measurements. Fig. 7 (right) shows the case where a threshold is applied to the reconstructed middle figure such that any value that is 10dB below the maximum is zeroed. This was done because we noticed that there could be scenarios where reconstructed pixels with very small values get magnified by some printers or monitors with certain gamma settings. A simple thresholding can avoid such cases.

Fig. 5. Our robotic platform – Pioneer 3-AT robot equipped with a servo control mechanism/fixture and a directional antenna.

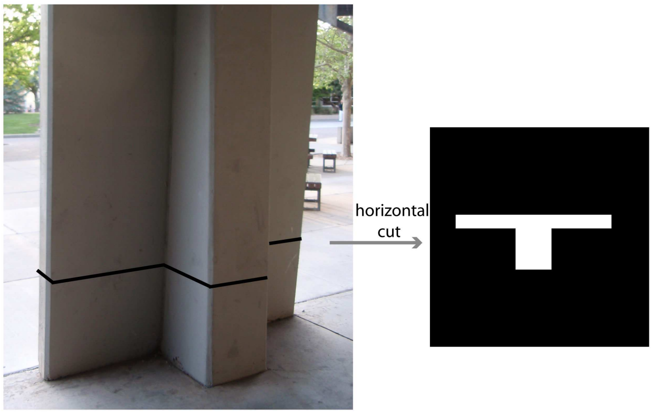

Fig. 6. (left) T-shaped column on the campus of the University of New Mexico and (right) a horizontal cut of it. Our robots reconstructed a 2D map of the column (horizontal cut), using our proposed framework.

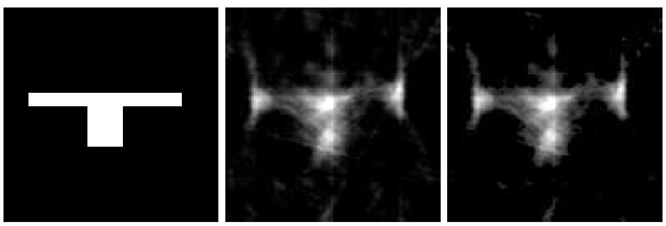

Fig. 7. Reconstruction of the T-shaped column of Fig. 6 on the campus of the University of New Mexico. The left figure shows a horizontal cut of the column. The middle figure shows our reconstruction of it, with 2 robots cooperatively making a very small number of wireless measurements. The T-shape can be easily detected in the reconstruction. The right figure shows the case where a threshold of 10dB is applied to the reconstructed middle figure such that any value that is 10dB below the maximum is zeroed.

V. CONCLUSIONS

In this paper we considered a mobile cooperative network that is tasked with building an obstacle map non-invasively. We proposed a framework that allowed the robots to build the map, by making a small number of wireless channel measurements. By exploiting the sparse representation of the map in space or spatial variations, we showed how the robots can map the obstacles efficiently. We proposed two cases of random and coordinated measurements, where the latter can result in the sparse sampling in the space or frequency domain. Our simulation results then showed the superior performance of the proposed framework. Finally, our experimental results confirmed the performance of our framework with two robots cooperatively mapping an obstacle on the campus of UNM.

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REFERENCES

[1] F. Dellaert, F. Alegre, and E. B. Martinson. Intrinsic localization and mapping with 2 applications: Diffusion mapping and macro polo localization. In *IEEE Intl. Conf. on Robotics and Automation*, volume 2, pages 2344 – 2349, 2003.

[2] R. Sim, G. Dudek, and N. Roy. A closed form solution to the single degree of freedom simultaneous localisation and map building (SLAM) problem. In *IEEE CDC*, volume 1, pages 191–196, 2000.

[3] R. Gartshore, A. Aguado, and C. Galambos. Incremental map building using an occupancy grid for an autonomous monocular robot. In *7th International Conference on Control, Automation, Robotics and Vision*, volume 2, pages 613–618, December 2002.

[4] Y. Mostofi and P. Sen. Compressive Cooperative Mapping in Mobile Networks. In *Proceedings of the 28th American Control Conference (ACC)*, pages 3397–3404, St. Louis, MO, June 2009.

[5] M. C. Wicks. Rf tomography with application to ground penetrating radar. In *Asilomar Conference on Signals, Systems and Computers*, pages 2017–2022, November 2007.

[6] A. M. Haimovich, R. S. Blum, and L. J. Cimini. Mimo radar with widely separated antennas. In *IEEE Signal Processing Magazine*, pages 116–129, January 2008.

[7] J. Wilson and N. Patwari. Radio tomographic imaging with wireless networks. accepted to appear in IEEE Trans. on Mobile Comp., 2010.

[8] M. Kanso and M. Rabbat. Compressed RF tomography for wireless sensor networks: Centralized and decentralized approaches. In *IEEE Intl. Conference on Distributed Computing in Sensor Systems*, June 2009.

[9] E. Candès, J. Romberg, and T. Tao. Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. on Information Theory*, 52(2):489–509, February 2006.

[10] D. L. Donoho. Compressed sensing. *IEEE Transactions on Information Theory*, 52(4):1289–1306, April 2006.

[11] D. Needell and R. Vershynin. Uniform uncertainty principle and signal recovery via regularized orthogonal matching pursuit, 2007. Preprint.

[12] F. Santosa and W. W. Symes. Linear inversion of band-limited reflection seismograms. *SIAM Journal on Scientific and Statistical Computing*, 7(4):1307–1330, 1986.

[13] R. Gribonval and M. Nielsen. Sparse representations in unions of bases. *IEEE Trans. on Information Theory*, 49(12):3320–3325, December 2003.

[14] E. J. Candès, J. Romberg, and T. Tao. Stable signal recovery from incomplete and inaccurate measurements. *Comm. Pure Appl. Math*, 59(8):1207–1223, 2005.

[15] http://www.dsp.ece.rice.edu/cs/.

[16] M. Rudelson and R. Vershynin. Sparse reconstruction by convex relaxation: Fourier and Gaussian measurements, 2006. Preprint.

[17] W. B. Johnson and J. Lindenstrauss (editors). *Handbook of the Geometry of Banach Spaces*, volume 1,2. Elsevier Science Ltd, North-Holland, Amsterdam, 2001.

[18] S. J. Szarek. Condition numbers of random matrices. *J. of Complexity*, 7(2):131–149, 1991.

[19] A. E. Litvak, A. Pajor, M. Rudelson, and N. Tomczak-Jaegermann. Smallest singular value of random matrices and geometry of random polytopes. *Advances in Mathematics*, 195(2):491–523, 2005.

[20] E. J. Candès. The restricted isometry property and its implications for compressed sensing. *Compte Rendus de l'Academie des Sciences*, 346:589–592, 2008.

[21] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

[22] ℓ_1 magic toolbox. http://www.acm.caltech.edu/l1magic/.

[23] S.J. Wright, R.D. Nowak, and M.A.T. Figueiredo. Sparse reconstruction by separable approximation. In *IEEE International Conference on Acoustics, Speech and Signal Processing*, pages 3373–3376, April 2008.

[24] M.A.T. Figueiredo, R.D. Nowak, and S.J. Wright. Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems. *IEEE Journal of Selected Topics in Signal Processing*, 1(4):586–597, Dec. 2007.

[25] Y. Nesterov. Gradient methods for minimizing composite objective function. *Center for Operations Research and Econometrics Discussion Paper*, (76), 2007.

[26] J. Tropp and A. Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. *IEEE Trans. on Information Theory*, 53(12):4655–4666, December 2007.

[27] Chengbo Li. *An Efficient Algorithm For Total Variation Regularization with Applications to the Single Pixel Camera and Compressive Sensing*. PhD thesis, RICE University, 2009.

[28] W. C. Jakes. *Microwave Mobile Communications*. Wiley-IEEE Press, New York, 1994.

[29] Y. Mostofi, A. Gonzalez-Ruiz, A. Ghaffarkhah, and D. Li. Characterization and Modeling of Wireless Channels for Networked Robotic and Control Systems - A Comprehensive Overview. In *Proceedings of 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, St. Louis, MO, October 2009.

[30] A. Goldsmith. *Wireless Communications*. Cambridge University Press, 2005.

[31] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic Imaging*. IEEE, 1988.