Image Enhancement: Histogram Processing

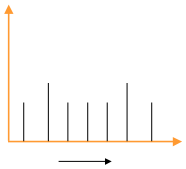
Reading:
Chapter 3 (Spatial domain)

Histogram Processing

- Histogram Equalization
- Histogram Specification/Matching

Histogram

$p(r_k) = n_k/n$



$r_k \in \{0, 1, 2, 3...., L-1\}$

n_k : # pixels with gray level r_k

n : # pixels in the image

Histogram

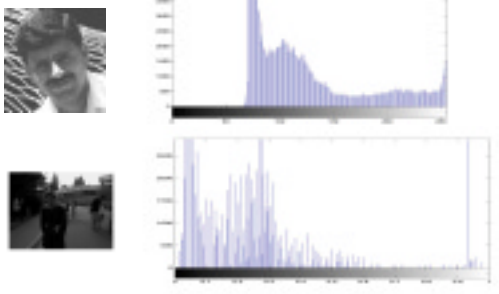
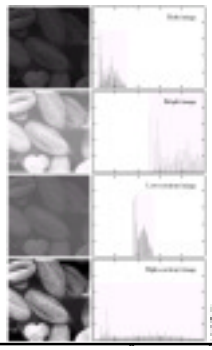


Figure 3.15: histograms



Histogram Modification

r : Input gray level $\in$ [0, 1]
s : Transformed gray level $\in$ [0, 1]

$s = T(r)$ T : Transformation function

Histogram Equalization

(i) $T(r)$ is single valued valued and monotonically increasing in $0 \le r \le 1$

(ii) $0 \le T(r) \le 1$ for $0 \le r \le 1$

$[0,\ 1] \xrightarrow{T} [0,\ 1]$

Inverse transformation : $T^{-1}(s) = r \qquad 0 \le s \le 1$

$T^{-1}(s)$ also satisfies (i) and (ii)

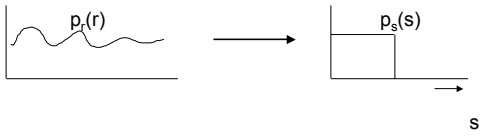
The gray levels in the image can be viewed as random variables taking values in the range [0,1].

Let $p_r(r)$: p.d.f. of input level r and let $p_s(s)$: p.d.f. of s

$s = T(r)$; $\therefore p_s(s) = p_r(r)\ dr/ds\Big|_{r=T^{-1}(s)}$ (from ECE 140)

Equalization (contd.)

We are interested in obtaining a transformation function T( ) which transforms an arbitrary p.d.f. to an uniform distribution



Consider $s = T(r) = \int_0^r p_r(w)\,dw \quad 0 \le r \le 1$

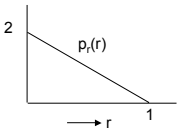
(Cumulative distribution function of r)

$p_s(s) = p_r(r) dr/ds\Big|_{r=T^{-1}(s)}$;

$$\frac{ds}{dr} = \frac{d}{dr}\left[\int_r^0 p_r(w)dw\right] = p_r(r)$$

$$\therefore p_s(s) = p_r(r)\frac{1}{p_r(r)}\Big|_{r=T^{-1}(s)} \equiv 1 \qquad 0 \le s \le 1$$

Equalization: Example



$$p_r(r) = \begin{cases} -2r+2 & 0 \le r \le 1 \\ 0 & \text{Else} \end{cases}$$

$$s = T(r) = \int_0^r (2-2w)\,dw = (2w - w^2)\Big|_0^r = 2r - r^2$$

$$\therefore r^2 - 2r + s = 0$$

Equalization (example: contd.)

$$r = \frac{+2 \pm \sqrt{4-4s}}{2} = 1 \pm \sqrt{1-s}$$

$r = T^{-1}(s) = 1 - \sqrt{1-s}$ as $r \in [0,1]$

$$p_s(s) = p_r(r)\frac{dr}{ds} = (-2r+2)\ \frac{d}{ds}(1-\sqrt{1-s})\Big|_{r=1-\sqrt{1-s}}$$

$$= (-2r+2)\left(\frac{-1}{2}(1-s)^{-1/2}(-1)\right)$$

$$= (-2+2\sqrt{1-s}+2)\frac{+1}{2\sqrt{1-s}} = 1 \qquad 0 \le r \le 1$$

Equalized Histograms

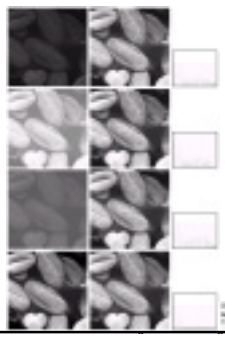
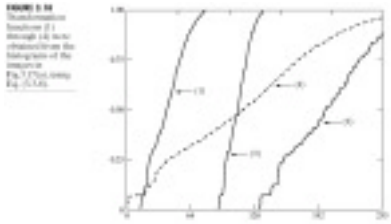


Fig 3.18: Transformation curves



Equalization: Discrete Case

$$p_r(r_k) = \frac{n_k}{n} \qquad 0 \le r_k \le 1 \quad ; \quad k = 0,1,\ldots,L-1$$

$L \rightarrow$ Number of levels

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} n_j / n$$

Discrete Case: Example

64x64 image; 8 gray levels.

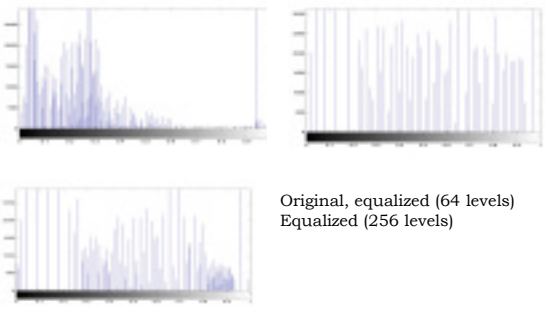
Notice that equalized histogram is not perfectly flat!

k	r_k	n_k	n_k/n	$S_k = \sum_{j=0}^{k} n_j/n$	$p_s(s_k)$
0	0	790	0.19	$0.19 \rightarrow 1/7 \rightarrow s_0$	0.19
1	1/7	1023	0.25	$0.44 \rightarrow 3/7 \rightarrow s_1$	0.25
2	2/7	850	0.21	$0.65 \rightarrow 5/7 \rightarrow s_2$	0.21
3	3/7	656	0.16	$0.81 \rightarrow 6/7 \rightarrow s_3$	0.24
4	4/7	329	0.08	$0.89 \rightarrow 6/7 \rightarrow s_3$	0.24
5	5/7	245	0.06	$0.95 \rightarrow 1 \rightarrow s_4$	0.11
6	6/7	122	0.03	$0.98 \rightarrow 1 \rightarrow s_4$	0.11
7	7/7	81	0.02	$1.0 \rightarrow 1 \rightarrow s_4$	0.11

Equalization: Image Examples

Original, Equalized (64)
Equalized (256)

..and their histograms



Original, equalized (64 levels)
Equalized (256 levels)

Histogram specification

Suppose $s = T(r) = \int_0^r p_r(w)\,dw$

$p_r(r) \rightarrow$ Original histogram ; $p_z(z) \rightarrow$ Desired histogram

Let $v = G(z) = \int_0^z p_z(w)\,dw$ and $z = G^{-1}(v)$

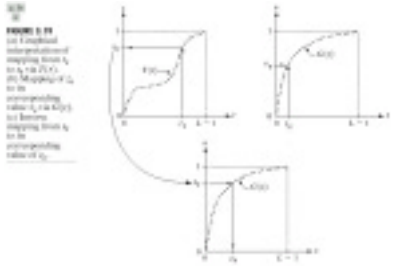
But s and v are identical p.d.f.

$\therefore z = G^{-1}(v) = G^{-1}(s) = G^{-1}(T(r))$

Question

- What happens when you apply equalization to an already equalized histogram
  - In the continuous case?
  - In the discrete case?

Fig 3.19: Matching



Matching: Summary

Steps:

(1) Equalize the levels of original image

(2) Specify the desired $p_z(z)$ and obtain G(z)

(3) Apply $z=G^{-1}(s)$ to the levels s obtained in step 1

Matching: an example

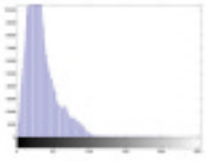
z_k	$p_z(z_k)$	$v_k = G(r_k)$	n_k	$p_z(z_k)$
$z_0 = 0$	0	0	0	0
$z_1 = 1/7$	0	0	0	0
$z_2 = 2/7$	0	0	0	0
$z_3 = 3/7$	0.15	$0.15 \leftrightarrow s_0 = 1/7$	790	0.19
$z_4 = 4/7$	0.2	$0.35 \leftrightarrow s_1 = 3/7$	1023	0.25
$z_5 = 5/7$	0.3	$0.65 \leftrightarrow s_2 = 5/7$	850	0.21
$z_6 = 6/7$	0.2	$0.85 \leftrightarrow s_3 = 6/7$	985	0.24
$z_7 = 1$	0.15	$1.0 \leftrightarrow s_4 = 1$	448	0.11

Histogram Matching: example

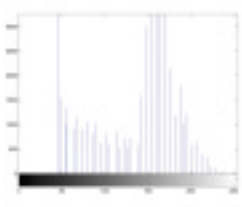
Original image
(Jenolan caves,
blue mountain,
Sydney, Ausralia)

Color to grayscale

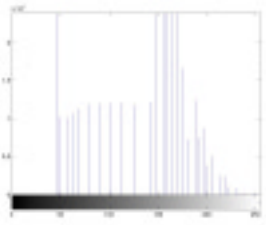
```
I=imread('sydney1.jpg);
I1=rgb2gray(I);
I1=imresize(I1,0.5);
Imhist(I1);
```



Desired & modified histograms



I2=histeq(I1,imhist(J));
Imhist(I2);



Imhist(J)
J=some image

Histogram modified image

Fig 3.20: Another example

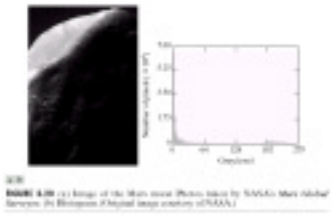


Fig 3.21

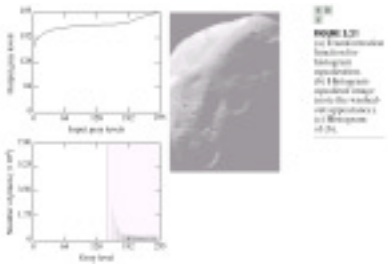


Fig 3.22