

Frequency Domain Processing

Frequency domain processing

- Homomorphic filtering

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Recap: Fig 4.1

FIGURE 4.1 The spectra of the function is the sum of the four functions shown in Figure 4.1(a). The periodic function is shown as a complex function of time and is not a single frequency.

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An example: Fig 4.2

FIGURE 4.2 (a) A rectangular function of frequency, and (b) its Fourier spectrum. (c) A filter function with unity gain, and (d) its Fourier spectrum.

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Basic Scheme

Frequency domain filtering operation

FIGURE 4.5 Basic steps for filtering in the frequency domain.

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An image and its DFT: Fig 4.4

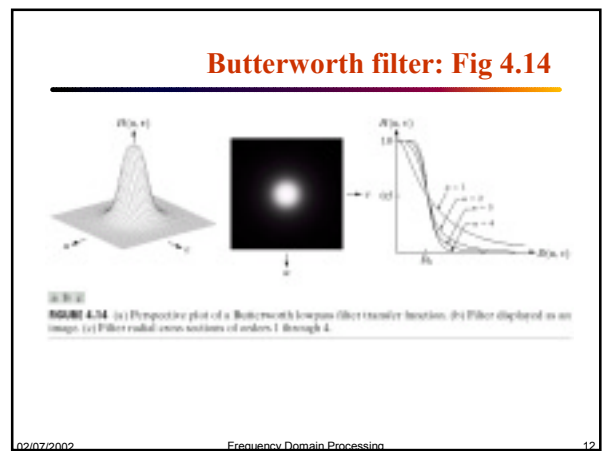
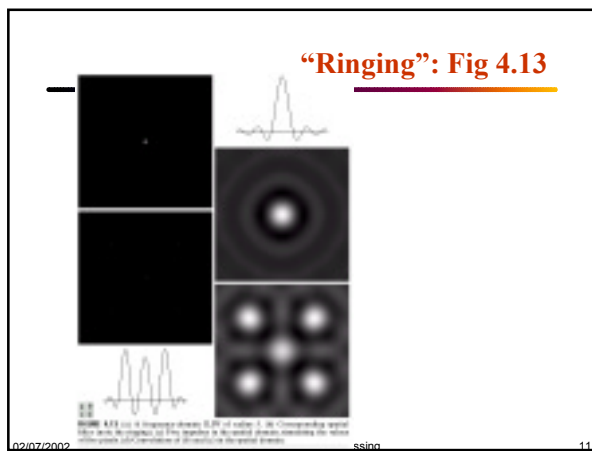
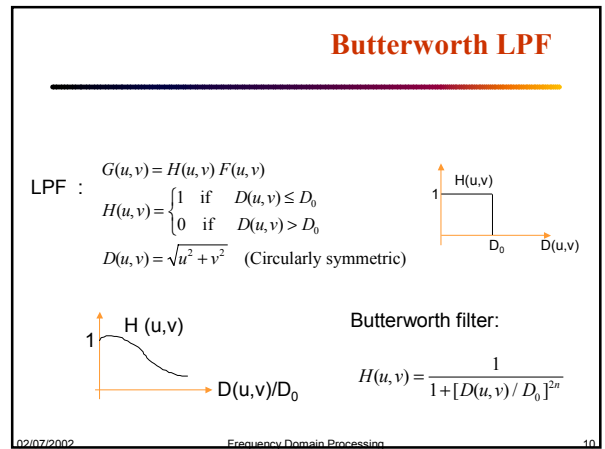
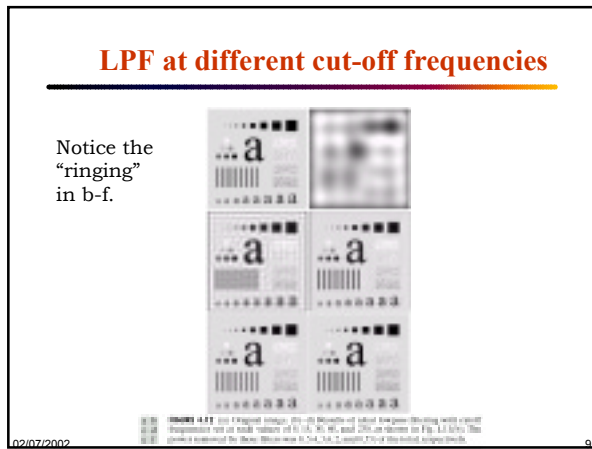
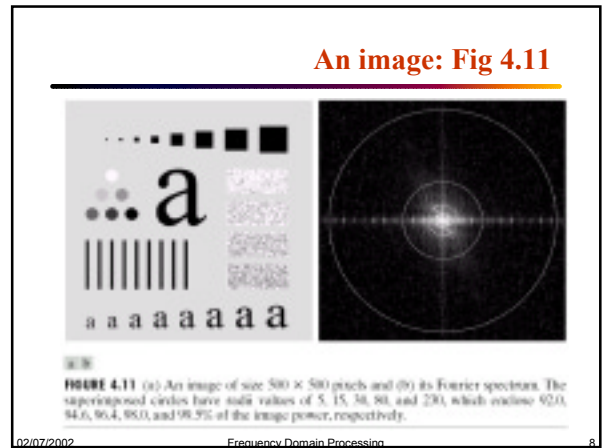
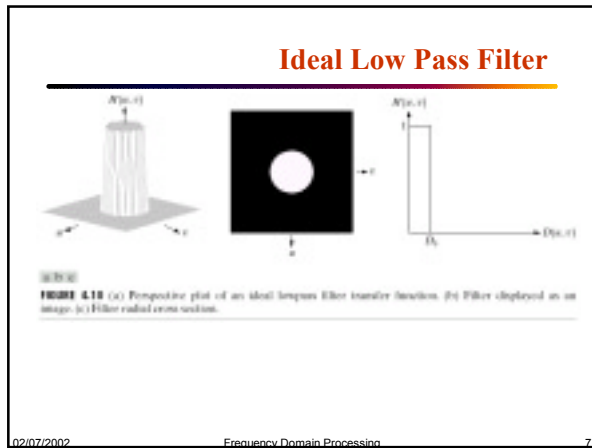
FIGURE 4.4 (a) A grayscale image of a mechanical part, and (b) its 2D Discrete Fourier Transform (DFT) magnitude spectrum.

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filtering

FIGURE 4.7 (a) A 3D surface plot of the DFT magnitude spectrum. (b) Result of DFT magnitude filtering the image in Fig 4.4(a). (c) A 3D surface plot of the DFT magnitude spectrum. (d) Result of DFT magnitude filtering the image in Fig 4.4(a).

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LPF example again



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Spatial representation of BWF

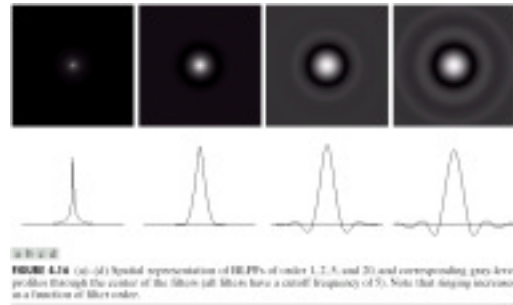


FIGURE 4.18 (a)–(d) Spatial representation of BWFs of order 1, 2, 3, and 20 and corresponding grey-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

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Gaussian LPF

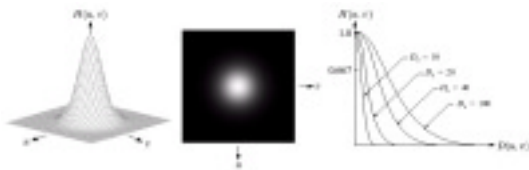


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross-sections for various values of D_0 .

$$H(u, v) = \exp(-D^2(u, v) / 2\sigma^2)$$

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Filtering using GLPF



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Another example

FIGURE 4.19

(a) Sample text of poor resolution (note broken characters in diagonal view). (b) Result of filtering with a GLPF (broken character appears more joined).

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1990 rather than the year 2000.



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Frequency domain processing HPF

$$\text{HPF} : H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

Butterworth filter:

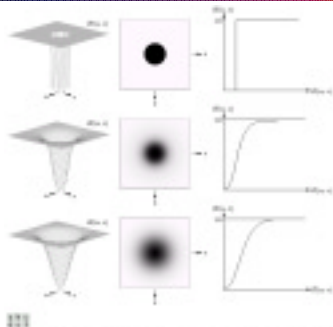
$$H(u, v) = \frac{1}{1 + [D_0 / D(u, v)]^{2n}}$$

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HPF: Fig 4.22



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FIGURE 4.22 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters and corresponding gray-level profiles.

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Spatial representation of HPF

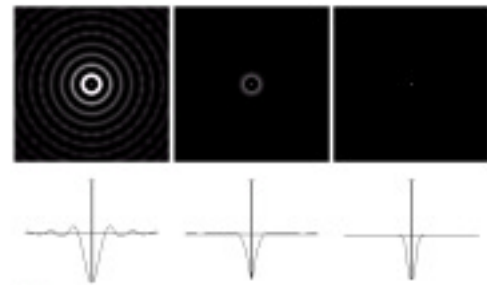


FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters and corresponding gray-level profiles.

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Ideal HPF



FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $d_0 = 15, 30, \text{ and } 90$, respectively. Problems with ringing are quite evident in (a) and (b).

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BWF



FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $d_0 = 15, 30, \text{ and } 90$, respectively. These results are much smoother than those obtained with an IHPF.

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GHPF



FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $d_0 = 15, 30, \text{ and } 90$, respectively. Compare with Figs. 4.24 and 4.25.

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Homomorphic filtering

$$\text{Consider } f(x,y) = i(x,y) \cdot r(x,y)$$

\uparrow \uparrow
 Illumination Reflectance

$$\text{Now } \mathfrak{I}\{f(x,y)\} \neq \mathfrak{I}\{i,r\}$$

So cannot operate on individual components directly

$$\text{Let } z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\mathfrak{I}\{z(x,y)\} = \mathfrak{I}\{\ln i\} + \mathfrak{I}\{\ln r\}$$

$$Z(u,v) = I + R; \text{ Let } S(u,v) = HZ = HI + HR$$

$$s(x,y) = \mathfrak{I}^{-1}\{HI\} + \mathfrak{I}^{-1}\{HR\}$$

$$\text{Let } i'(x,y) = \mathfrak{I}^{-1}\{HI\} ; r'(x,y) = \mathfrak{I}^{-1}\{HR\}$$

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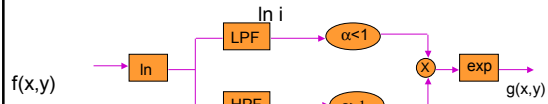
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Homomorphic filtering (contd.)

$$\begin{aligned} \therefore s(x,y) &= i' + r' \\ g(x,y) &= \exp(s(x,y)) \\ &= \exp i' + \exp r' \\ &= i_o(x,y) r_o(x,y) \end{aligned}$$

In practice: $i \rightarrow$ slowly varying \Rightarrow LF
 $r \rightarrow$ fast varying \Rightarrow HF



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Homomorphic filtering (cont.)



FIGURE 4.31
Homomorphic filtering approach for image enhancement.



FIGURE 4.32
Cross section of a smoothly varying filter function, $M(x,y)$, in the distance from the origin of the complex plane.

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An example

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FIGURE 4.33
(a) Original image, (b) image processed by homomorphic filtering (note details inside shadow). (Workshop.)



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