Linear Systems: Discrete case & 2D

January 14, 2003

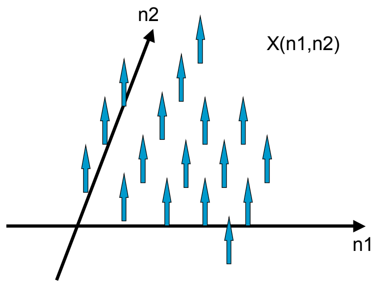
Linear systems-review

Part 1: Review from G&W (continuous case)

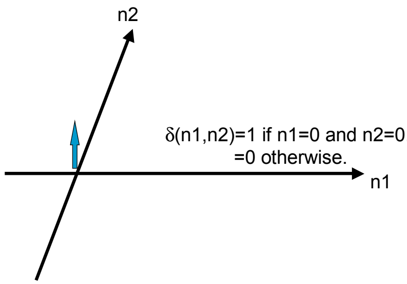
Part 2: Discrete case & 2D

- 2D impulse function
- Line function
- Step function
- Linear systems and Shift invariance
- Impulse Response of LSI Systems
- 2-D Convolution

2-D Systems

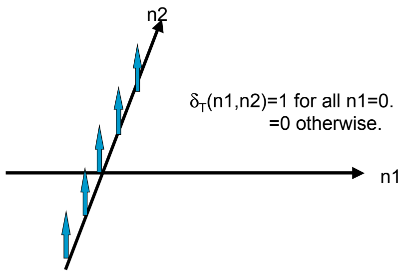


Impulse Function(Kronecker Delta)



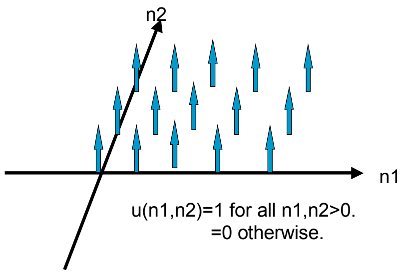
$\delta(n1,n2)=1$ if n1=0 and n2=0.
=0 otherwise.

Line Impulse



$\delta_T(n1,n2)=1$ for all n1=0.
=0 otherwise.

Unit Step Function



u(n1,n2)=1 for all n1,n2>0.
=0 otherwise.

“ System”

An input-output relationship is called a system if there is a unique output for any given input.

Y(n1,n2) = T[X(n1,n2)]

Linear Systems

The linearity of a system T is defined as

Linearity:

T[a X_1(n1,n2) + b X_2(n1,n2)] = a Y_1 + b Y_2

(i.e., principle of superposition holds).

Are these linear?

(a) y(m,n) = x(m,n) g(m,n)

(b) y(m,n) = $[x(m,n)]^2$

Linear Shift Invariant Systems

Shift Invariance:

T[X(m-k, n-l)] = Y(m-k, n-l) where

Y(m,n) = T[X(m,n)].

A LSI system is completely characterized by its response to the impulse function δ(m,n).

Convolution

Let $h(n_1,n_2) = T[\delta(n_1,n_2)]$; $y(n_1,n_2) = T[x(n_1,n_2)]$; then

$h(n_1- k_1 ,n_2 - k_2) = T[\delta(n_1- k_1 ,n_2 - k_2)]$, and

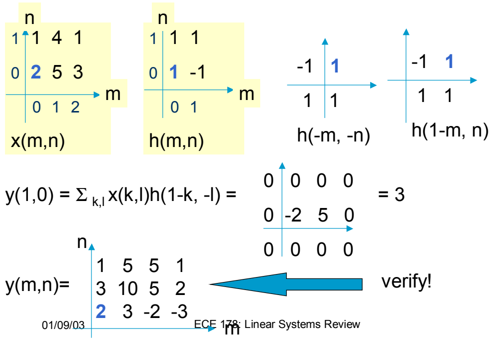
$$y(n_1,n_2) = T\left[\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x(k_1,k_2)\,\delta(n_1-k_1,n_2-k_2)\right]$$

$$= \sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x(k_1,k_2)\;T[\delta(n_1-k_1,n_2-k_2)]$$

$$= \sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x(k_1,k_2)\,h(n_1-k_1,n_2-k_2)$$

$$y(n_1,n_2) = h(n_1,n_2) * x(n_1,n_2)$$

Convolution: example

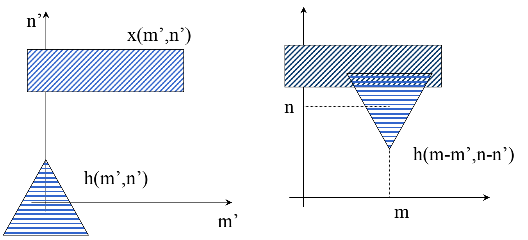


y(1,0) = Σ $_{k,l}$ x(k,l)h(1-k, -l) = = 3

y(m,n)=

verify!

Discrete Convolution in 2D



output=sum of the product of the two in the overlapped region.