UNIVERSITY of CALIFORNIA
Santa Barbara
Department of Electrical and Computer Engineering

ECE 178   **Midterm Exam II**   Date: February 27, 2003.
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TAs: J. Tešic, M. Zuliani   Max Points: 100

SOLUTIONS

1. (25) Consider a spatial filtering operation which averages the four nearest neighbors of a pixel within a 3x3 window. What does the filter look like in the frequency domain?

$$y(m, n) = \frac{1}{4}[x(m-1, n) + x(m+1, n) + x(m, n-1) + x(m, n+1)]$$

Frequency representation of the filter is:

$$Y(k, l) = DFT\{y(m, n)\} = \frac{1}{4}[DFT\{x(m-1, n)\} + DFT\{x(m+1, n)\} + DFT\{x(m, n-1)\} + DFT\{x(m, n+1)\}]$$

Assume that the dimension of image is $N \times N$. 2D DFT has the translation property (Eq. 4.6-2, textbook page 195):

$$x(m - m_0, n - n_0) \Leftrightarrow X(k, l)e^{-j2\pi\frac{m_0 k}{N}}e^{-j2\pi\frac{n_0 l}{N}}$$

Therefore: $Y(k, l) = \frac{1}{4}X(k, l)[e^{-j2\pi\frac{k}{N}} + e^{j2\pi\frac{k}{N}} + e^{-j2\pi\frac{l}{N}} + e^{j2\pi\frac{l}{N}}]$

In the frequency domain, filter representation is:

$$\boxed{Y(k, l) = \frac{1}{2}X(k, l)[\cos(2\pi\frac{k}{N}) + \cos(2\pi\frac{l}{N})]}$$

2. (25) An image has a histogram $h(b) = \frac{3}{2}\sqrt{b}$, $b \in [0,1]$. What transformation of this image results in equilized histogram?

The density function is valid, and the histogram equalization transformation is:

$$s = T(b) = \int_0^b h(b)db = \int_0^b \frac{3}{2}\sqrt{b}db = \frac{3}{2}\int_0^b b^{\frac{1}{2}}db = \frac{3}{2}\frac{b^{\frac{3}{2}}}{\frac{3}{2}} = b^{\frac{3}{2}} = b\sqrt{b}$$

Therefore, our transformation is:

$$\boxed{s = b\sqrt{b}, \quad s \in [0,1]}$$

Check: $b = 0 \Rightarrow s = 0$, $b = 1 \Rightarrow s = 1$. Limits are handled correctly.

3. (20) Consider a 1-D signal shown below that is being processed by a median filter of length 2M+1. What is the constraint on the width N for the signal to go through the median filter without any change? (establish a relationship between M and N for the signal to be invariant to this median filtering)
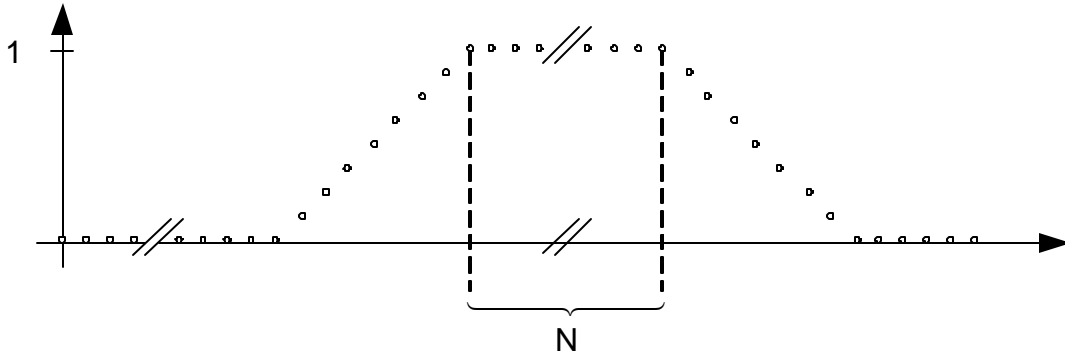

Observed signal is monotonic on all three segments.

Note that, if signal is monotone inside $2M+1$ median filter window, the pixel remains *unchanged* ! Also note that the result of the median filtering operation does nor depend on the direction in which it is applied. Due to monotonicity, there is exactly M values smaller or equal than x and exactly M values larger or equal than x:

$$\underbrace{\square ... \square}_{M} \, \boxed{x} \, \underbrace{\square ... \square}_{M}$$
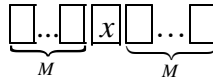
Therefore, for the pixel in our signal to remain unchanged, we need to keep the monotonicity inside the median filter. The bound on the filter length gets tighter as we approach to first pixel with value 1. We have to define M, so that the following does not occur:
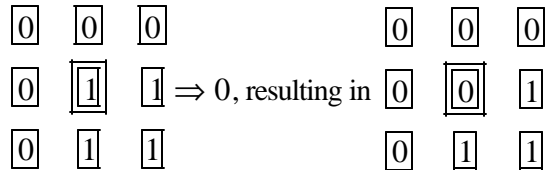
$$\underbrace{\boxed{\leq 1}..\boxed{\leq 1}}_{M} \overbrace{\boxed{1}\boxed{1}...\boxed{1}}^{N} \underbrace{\boxed{\leq 1}..\boxed{\leq 1}}_{M+1}$$
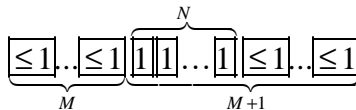
Therefore, $M+1 \leq N$. Observe that, for the flat part of the signal, values remain unchanged for $M+1 \leq N$. Check the result of median filtering if applied on the Nth pixel with value 1 for $M+1 \leq N$:

$$\underbrace{\boxed{1}...\boxed{1}}_{M} \boxed{1} \underbrace{\boxed{\leq 1}..\boxed{\leq 1}}_{M}$$
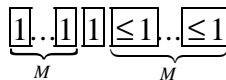
The value remains unchanged. Therefore, this signal is going to be invariant to median filtering if: $\boxed{M+1 \leq N}$.
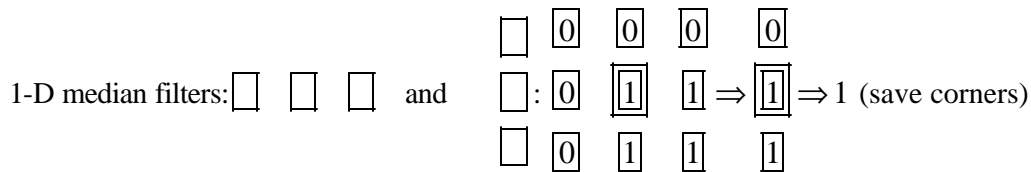
4. (20) A particular class of images consists of images of rectangular objects with the sides oriented parallel to the x and y axes. The imaging system introduces the undesirable impulse noise. Design a suitable enhancement technique which preserves *edges* <u>and</u> *corners* of the rectangles. State all your assumptions clearly and make sure to illustrate with an example that the proposed scheme preserves <u>both</u> edges and corners.

Impulse noise is removed using median filtering. Edges are saved if we use 1-D and 2-D median filtering. However, 2D median filtering does not save corners of the rectangle:

$$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \boxed{1} & 1 \\ 0 & 1 & 1 \end{array} \Rightarrow 0, \text{ resulting in } \begin{array}{ccc} 0 & 0 & 0 \\ 0 & \boxed{0} & 1 \\ 0 & 1 & 1 \end{array}$$
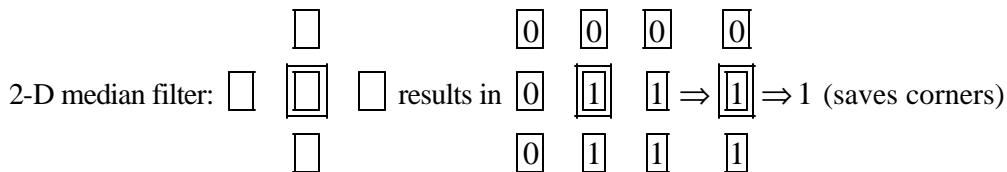
Use of 1-D median filters results in suitable enhancement technique for this particular problem. There are two ways of implementing 1-D median filters

   (a) Apply 2 different (horizontal and vertical) 1-D median filters separately (ordering is not important).

1-D median filters: $\square\ \square\ \square$ and $\square$: $\begin{array}{cccc} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 1 & \Rightarrow \boxed{1} & \Rightarrow 1 \text{ (save corners)} \\ & 0 & 1 & 1 & 1 \end{array}$

   (b) Apply cross-median filtering in 2D:

2-D median filter: $\begin{array}{c} \square \\ \square\ \boxed{\square}\ \square \\ \square \end{array}$ results in $\begin{array}{cccc} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 1 & \Rightarrow \boxed{1} & \Rightarrow 1 \text{ (saves corners)} \\ & 0 & 1 & 1 & 1 \end{array}$

5. (10) State of the following are TRUE or FALSE:

(1) The 2D DFT of a symmetric matrix results in real coefficients only. **FALSE**

  (2) Repeated application of histogram equalization results in a constant intensity image. **FALSE**

(3) Median filtering removes salt & pepper noise in images. **TRUE**

(4) Average or mean filtering is equivalent to low pass filtering. **TRUE**

(5) Exactly 2 out of these 5 statements are TRUE. **AMBIGUOUS**