ECE 178: Image Processing REVIEW

Lecture #20

March 13, 2003

Course Outline

- Introduction
- Digital Images
- Image Transforms
- Sampling and Quantization
- Image Enhancement
- Image/Video Coding
  - JPEG
  - MPEG

Relationship between pixels

- Neighborhoods
  - 4-neighbors (N,S,W,E pixels) == $N_4(p)$.
  - Diagonal neighbors == $N_D(p)$.
  - 8-neighbors: include diagonal pixels == $N_8(p)$.
- Connectivity -> to trace contours, define object boundaries, segmentation.
  - V: Set of gray level values used to define connectivity; e.g., V={1}.

Connected components

- If p and q are in S, p is connected to q in S if there is a path from p to q entirely in S.
- Connected component: Set of pixels in S that are connected; There can be more than one such set within a given S.
- Labelling of connected components.

Distance Measures

- What is a Distance Metric?

  For pixels *p*,*q*, and *z*, with coordinates (*x*,*y*), (*s*,*t*), and (*u*,*v*), respectively:

$$D(p,q) \geq 0 \quad (D(p,q) = 0 \text{ iff } p = q)$$

$$D(p,q) = D(q,p)$$

$$D(p,z) \leq D(p,q) + D(q,z)$$

Linear systems-review

- 2D impulse function
- Line function
- Step function
- Linear systems and Shift invariance
- Impulse Response of LSI Systems
- 2-D Convolution

2-D Digital Transforms

Week II-III

2D FT: Properties

Linearity: $a\ f(x,y) + b\ g(x,y) \Longleftrightarrow a\ F(u,v) + b\ G(u,v)$

Convolution: $f(x,y) \star g(x,y) = F(u,v)\ G(u,v)$

Multiplicationn: $f(x,y)\ g(x,y) = F(u,v) \star G(u,v)$

Separable functions: Suppose $f(x,y) = g(x)\ h(y)$, Then
$F(u,v)=G(u)H(v)$

Shifting: $f(x \pm x_0, y \pm y_0) \longleftrightarrow \exp[2\pi\ (x_0 u \pm y_0 v)]\ F(u,v)$

2-D DFT

Often it is convenient to consider a symmetric transform:

$$v(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u(n) W_N^{\ kn} \quad \text{and}$$

$$u(n) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} v(k) W_N^{\ -kn}$$

In 2-D: consider a NXN image

$$v(k,l) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} u(m,n)\, W_N^{\ km} W_N^{\ ln},$$

$$u(m,n) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} v(k,l)\, W_N^{\ -km-ln}$$

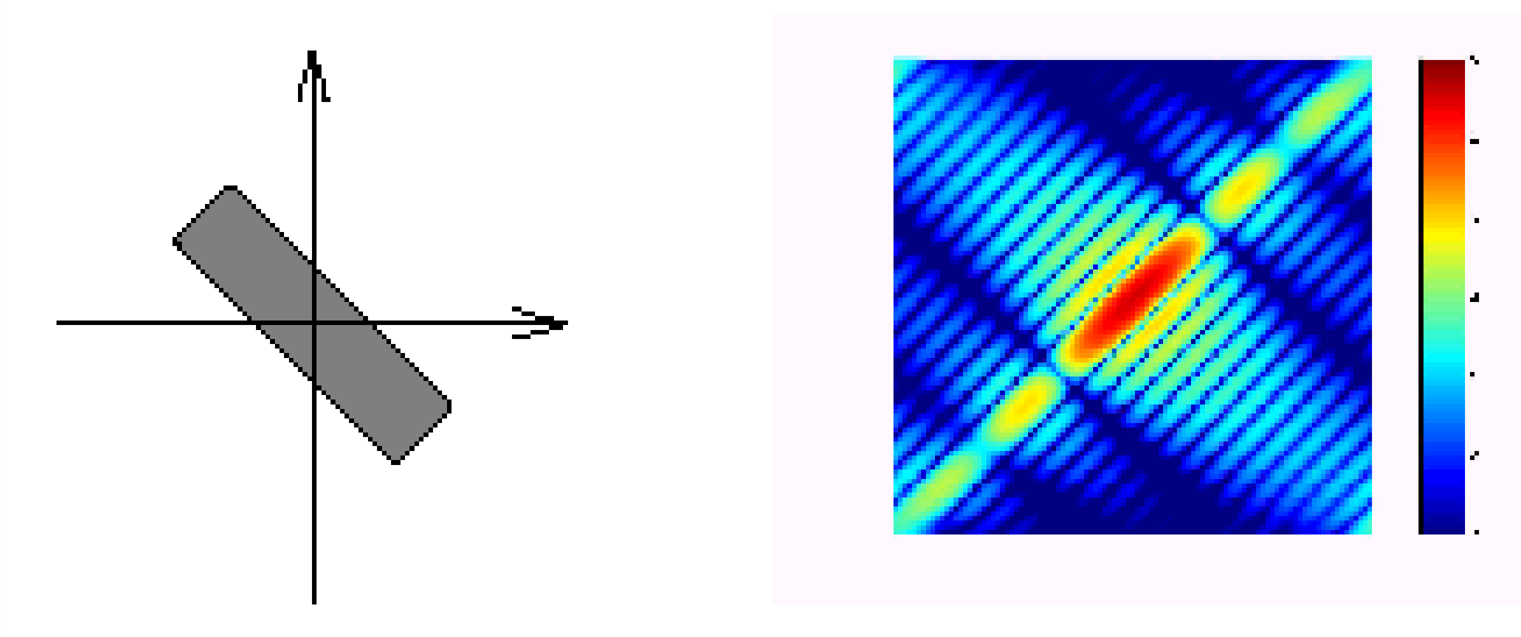
2D DFT Properties: Periodicity & Conjugate Symmetry

$$u(m,n) \overset{F}{\longleftrightarrow} v(k,l)$$

$$v(k,l) = v(k+N, l) = v(k, l+N) = v(k+N, l+N)$$

If $u(m,n)$ is real, $v(k,l)$ also exhibits conjugate symmetry
$v(k,l) = v^* (-k, -l)$ or $| v(k,l) | = | v(-k, -l) |$

Rotation



Linear Convolution using DFT

Extended sequences

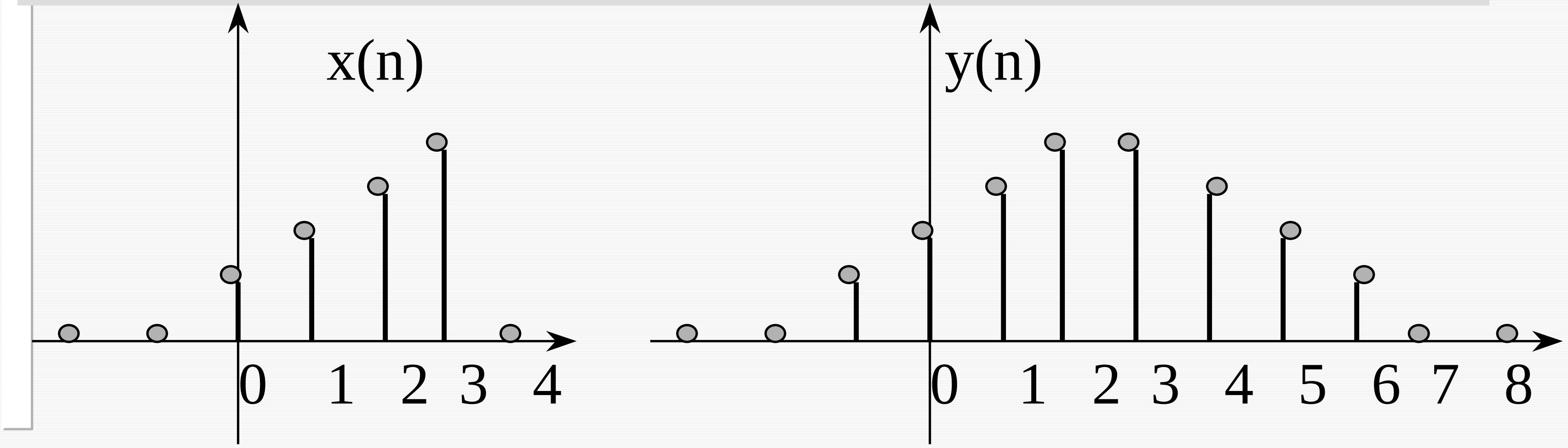
Circular convolution

Discrete Cosine Transform

Consider 1 - D first; Let x(n) be a N point sequence $0 \le n \le N - 1$.

$$x(n) \leftrightarrow \begin{matrix} \text{2 - N point} \\ y(n) \end{matrix} \overset{DFT}{\longleftrightarrow} \begin{matrix} 2 - N\ \text{point} \\ Y(u) \end{matrix} \leftrightarrow \begin{matrix} N - \text{point} \\ C(u) \end{matrix}$$

$$y(n) = x(n) + x(2N - 1 - n) = \begin{cases} x(n), & 0 \le n \le N - 1 \\ x(2N - 1 - n), & N \le n \le 2N - 1 \end{cases}$$



Why DCT?

- Blocking artifacts less pronounced in DCT than in DFT.
- Good approximation to the Karhunen-Loeve Transform (KLT) but with basis vectors fixed.
- DCT is used in JPEG image compression standard.

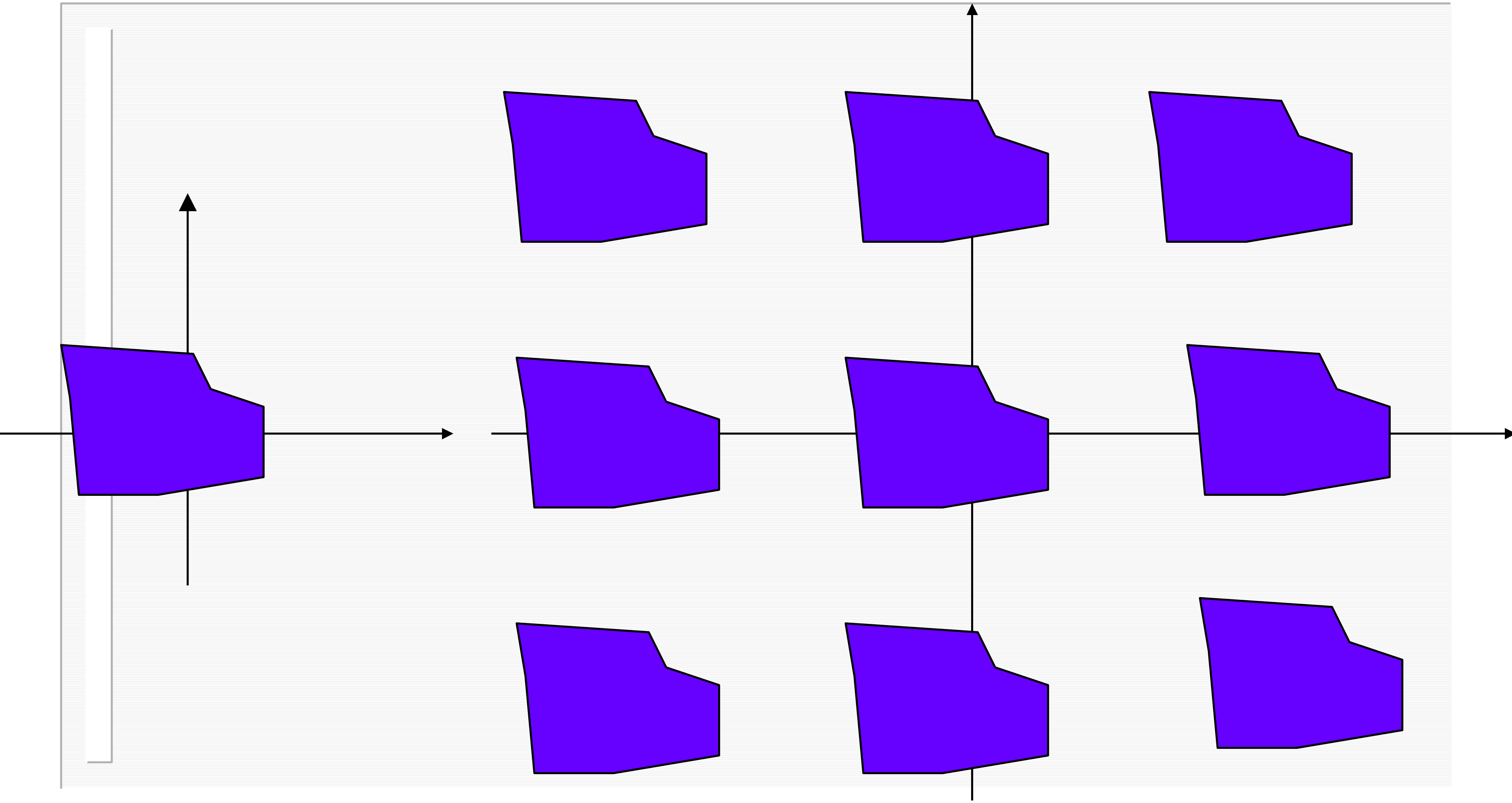
Sampling and Quantization

WEEK IV

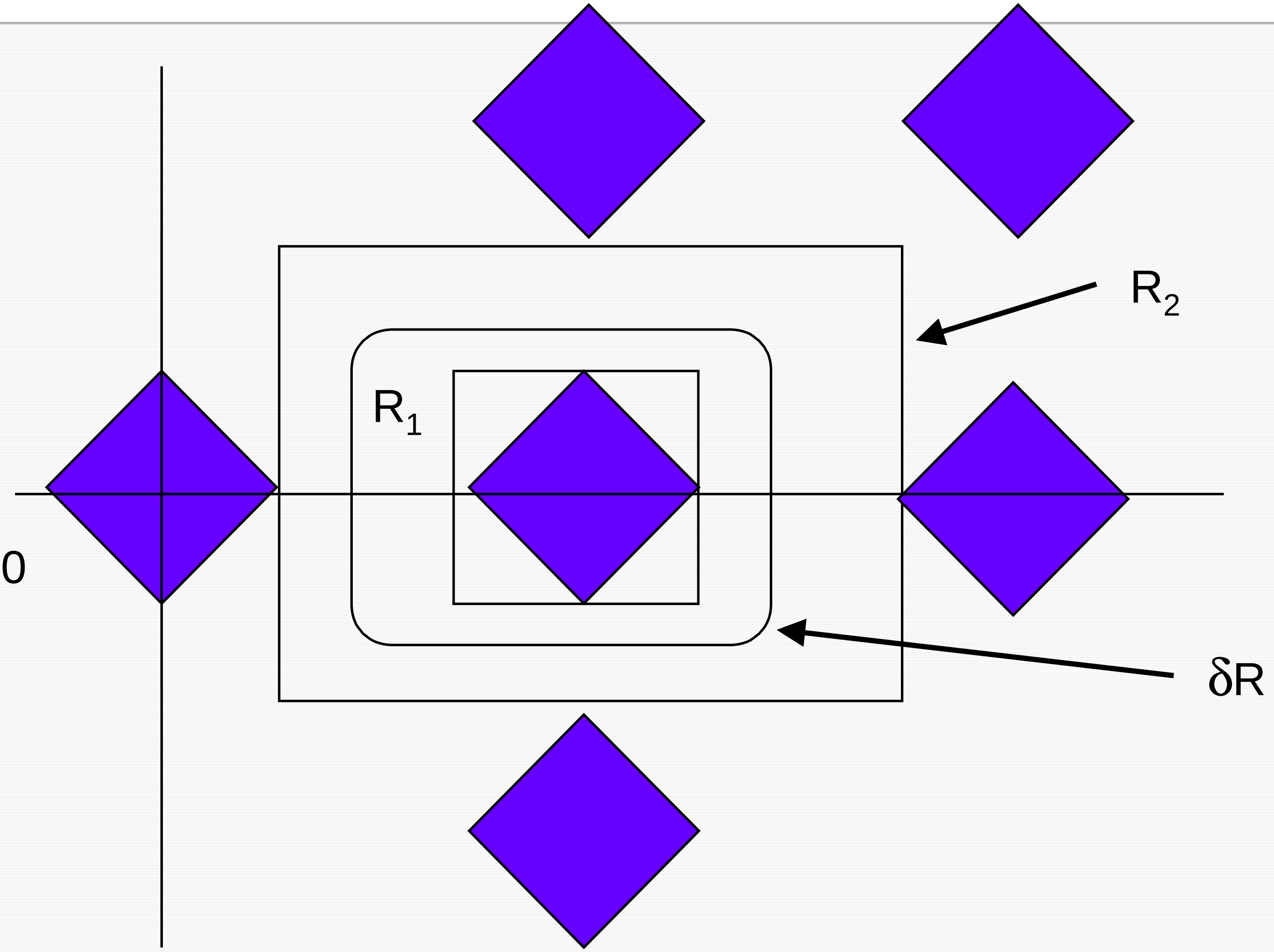
Sampling and Quantization

- 2D sampling theorem
- Optimal quantizer
  - Uniform quantizer

Sampled Spectrum: Example



Reconstruction



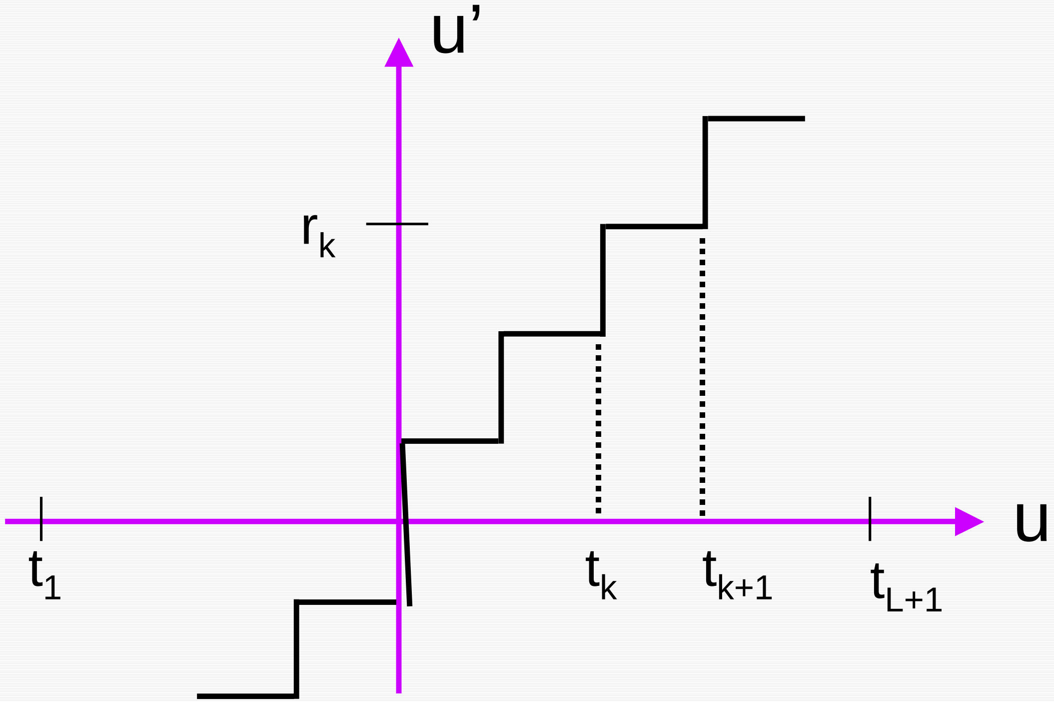
Quantization

- Optimal quantizer
- Uniform Quantizer

Image Quantization

$$u \xrightarrow{\text{Quantizer}} u' \; \varepsilon \; \{r_1,r_2,r_3,....,r_L\}$$

(continuous)



MMSE Quantizer

Minimise the mean squared error, MSE = Expected value of $(u-u')^2$ given the number of quantization levels L.

Assume that the density function $\mathbf{p_u(u)}$ is known (or can be approximated by a normalised histogram).

Note that for images, u==image intensity. $\mathbf{p_u(u)}$ is the image intensity ditribution.

Optimum transition/reconst.

(1) Optimal transition levels lie halfway between the optimum reconstruction levels.

(2) Optimum reconstruction levels lie at the center of mass of the probabality density in between the transition levels.

(3) A and B are simultaneous non-linear equations (in general)

→ Closed form solutions normally dont exist → use numerical techniques

Image Enhancement

WEEK V-VI

Image Enhancement

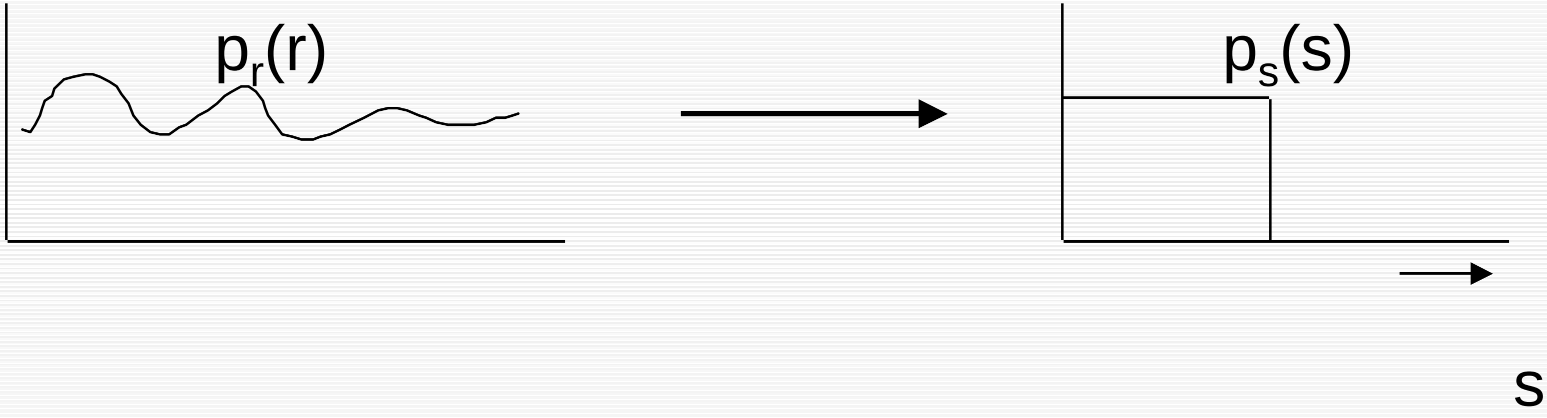
- Point Operations
- Histogram Modification
- Average and Median Filtering
- Frequency domain operations
- Homomorphic Filtering
- Edge enhancement

Histogram Modification

- Histogram Equalization
- Histogram Specification

Equalization (contd.)

We are interested in obtaining a transformation function T( ) which transforms an arbitrary p.d.f. to an uniform distribution



Histogram specification

Suppose $s = T(r) = \int_0^r p_r(w)\,dw$

$p_r(r) \rightarrow$ Original histogram ; $p_z(z) \rightarrow$ Desired histogram

Let $v = G(z) = \int_0^z p_z(w)\,dw$ and $z = G^{-1}(v)$

But s and v are identical p.d.f.

$\therefore z = G^{-1}(v) = G^{-1}(s) = G^{-1}(T(r))$

Steps:

(1) Equalize the levels of original image

(2) Specify the desired $p_z(z)$ and obtain $G(z)$

(3) Apply $z=G^{-1}(s)$ to the levels s obtained in step 1

Example Problem

Which transformation should be applied in order to change the histogram $h(b) = \frac{3}{2}\sqrt{b}$ into $t(b) = \frac{\pi}{2}\sin(\pi b)$?

Median filtering

Replace f (x,y) with median [f (x' , y')]

(x' , y') E neighbourhood

- Useful in eliminating intensity spikes. (salt & pepper noise)
- Better at preserving edges.

Example:

10	20	20
20	15	20
25	20	100

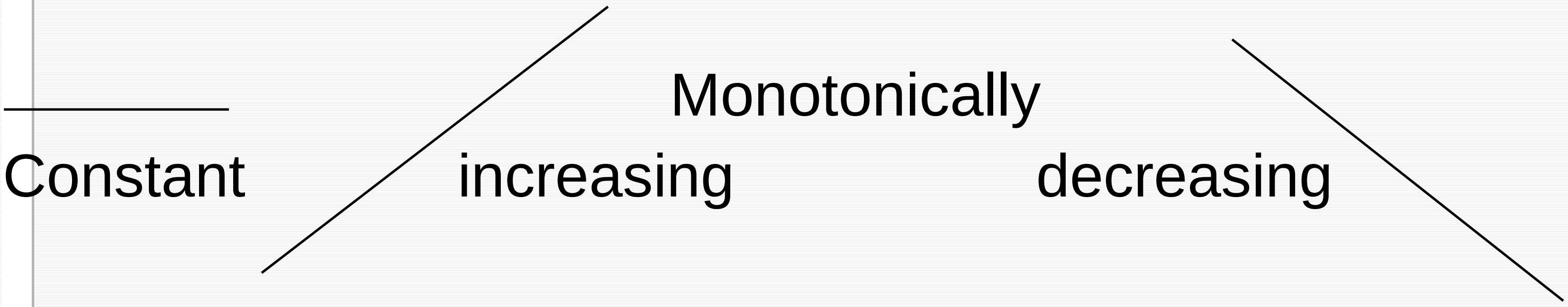
→ (10,15,20,20,20,20,20,25,100)

Median=20

So replace (15) with (20)

Invariant Signals

Invariant signals to a median filter:



Monotonically

Constant

increasing

decreasing

length?

Homomorphic filtering

(not discussed during class lecture)

Consider f (x,y) = i (x,y) . r (x,y)

Illumination Reflectance

Now $\Im\{f(x,y)\} \neq \Im\{i.r\}$

So cannot operate on individual components directly

Let $z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$\Im\{z(x,y)\} = \Im\{\ln i\} + \Im\{\ln r\}$$

$Z(u,v) = I + R$; Let $S(u,v) = HZ = HI + HR$

$$s(x,y) = \Im^{-1}\{HI\} + \Im^{-1}\{HR\}$$

Let $i'(x,y) = \Im^{-1}\{HI\}$; $r'(x,y) = \Im^{-1}\{HR\}$

IMAGE COMPRESSION

Week VIII-IX

Image compression

Objective: To reduce the amount of data required to represent an image.

Important in data storage and transmission

- Progressive transmission of images (internet, www)
- Video coding (HDTV, Teleconferencing)
- Digital libraries and image databases
  - Medical imaging
  - Satellite images

IMAGE COMPRESSION

- Data redundancy
- Self-information and Entropy
- Error-free and lossy compression
- Huffman coding, Arithmetic coding
- Predictive coding
- Transform coding

Video Coding

- Motion compensation
- H.261, MPEG-1 and MPEG-2

The End