

Linear Systems: Discrete case & 2D

January 14, 2003

Linear systems-review

Part 1: Review from G&W (continuous case)

Part 2: Discrete case & 2D

2D impulse function

Line function

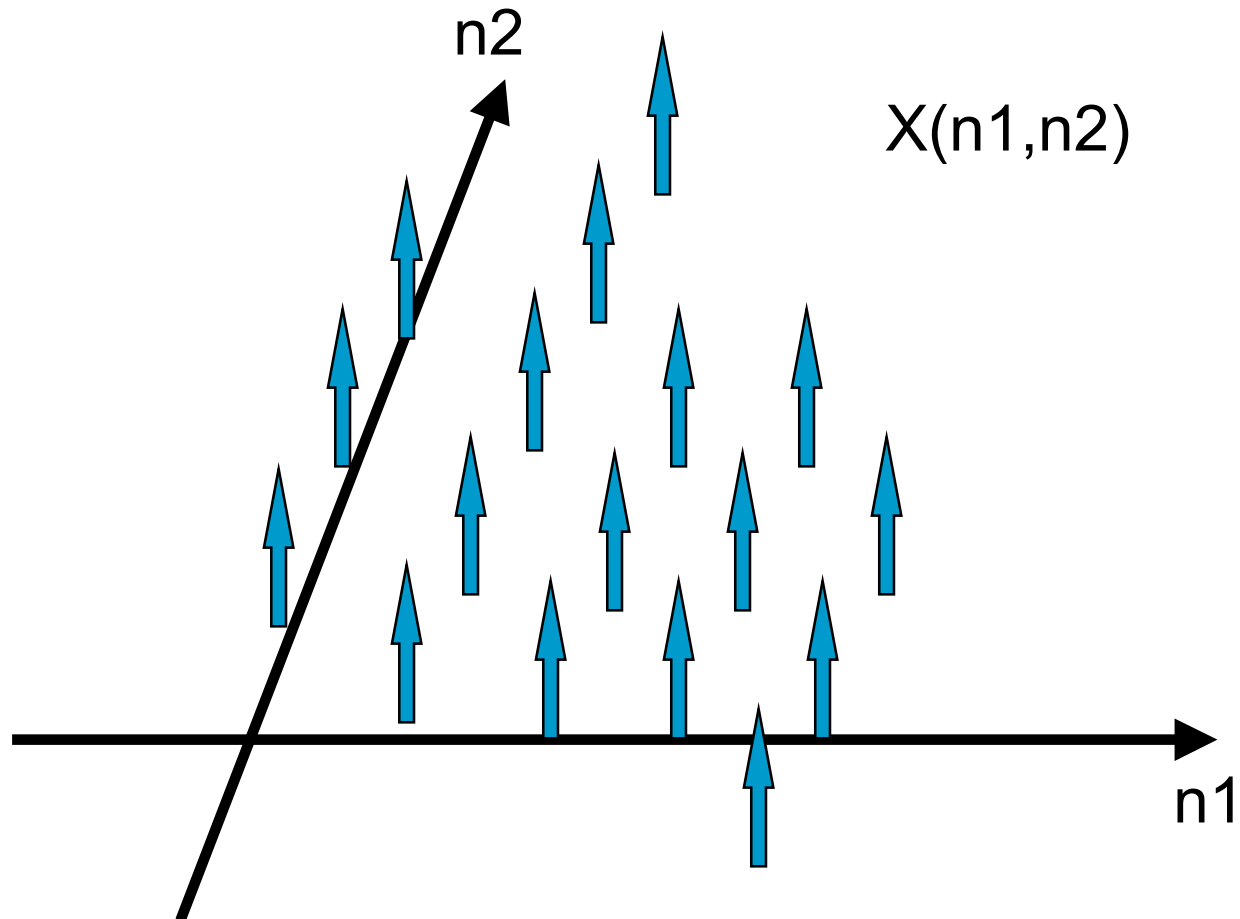
Step function

Linear systems and Shift invariance

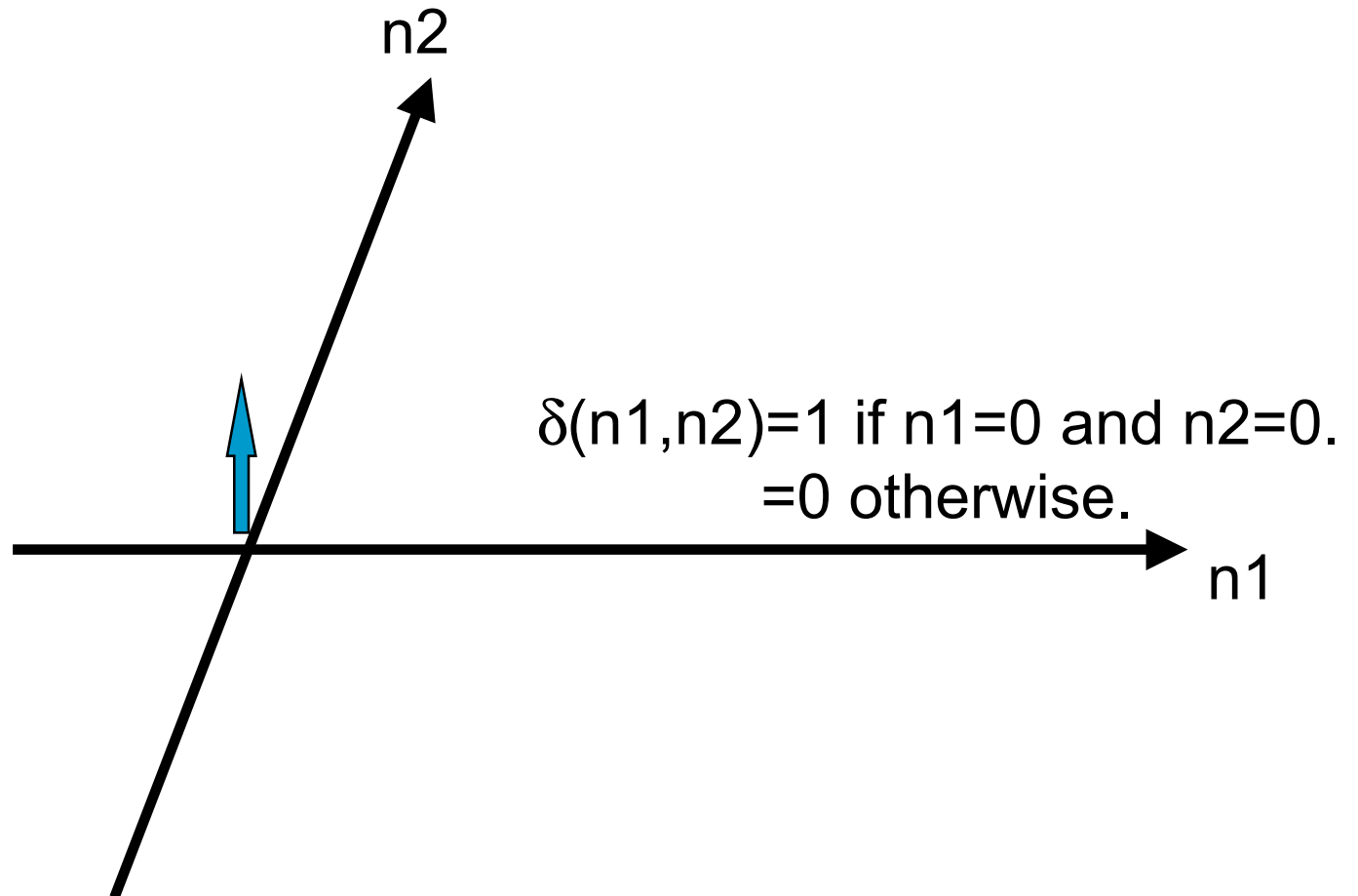
Impulse Response of LSI Systems

2-D Convolution

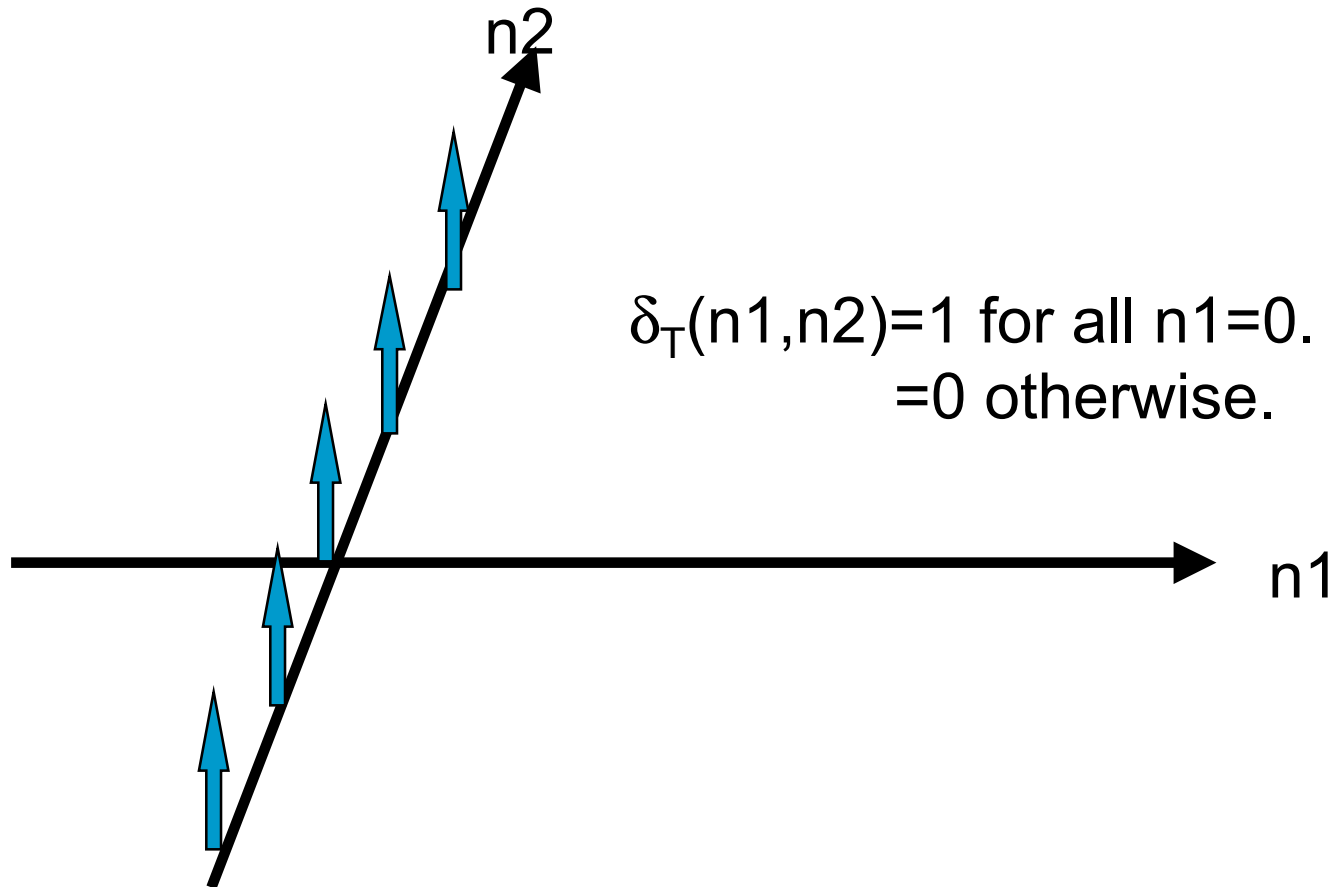
2-D Systems



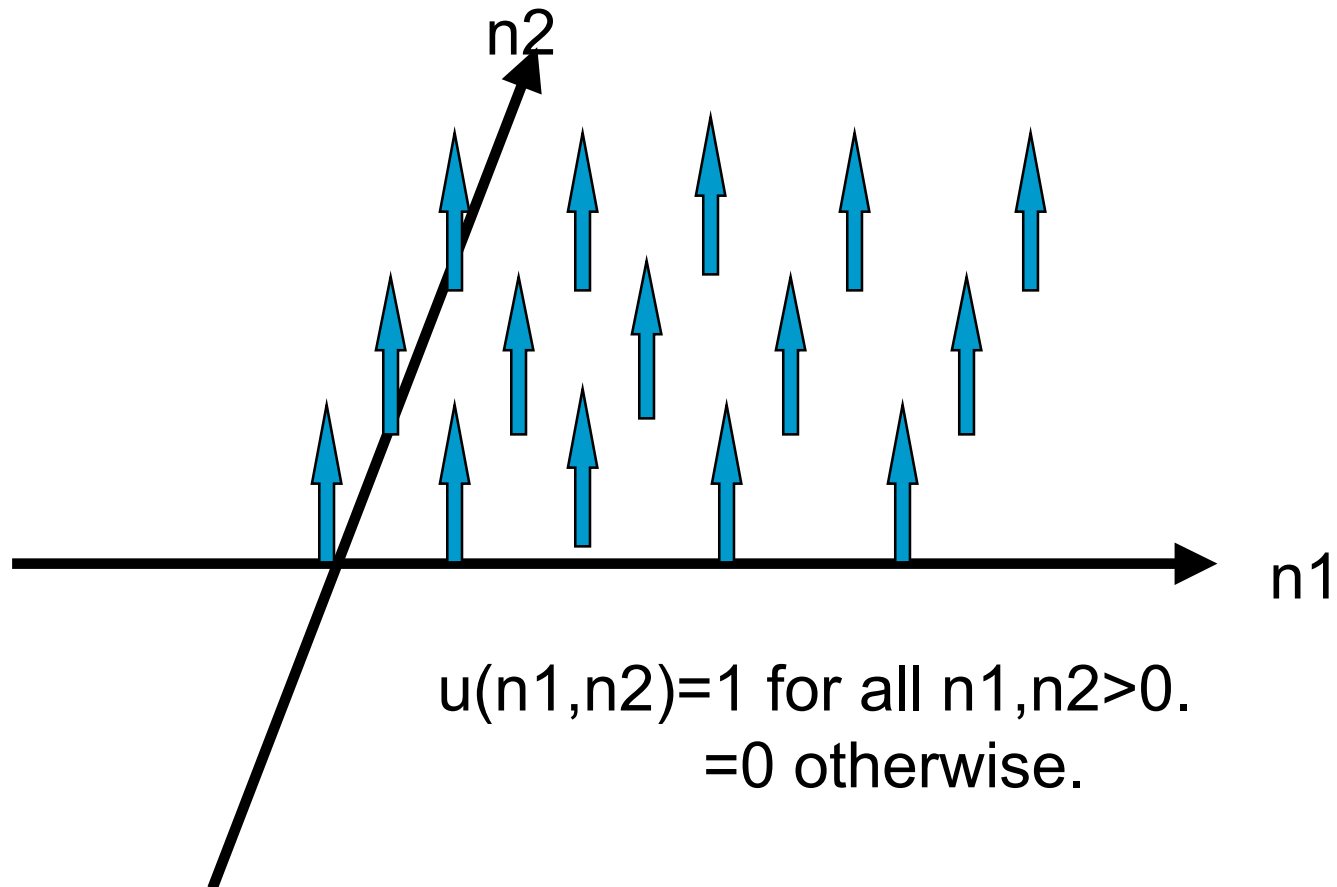
Impulse Function(Kronecker Delta)



Line Impulse



Unit Step Function



“System”

An input-output relationship is called a system if there is a unique output for any given input.

$$Y(n_1, n_2) = T[X(n_1, n_2)]$$



Linear Systems

The linearity of a system T is defined as

Linearity:

$$T[a X_1(n_1, n_2) + b X_2(n_1, n_2)] = a Y_1 + b Y_2$$

(i.e., principle of superposition holds).

Are these linear?

(a) $y(m, n) = x(m, n) g(m, n)$

(b) $y(m, n) = [x(m, n)]^2$

Linear Shift Invariant Systems

Shift Invariance:

$$T[X(m-k, n-l)] = Y(m-k, n-l) \text{ where} \\ Y(m,n) = T[X(m,n)].$$

A LSI system is completely characterized by its response to the impulse function $\delta(m,n)$.

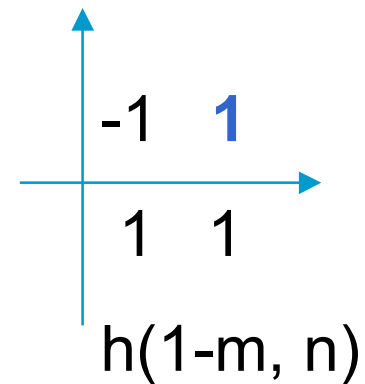
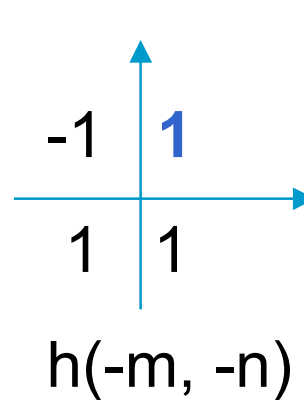
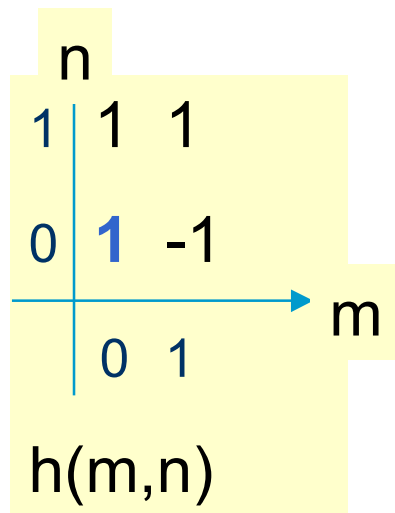
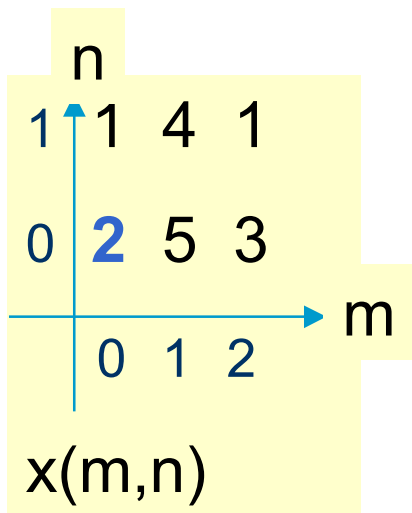
Convolution

Let $h(n_1, n_2) = T[\delta(n_1, n_2)]$; $y(n_1, n_2) = T[x(n_1, n_2)]$; then
 $h(n_1 - k_1, n_2 - k_2) = T[\delta(n_1 - k_1, n_2 - k_2)]$, and

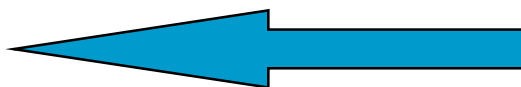
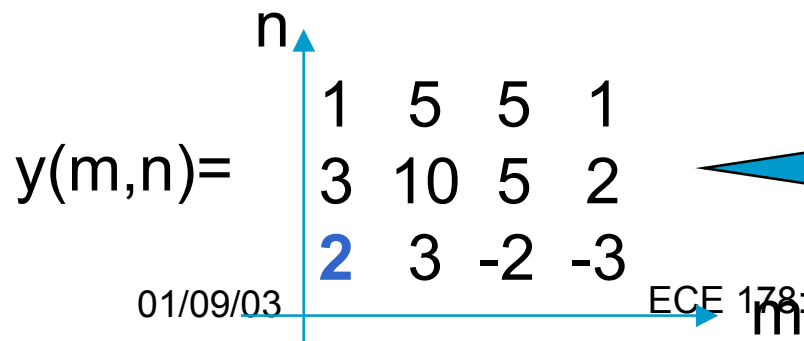
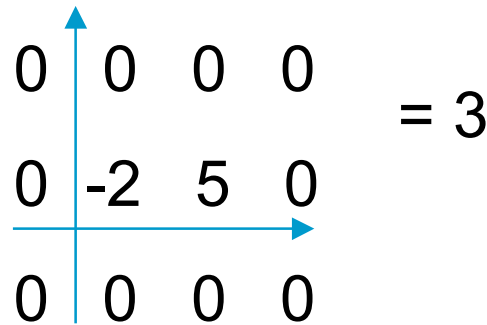
$$\begin{aligned} y(n_1, n_2) &= T \left[\sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2) \delta(n_1 - k_1, n_2 - k_2) \right] \\ &= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2) T[\delta(n_1 - k_1, n_2 - k_2)] \\ &= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2) \end{aligned}$$

$$y(n_1, n_2) = h(n_1, n_2) * x(n_1, n_2)$$

Convolution: example

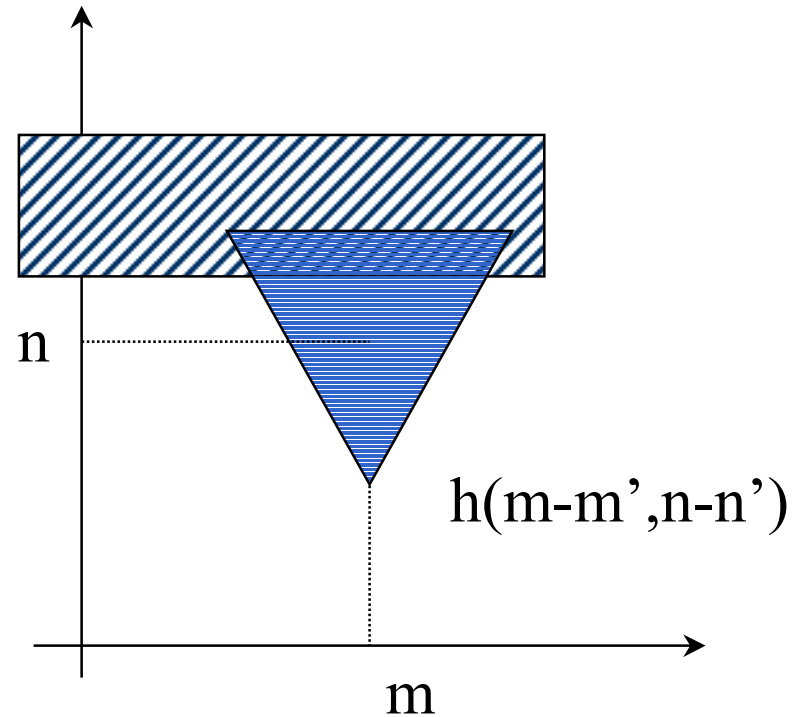
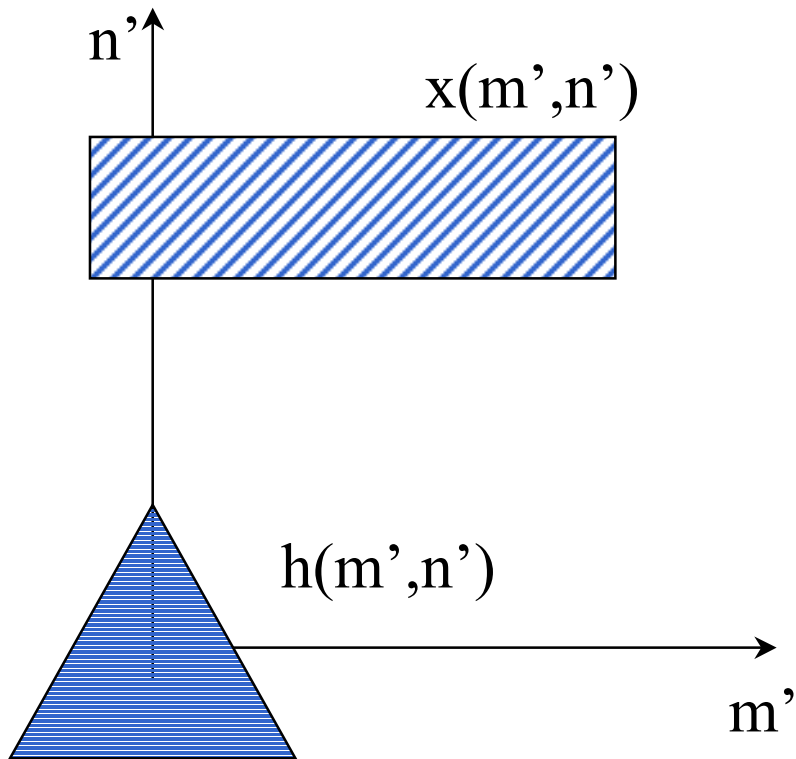


$$y(1,0) = \sum_{k,l} x(k,l)h(1-k, -l) =$$



verify!

Discrete Convolution in 2D



output = sum of the product
of the two in the overlapped region.