Fault-Tolerant Computing

Software Design Methods
About This Presentation

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<tr>
<td>First</td>
<td>Dec. 2006</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Agreement and Adjudication
Abe, I just noticed – our marriage license has an expiration date! And it’s today! Do you know anything about this, Abe? . . . Abe? . . . Abe?

“Next case: the Internet economy versus millions of investors who should have known better.”

Software Specification

1. As Management Requested It
2. As Specified in the Project Request
3. As Designed By The Senior Analyst
4. As Produced By The Programmers
5. As Installed
6. What The User Wanted

“May I see the manager? I have a small problem...”

“Would you like to make it a large for only twenty-five cents!”

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“Drink 8 Glasses of Water Everyday”

“It was the only mission statement everyone could agree on.”

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Multilevel Model of Dependable Computing

Level → Component Logic Information System Service Result

Ideal → Defective → Faulty → Erroneous → Malfunctioning → Degraded → Failed

Unimpaired Low-Level Impaired Mid-Level Impaired High-Level Impaired

Legend: Entry → Deviation ← Remedy ⊞ Tolerance
Introduction to Voting

Voting schemes and associated terminology in dependable computing were originally derived from concepts in sociopolitical elections.

With inputs drawn from a small set of integers, the similarity between the two domains is strong.

**Example:** Radar image analysis used to classify approaching aircraft type as civilian (0), fighter (1), bomber (2).

If three independent units arrive at the conclusions \(<1, 1, 2>\), then the presence of a fighter plane may be assumed.

Option or candidate 1 “wins” a majority of the vote.

With a large or infinite input domain, voting takes on a new meaning.

**Example:** There is no strict majority when distance of an approaching aircraft, in km, is indicated as \(<12.5, 12.6, 14.0>\), even though the good agreement between 12.5 and 12.6 could lead to a 12.55 km estimate.
A General Framework for Voting

Virtually all voting schemes of practical interest can be formulated in terms of the generalized weighted voting model, as follows:

Given $n$ input data objects $x_1, x_2, \ldots, x_n$ and associated nonnegative real votes $v_1, v_2, \ldots, v_n$, with $\sum v_i = V$, compute output $y$ and its vote $w$ such that $y$ is “supported by” a set of input objects with votes totaling $w$, where $w$ satisfies a condition associated with the voting subscheme.

Possible voting subschemes:

- **Unanimity** $w = V$
- **Majority** $w > V/2$
- **Supermajority** $w \geq 2V/3$
- **Byzantine** $w > 2V/3$
- **Plurality** $(w$ for $y) \geq (w$ for any $z \neq y)$
- **Threshold** $w > \text{a preset lower bound}$

![Diagram of generalized weighted voter](image)
A Taxonomy of Voting Schemes

One can classify generalized weighted voting schemes into $2^4 = 16$ categories based on dichotomies associated with input data (the $x_i$s), output data ($y$), input votes (the $v_i$s), and output vote ($w$).

### Input
- **Data**
  - Exact/Inexact
- **Vote**
  - Oblivious/Adaptive

### Output
- **Consensus/Mediation**
- **Threshold/Plurality**

First entry in each box is the simpler of the two.

- A term such as “threshold voting” stands for 8 different methods.
- Total support from a subset of inputs (quorum), or shades of support from all inputs.
- Support exceeds a lower bound, or is max over all possible outputs.

Input objects inflexible, or representing flexible “neighborhoods”.

Input weights set at design time, or allowed to change dynamically (adjustable/variable).
Approximate Voting

The notion of an input object “supporting” a particular output (akin to a hypothesis supporting an end result or conclusion) allows us to treat approximate and exact voting in the same way.

**Example 1:** Input objects are points in the 2D space and the level of “support” between them is a function of their Euclidean distance.

**Example 2:** Input objects are conclusions of character recognizers as to the identity of a character, with varying degrees of mutual support.

![Diagram](image)

- Largest maximal compatible class
- Median selection from the maximal mutually supportive set of inputs
- Proximity limit

Dec. 2006
Agreement and Adjudication
Slide 9
Generalized Median Voting

To find the median of a set of numbers, repeatedly remove largest and smallest numbers, until only one or two remain.

If we replace “largest and smallest numbers” by “the two inputs that are furthest apart,” we can use an arbitrary distance metric in our screening.

A distance metric is any metric (mapping of pairs of inputs into real values) that satisfies the three conditions:

- **Isolation**\[ d(x, y) = 0 \text{ iff } x = y \]
- **Symmetry**\[ d(x, y) = d(y, x) \]
- **Triangle inequality**\[ d(x, y) + d(y, z) \geq d(x, z) \]

For example, the Hamming distance satisfies these conditions.
Approval Voting

Approval voting was introduced to prevent the splitting of votes among several highly qualified candidates from leading to the election of a less qualified candidate in plurality voting.

In approval voting, a voter divides the set of candidates into two subsets of “qualified” and “not qualified” and indicates approval of the first subset.

In the context of computing, approval voting is useful when a question has multiple answers or when the solution process is imprecise or fuzzy.

Example question: What is a safe setting for a particular parameter in a process control system?

When the set of approved values constitute a continuous interval of real values, we have “interval” inputs and “interval” voting.
Interval Voting

Inputs to the voting process are intervals, representing approved values. How should the voting result be derived from the input intervals?

Various combining rules can be envisaged. If there is overlap among all intervals, then the decision is simple. Depending on context, it may make sense to consider greater levels of approval near the middle of each interval or to associate negative approval levels outside the approved intervals.

Combining two 3D intervals:
- FLIR uncertainty region
- Radar uncertainty region
- Intersection

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The Impossibility of Perfect Voting

Properties of an ideal voting scheme:

1. No big brother
   (voters free to express preference)
2. Independence of irrelevant alternatives
   (preference for one candidate over another is independent of all others)
3. Involvement
   (every outcome is possible)
4. No dictatorship or antidictatorship
   (outcome not always conforming to, or opposite of, one voter’s view)

**Arrow’s Theorem:**
No voting scheme exists that satisfies all four conditions

**True majority voting scheme:**
Each voter orders all the candidates; no circular preference allowed
Choose a candidate who beats every other one in pairwise competitions
(both simple majority and plurality rules fail to choose a candidate)
Distributed Voting: The Agreement Problem

**Problem:** Derive a highly reliable value from multiple computation results or stored data replicas at multiple sites.

**Key challenge:** Exchange data among nodes so that all healthy nodes end up with the same set of values; this guarantees that running the same decision process on the healthy nodes produces the same result.

Errors are possible in both data values and in their transmission between sites.

Agreement algorithms generally use multiple rounds of communication, with values held at each site compared and filtered, until the set of values held at all sites converge to the same set.
Byzantine Failures in Distributed Voting

Three sites are to collect three versions of some parameter and arrive at consistent voting results

Assume median voting

```
Data capture (sensor, A/D, . . . )

Input

Processing

120, 121, 122

↓

120

↓

122

120, 121, 122

↓

120

↓

122

120, 122, 158

↓

120

↓

122
```
The Interactive Consistency Algorithm

ICA(0) [no failure]
1. The transmitter sends its value to all other $n - 1$ nodes
2. Each node uses the value received from the transmitter, or a default value $\Phi$ if it received no value

ICA($f$), $f > 0$ [f failures]
1. The transmitter sends its value to all other $n - 1$ nodes
2. Let $v_i$ be the value received by node $i$ from the transmitter, or a default value $\Phi$ if it received no value; node $i$ then becomes the transmitter in its own version of ICA($f - 1$), sending its value to $n - 2$ nodes
3. For each node $i$, let $v_{ij}$ be the value it received from node $j$, or a default value $\Phi$ if it received no value from node $j$. Node $i$ then uses the value majority($v_{i,1}, v_{i,2}, \ldots, v_{i,i-1}, v_{i,i+1}, \ldots, v_{i,n}$)
Correctness and Performance of ICA

**Theorem 1:** With ICA($f$), all nonfailed nodes will agree on a common value, provided that $n \geq 3f + 1$ (proof is by induction on $f$)

ICA works correctly, but it needs an exponential number of messages:

$$(n-1) + (n-1)(n-2) + (n-1)(n-2)(n-3) + \ldots + (n-1)(n-2) \ldots (n-m)$$

More efficient agreement algorithms exist, but they are more difficult to describe or to prove correct; $f + 1$ rounds of message exchange is the least possible, so some algorithms trade off rounds for # of messages

**Theorem 2:** In a network $G$ with $f$ failed nodes, agreement is possible only if the connectivity is at least $2f + 1$
The Byzantine Generals Problem

A general and \( n - 1 \) lieutenants lead \( n \) divisions of the Byzantine army camped on the outskirts of an enemy city.

The \( n \) divisions can only communicate via messengers.

We need a scheme for the generals to agree on a common plan of action (attack or retreat), even if some of the generals are traitors who will do anything to prevent loyal generals from reaching agreement.

The problem is nontrivial even if messengers are totally reliable.

With unreliable messengers, the problem is very complex.
Byzantine Generals with Reliable Messengers

With $f$ Byzantine failures, $\geq 3f + 1$ nodes needed to reach agreement
Byzantine Resiliency

To tolerate \( f \) Byzantine failures:

We need \( 3f + 1 \) or more FCRs (fault containment regions)

FCRs must be interconnected via at least \( 2f + 1 \) disjoint paths

Inputs must be exchanged in at least \( f + 1 \) rounds

Corollary 1: Simple 3-way majority voting is not Byzantine resilient

Corollary 2: Because we need \( 2f + 1 \) good nodes out of a total of \( 3f + 1 \) nodes, at least a fraction \( \frac{2f + 1}{3f + 1} = \frac{2}{3} + \frac{1}{9f + 3} \) of the nodes must be healthy

Anything greater than a supermajority (2/3) will do