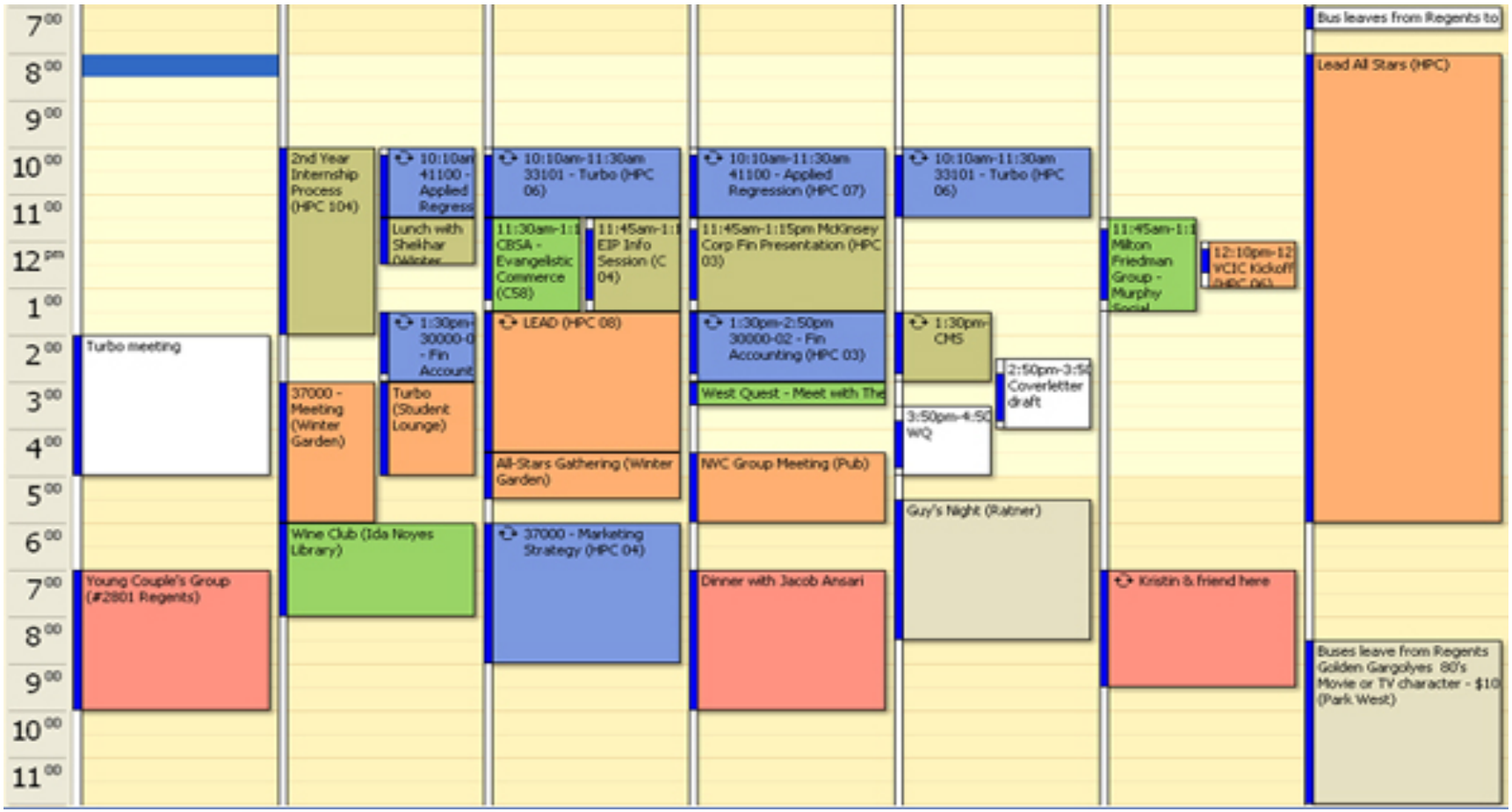
Task Scheduling

A Lecture in CE Freshman Seminar Series:
Ten Puzzling Problems in Computer Engineering



About This Presentation

This presentation belongs to the lecture series entitled "Ten Puzzling Problems in Computer Engineering," devised for a ten-week, one-unit, freshman seminar course by Behrooz Parhami, Professor of Computer Engineering at University of California, Santa Barbara. The material can be used freely in teaching and other educational settings. Unauthorized uses, including any use for financial gain, are prohibited.

Edition	Released	Revised	Revised
First	May 2007		

Mini-Sudoku Puzzle

Complete entries in this chart so that numbers 1-6 appear without repetition in each row, each column and each 2 × 3 block

	2				
		4			1
6		3	1		
		5	3		6
4			6		
				3	

Standard Sudoku consists of a 9 × 9 chart, but this mini version is good for a quick fix

USA Today carries a daily mini-Sudoku at its site: http://puzzles.usatoday.com

Sudoku isn't a math puzzle: Can substitute letters A-F or any other six symbols for numbers 1-6

Mini-Sudoku Puzzle: Solution Method

Complete entries in this chart so that letters A-F appear without repetition in each row, each column and each 2 × 3 block

	B		D		C
C		D			A
F	D	C	A		
B	A	E	C	D	F
D	C		F	A	
				C	D

To continue from here, write down all possible choices in the remaining blank boxes and see whether the resulting info leads to more progress

SuDoKu: abbr. in Japanese for “numbers must be single.” Euler may have invented it; Howard Garns (US) & Wayne Gould (HK) popularized it in modern times

Sudoku Puzzle: Easy Example

Complete entries in this chart so that numbers 1-9 appear without repetition in each row, each column and each 3 × 3 block

Many newspapers carry these puzzles; there are also many collections in book form

Sudoku puzzles of varying difficulties (easy, medium, hard, evil) are available at http://www.websudoku.com and several other Web sites, such as USA Today's site http://puzzles.usatoday.com

5			6	7	4			2
4		7	9		5	6		1
	3			8			7	
6		4				1		3
	9			5			6	
2		9	8		3	4		5
8			5	9	7			6

Sudoku Puzzle Solution Method

Strategy 1: Identify a missing number from a row, column, or block; if you can exclude all but one cell for that number, then write it down

					8			
5			6	7	4			2
4		7	9		5	6		1
1	3	25	4	8	6	25	7	9
6	58	4	7	2	9	1	58	3
7	9	28	3	5	1	28	6	4
2	7	9	8	6	3	4	1	5
8	4	1	5	9	7	23	23	6
					2			

7, 8, 9 missing from this column

1, 2, 3, 4 missing from this row

Strategy 2: When you can't make progress by Strategy 1, write down all candidate numbers in the cells and try to eliminate a number of options via reasoning. For example if xy, xy, xyz are candidates in three cells of a block, then the cell marked xyz must hold z

Sudoku Puzzle: Hard Example

Complete entries in this chart so that numbers 1-9 appear without repetition in each row, each column and each 3 × 3 block

			6		8			
		4						5
		1		5			6	4
	3	9	4					1
				2				
5					1	7	9	
8	5			3		1		
9						2		
			9		6			

Hard puzzles typically have fewer entries supplied, with each row, column, or block containing only a few entries

Hard puzzles may have handles or starting points (5 in the top left block or 9 in center and lower right blocks)

Variations on Sudoku

Other sizes (e.g., 6 × 6, with 2 × 3 blocks; or 16 × 16, with 4 × 4 blocks)

Combining this 2000s phenomenon with Rubik's cube of the 1980s . . .

or with the age-old sliding 15 puzzle

Task Scheduling Problem

We have a set of of tasks	Numbers in Sudoku puzzle
There are some “processors” that can execute tasks	Cells in Sudoku puzzle can hold numbers
Assign tasks to processors so as to meet certain constraints	Place numbers in cells while honoring some constraints

A task may fit only some processors
Tasks may have prerequisites tasks
Preemption may (not) be allowed
Tasks may have deadlines
Shortest schedule may be required

Use only numbers 1-9
Some numbers already placed
Different numbers in each row
Different numbers in each column
Different numbers in each block

Virtually all instances of the task scheduling problem are difficult (NP-complete), just like Sudoku

Resource Allocation Problem

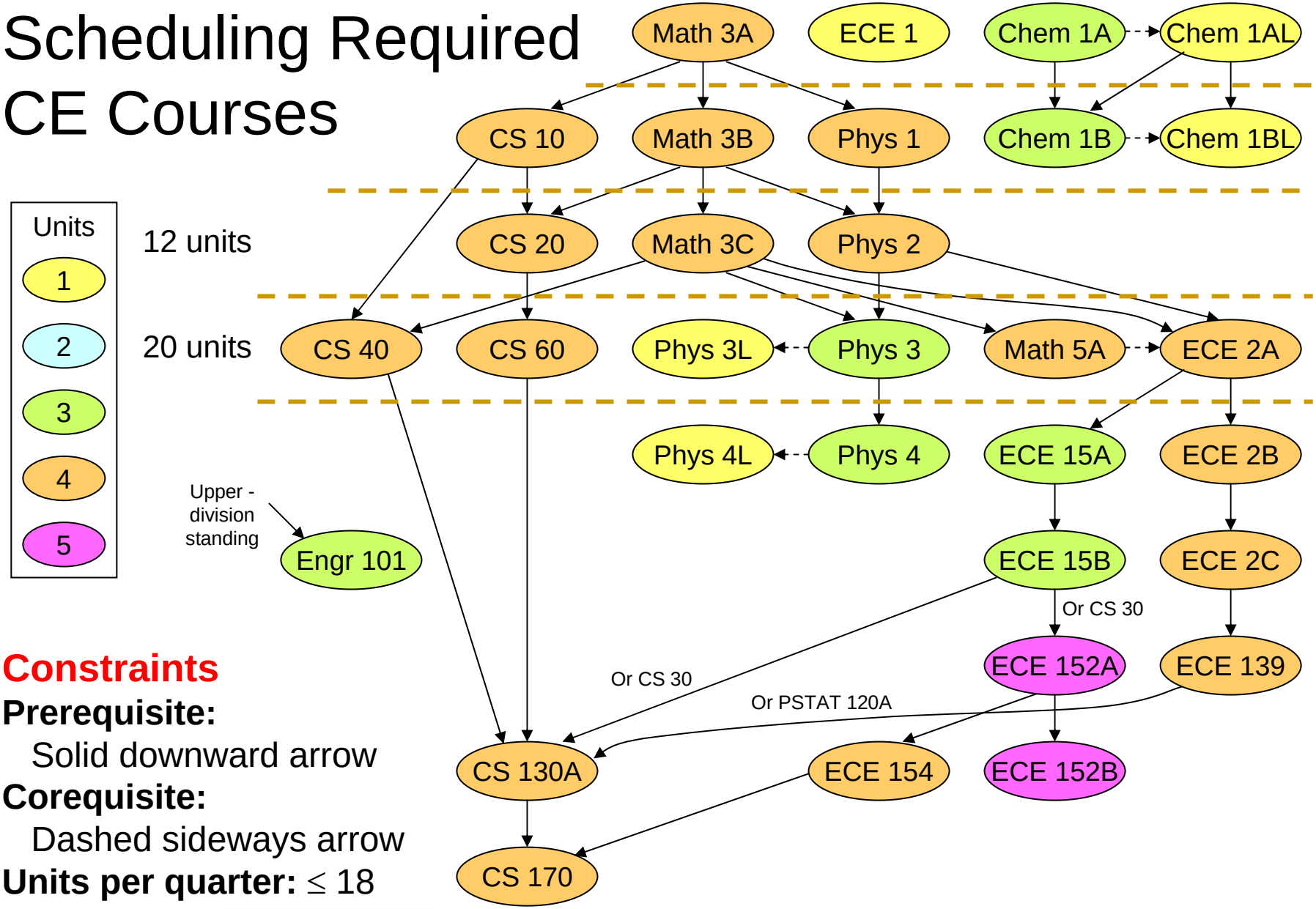
We have a set of of resources	Numbers in Sudoku puzzle
There are “locations” where resources may be placed	Cells in Sudoku puzzle can hold numbers
Assign resources to locations to meet certain constraints	Place numbers in cells while honoring some constraints

A resource may fit only some locations
Resources must be “easily” accessible
Resource mobility may (not) be allowed
Resource cost may differ by location
Lowest-cost assignment may be required

Use only numbers 1-9
Some numbers already placed
Different numbers in each row
Different numbers in each column
Different numbers in each block

Virtually all instances of the resource allocation problem are difficult (NP-complete), just like Sudoku

Scheduling Required CE Courses



Constraints

Prerequisite:

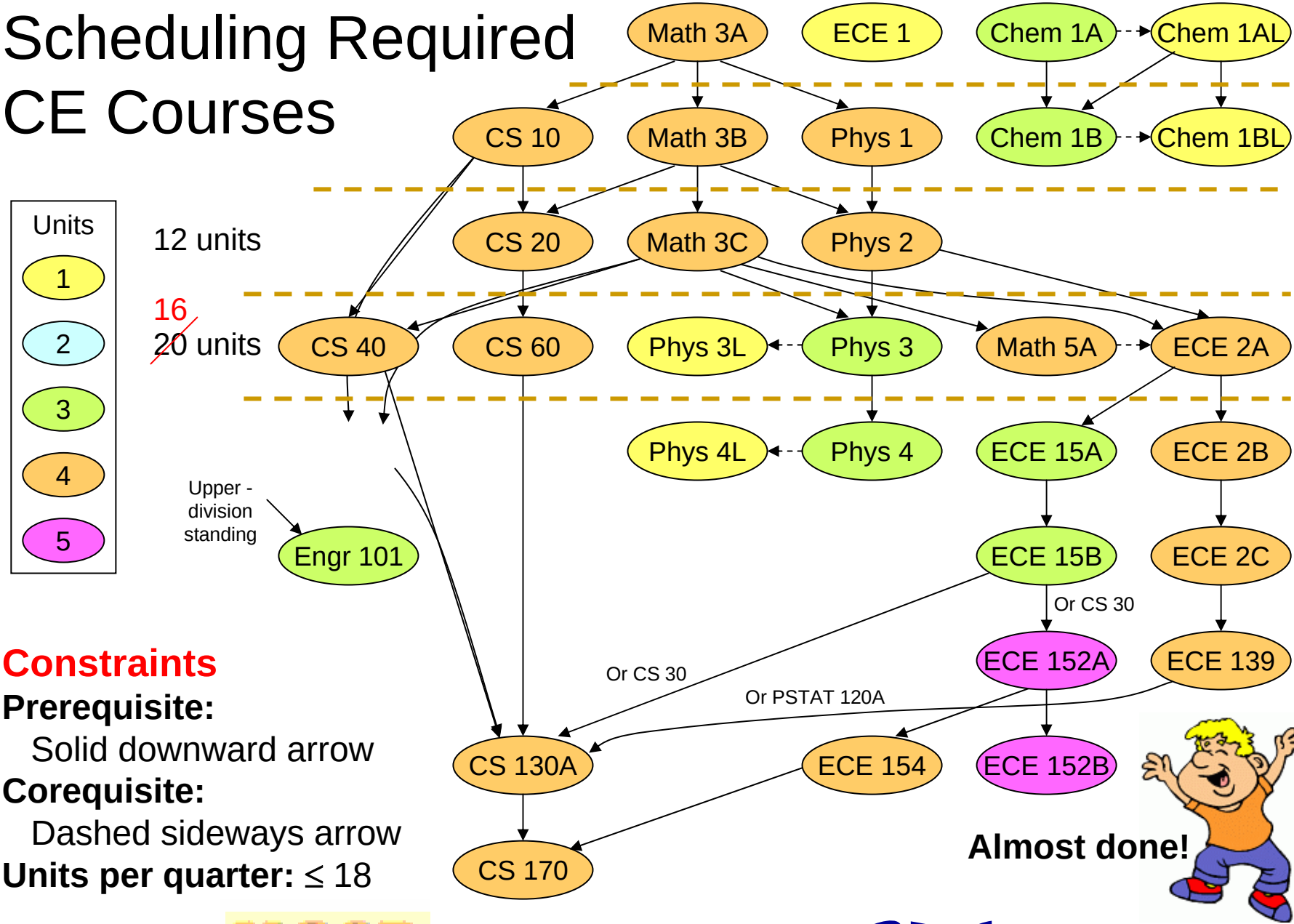
Solid downward arrow

Corequisite:

Dashed sideways arrow

Units per quarter: ≤ 18

Scheduling Required CE Courses



Constraints

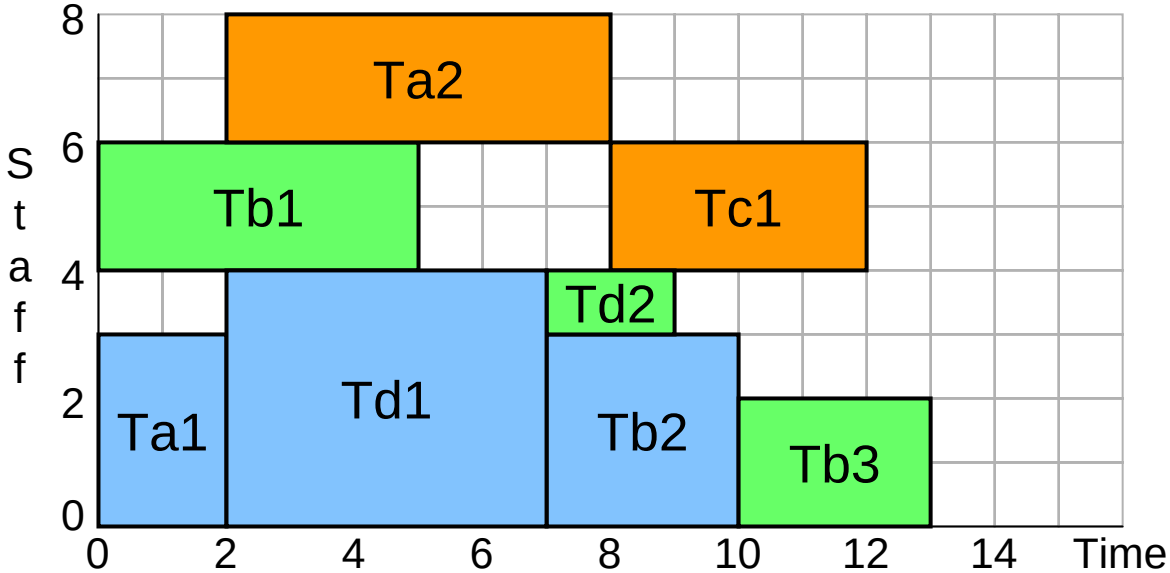
Prerequisite:
Solid downward arrow

Corequisite:
Dashed sideways arrow

Units per quarter: ≤ 18

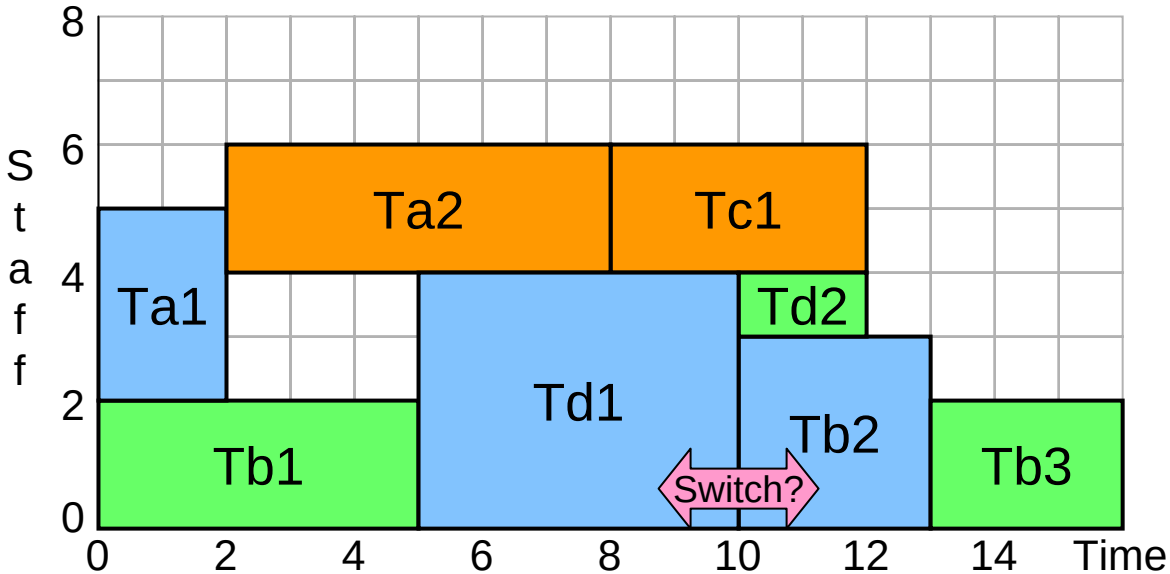
Almost done!

Job-Shop Scheduling

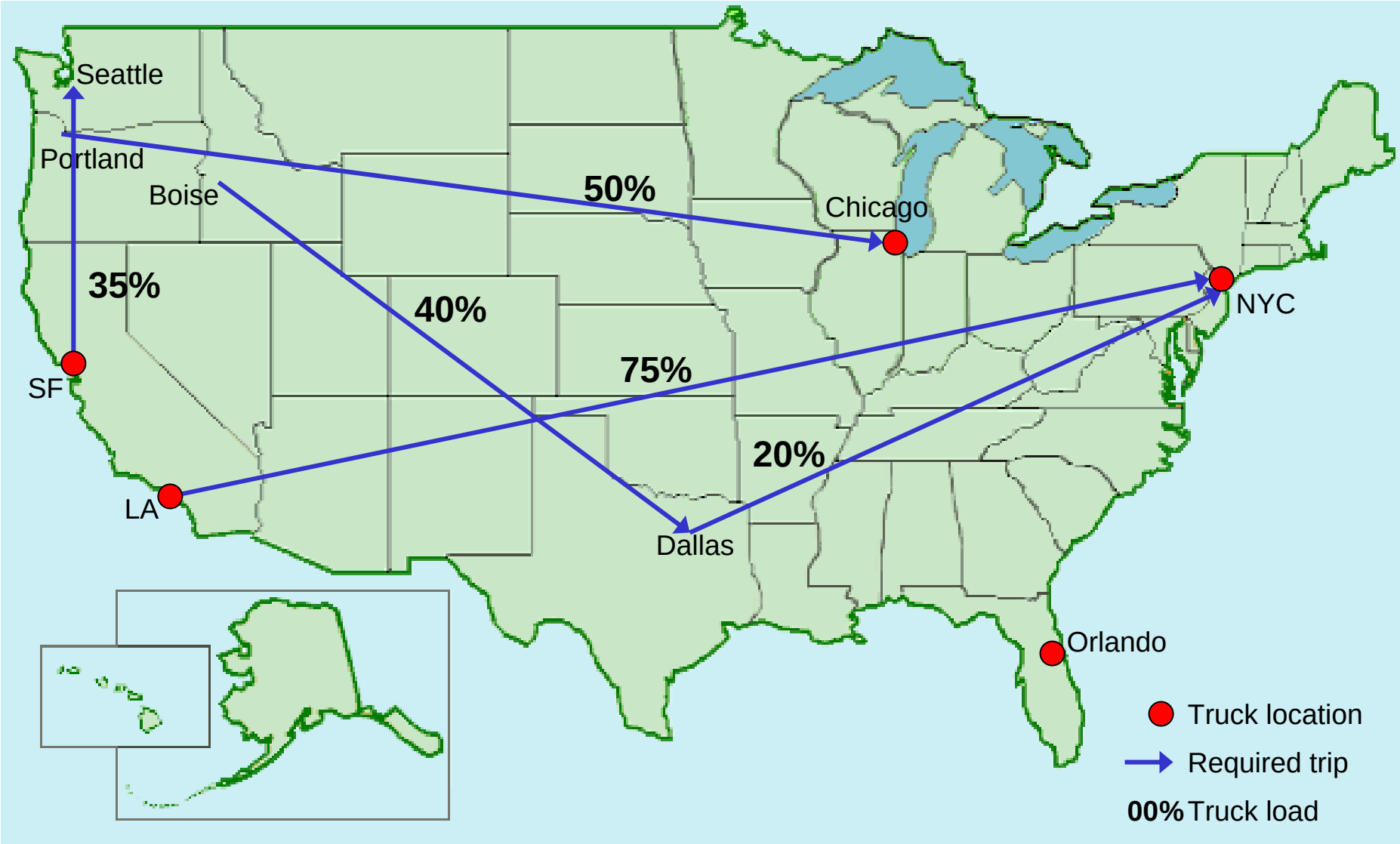


Job	Task	Machine	Time	Staff
Ja	Ta1	M1	2	3
Ja	Ta2	M3	6	2
Jb	Tb1	M2	5	2
Jb	Tb2	M1	3	3
Jb	Tb3	M2	3	2
Jc	Tc1	M3	4	2
Jd	Td1	M1	5	4
Jd	Td2	M2	2	1

M1 M2 M3



Truck Scheduling



Multiprocessor Scheduling

Task graph with unit-time tasks

Here's a heuristic known as list scheduling:

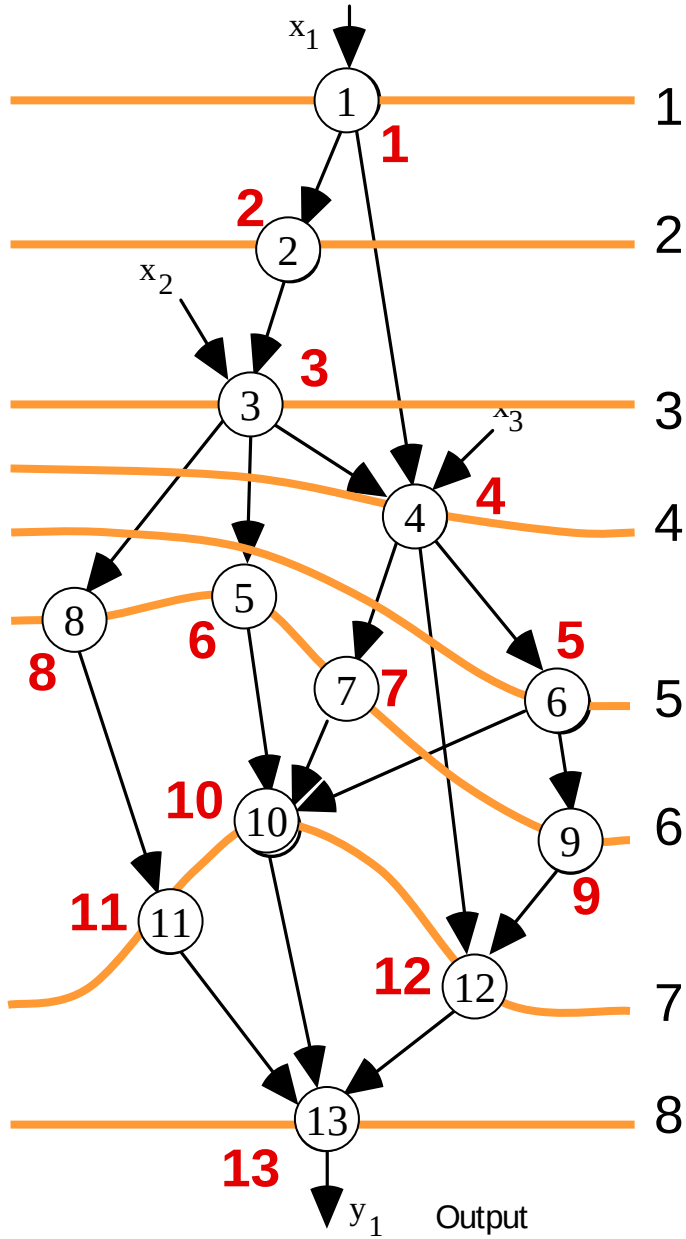
1. Find the depth T_∞ of the task graph
2. Take T_∞ as a goal for the running time T_p
3. Determine the latest possible start times
4. Assign priorities in order of latest times

$T_\infty = 8$ (execution time goal)

Latest start times: see the layered diagram

Priorities: shown on the diagram in red

When two tasks have the same "latest start time," a secondary tie-breaking rule is used



Assignment to Processors

Tasks listed in priority order

1* 2 3 4 6 5 7 8 9 10 11 12 13

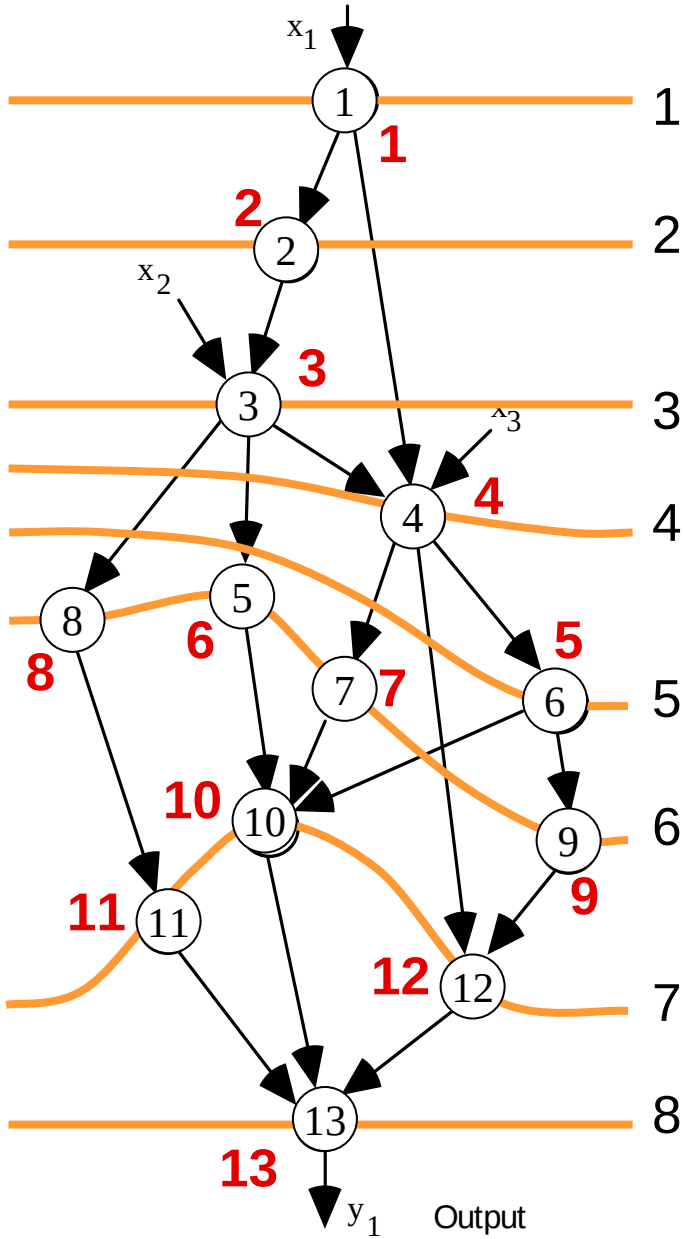
P_1	1	2	3	4	6	5	7	8	9	10	11	12	13

P_1	1	2	3	4	6	8	10	12	13
P_2				5	7	9	11		

P_1	1	2	3	4	6	9	12	13
P_2				5	7	10		
P_3				8	11			

1 2 3 4 5 6 7 8 9 10 11 12 13

Time Step



Even in this simple case of unit-time tasks, multiprocessor scheduling remains difficult with as few as 3 processors