

# ECE 137 A Mid-Term Exam

Thursday February 6, 2014

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 2 problems on this exam, and you have 75 minutes.

Use any and all reasonable approximations (5% accuracy is fine. ) , *AFTER STATING and approximately Justifying them.*

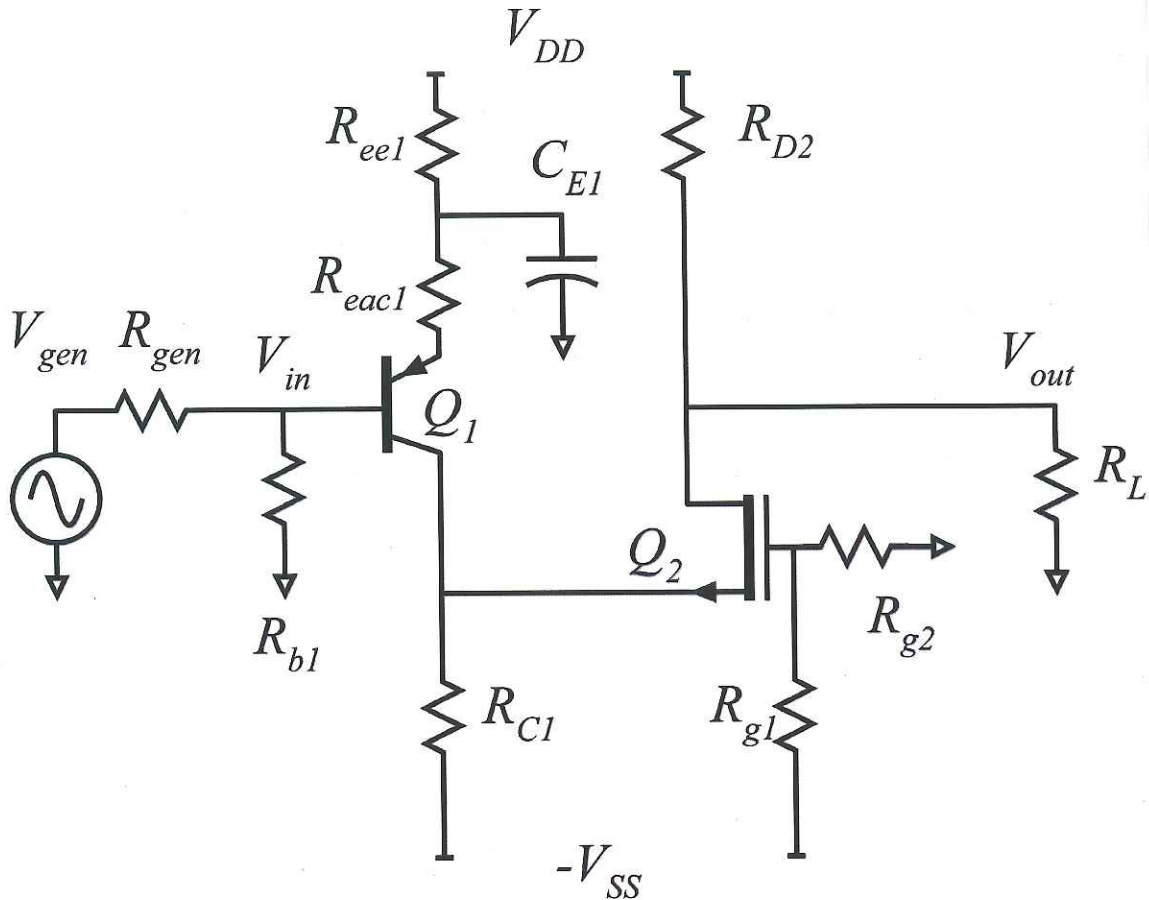
Name: \_\_\_\_\_

*Solution B*

Part	Points Received	Points Possible
1a		7
1b		7
1c		6
1d		15
1e		15
1f		6
1g		14
2a		12
2b		13
2c		5
TOTAL		100

**Problem 1, 70 points**

You will be working on the circuit below:



Q1:  $\beta = 50$ ,  $V_A = 50$  V

Q2: Velocity-limited  $V_{th} = 0.2$  V,  $1/\lambda = \text{infinity}$ ,  $\Delta V = L_g v_{th} / \mu = 0.1$  V,  $c_{ox} v_{th} W_g = 5$  mA/V

The supplies are +3V and -3V

$R_{gen} = 1000$  Ohms,  $R_L = 5,000$  Ohms,  $R_{g2} = 50$  kOhms,  $R_{eac1} = 37$  Ohms,  $R_{b1} = 10$  kOhms

$C_{e1}$  is very large (AC short-circuit)

Part a, 7 points

DC bias.

$V_{in}$  is at (approximately) zero volts DC.

The gate of Q2 is to be biased at -2 Volts

The drain is to be biased at zero volts.

Q1 is to be biased at 2 mA emitter current

Q2 is to be biased at 1 mA drain current.

Find the following:

$R_{c1} = 167\Omega$     $R_{g1} = 25k\Omega$     $R_{e1} = 1.11k\Omega$

$R_{g2} =$  \_\_\_\_\_    $R_{d2} = 3k\Omega$

① 
$$I_{D2} = 1mA = (5mA/V)(V_{gs} - V_{t2} - \Delta V)$$
$$1mA = (5mA/V)(V_{gs} - 0.2V - 0.1V)$$
$$V_{gs1} = 0.2V + 0.1V + 1mA/(5mA/V)$$
$$= 0.5V.$$

① 
$$V_{s1} = -0.5V - 2V = -2.5V$$

① 
$$R_{c1} \text{ carries } 3mA \text{ (} I_{c1} + I_{D2} \text{)}$$
$$R_{c1} = \frac{3V - 2.5V}{3mA} = 167\Omega.$$

① 
$$\text{Base of } Q1 \text{ @ } 0V \rightarrow V_{e1} = 0.7V.$$

① 
$$R_{E1} = \frac{3V - 0.7V}{2mA} - R_{e1} = 1.15k\Omega - 37\Omega = 1.11k\Omega.$$

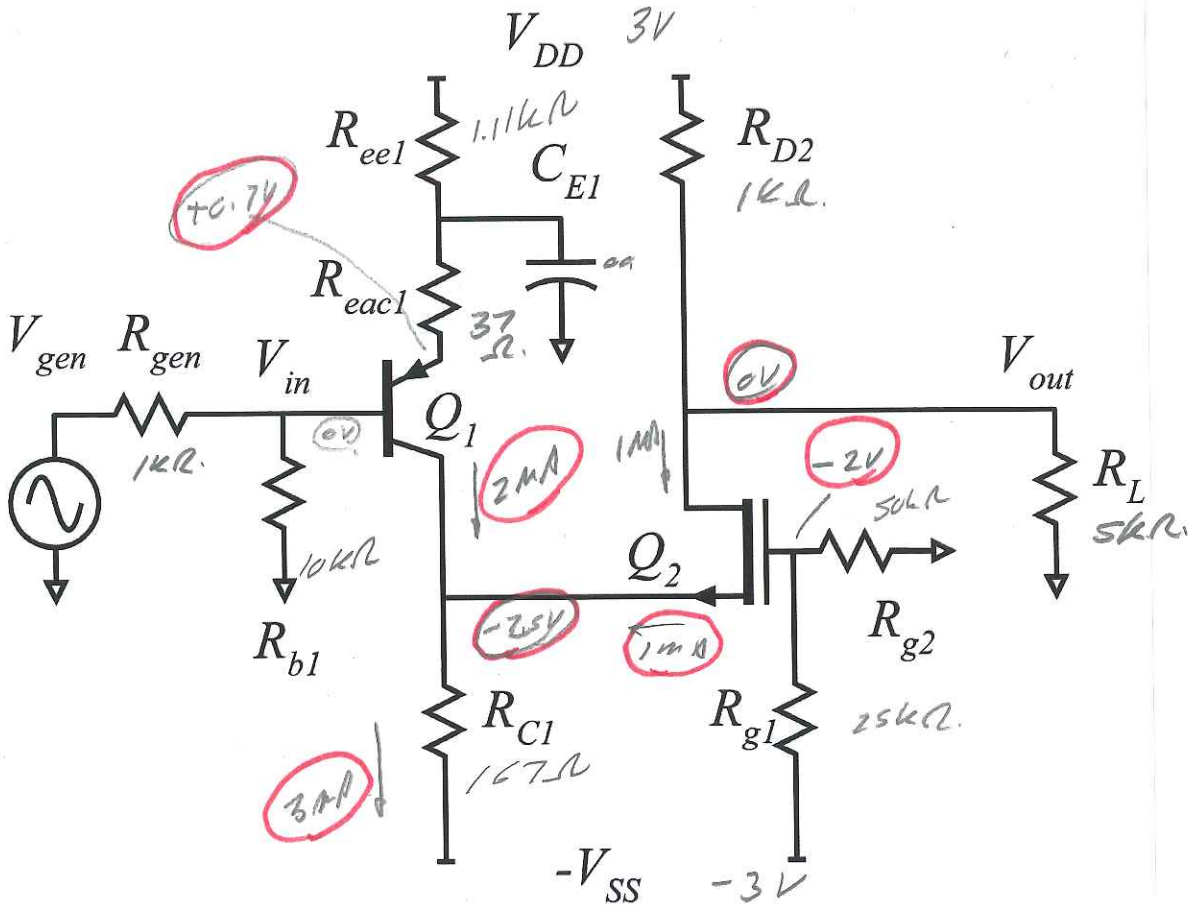
① 
$$R_{D2} = \frac{3V - 0V}{1mA} = 3k\Omega$$

① 
$$R_{g2} \text{ carries } 2V / 50k\Omega = 40\mu A$$
$$R_{g1} = \frac{3V - 2V}{40\mu A} = 25k\Omega$$

$Q1: \beta = 50, V_A = 50V$   
 $Q2: V_{th} = 0.2V, I = 0V, \Delta V = 0.1V, 5mA/V.$

Part b, 7 points

DC bias



On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors

Part c. 6 points

Find the small signal parameters of Q1 and Q2.

Transistor Q1:  $g_m = \underline{77 \text{ mS}}$   $R_{ce} = \underline{50 \text{ k}\Omega}$   $R_{be} = \underline{1.3 \text{ k}\Omega}$

Transistor Q2:  $g_m = \underline{5 \text{ mS}}$   $R_{ds} = \underline{\infty \Omega}$

Q1:  $I_C = 2 \text{ mA}$ ,  $V_A = 100 \text{ V}$ ,  $\beta = 100$

1.5  $\left[ g_m = \frac{2 \text{ mA}}{26 \text{ mV}} = 76.9 \text{ mS} = \frac{1}{13 \Omega}$

1.5  $\left[ R_{be} = \beta / g_m = 100 \cdot 13 \Omega = 1.3 \text{ k}\Omega$

1  $\left[ R_{ce} = \frac{V_{CE} + V_A}{I_C} \approx \frac{V_A}{I_C} = \frac{100 \text{ V}}{2 \text{ mA}} = 50 \text{ k}\Omega$

Q2:  $I_D = 1 \text{ mA}$

1  $\left[ g_m = 5 \text{ mA/V} = 5 \text{ mS}$

1  $\left[ R_{ds} = \infty \text{ because } \mu = 0 \text{ V}^{-1}$

Part d, 15 points.

Find the small signal voltage gain ( $V_{d2}/V_{s2}$ ) of Q2 and Q2's small-signal input resistance.

$$V_{d2}/V_{s2} = \underline{9.38}$$

$$R_{in,q2} = \underline{200\Omega}$$

3.5 [ Q2: common gate.

3.5 [ BECAUSE  $r_{DS} = \infty$ ,  $R_{in2} = 1/g_{m2} = 200\Omega$

3.5 [  $R_{Leg2} = R_{D2} \parallel r_L = 3k\Omega \parallel 5k\Omega = 1.88k\Omega$ .

$$3.5 \left[ \frac{V_{D2}}{V_{S2}} = \frac{R_{Leg2}}{R_{in2}} = \frac{1.88k\Omega}{200\Omega} = 9.38 \right]$$

Part e, 15 points

Find the small signal voltage gain ( $V_{c1}/V_{b1}$ ) of Q1 and the \*\*\* amplifier \*\*\* input resistance.

$V_{c1}/V_{b1} = \underline{\underline{-1.82}}$

$R_{in, amplifier} = \underline{\underline{3.33 \text{ k}\Omega}}$

2 [ Q1 - common emitter with degeneration

$1/g_m = 13 \Omega, R_{EAC1} = 37 \Omega$

2 [  $R_{out, Q1} = R_{C1} (1 + g_m R_{EAC1}) = 50 \text{ k}\Omega (1 + 37/13)$   
= very large, neglect

3 [  $R_{Leg1} = R_{C2} \parallel R_{C1} \parallel R_{out, Q1}$   
=  $200 \Omega \parallel 167 \Omega = 91.0 \Omega$

3 [  $A_{v1} = \frac{-R_{Leg1}}{1/g_m + R_{EAC1}} = \frac{-91 \Omega}{13 \Omega + 37 \Omega} = -1.82$

3 [  $R_{in, Q1} = \beta (R_{EAC1} + 1/g_m) = 100 \cdot (37 \Omega + 13 \Omega)$   
=  $5 \text{ k}\Omega$

2. [  $R_{in, amp} = R_{in, Q1} \parallel R_{B1} = 5 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 3.33 \text{ k}\Omega$



Part f, 6 points

Find  $(V_{out}/V_{in})$ ,  $(V_{in}/V_{gen})$  and  $(V_{out}/V_{gen})$

$$(V_{out}/V_{in}) = \underline{\underline{-17.1}}$$

$$(V_{in}/V_{gen}) = \underline{\underline{0.770}}$$

$$(V_{out}/V_{gen}) = \underline{\underline{-13.1}}$$

$$2 \left[ V_{in}/V_{gen} = \frac{R_{in}}{R_{in} + R_{gen}} = \frac{3.33k\Omega}{1k\Omega + 3.33k\Omega} = 0.770 \right]$$

$$2 \left[ V_{out}/V_{in} = A_{v1} \cdot A_{v2} = (-1.82)(9.38) = -17.1 \right]$$

$$2 \left[ V_{out}/V_{gen} = (V_{out}/V_{in}) \cdot (V_{in}/V_{gen}) = -17.1 \cdot (0.770) = -13.1 \right]$$



Part g, 14 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. Give the sign (+ or -) in your answers below.

Cutoff of Q1; Maximum  $\Delta V_{out}$  resulting = -1.71V

Saturation of Q1; Maximum  $\Delta V_{out}$  resulting = +25.3V

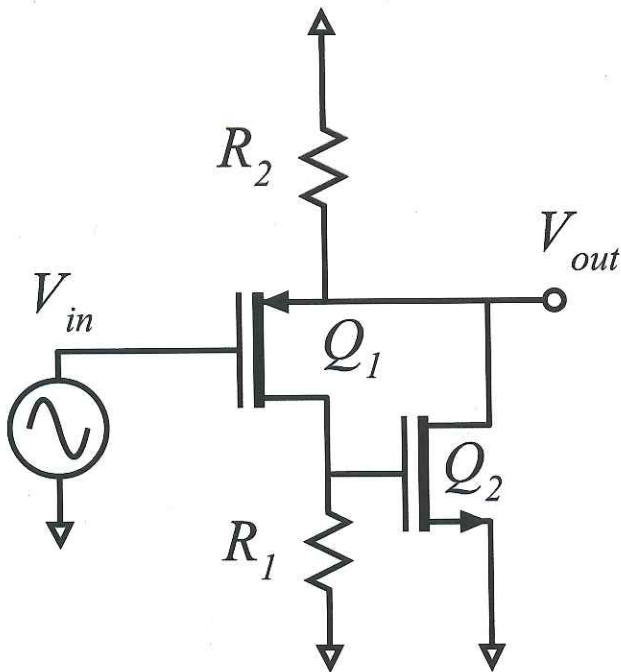
Cutoff of Q2; Maximum  $\Delta V_{out}$  resulting = +1.88V

Knee voltage of Q2; Maximum  $\Delta V_{out}$  resulting = -2.4V

Note:  $A_{v2} = 9.38$

- 3.5 Cutoff Q1:
  - 1.  $[\Delta I_{C1} / \max = 2 \mu A - 0 \mu A = 2 \mu A \text{ decrease}]$
  - 1.  $[R_{Eq1} = 91 \Omega]$
  - 1.  $[\Delta V_{C1} / \max = 91 \Omega \cdot 2 \mu A = 182 \text{ mV}, \downarrow \text{negative-going}]$
  - 1/2  $[\Delta V_{out} = 182 \text{ mV} \cdot A_{v2} = 182 \text{ mV} \cdot 9.38 = -1.71 \text{ V}]$
- 3.5 Saturation Q1:
  - 1.  $[V_{CEQ} = 0.7 \text{ V} + 2.5 \text{ V} = 3.2 \text{ V}, V_{CE\text{set}} \approx 0.5 \text{ V}]$
  - 1.  $[\Delta V_{CE} / \max = 3.2 \text{ V} - 0.5 \text{ V} = 2.7 \text{ V}]$
  - 1.  $[ \text{neglecting } \Delta V_{CE}, \Delta V_{C} = +2.7 \text{ V} \uparrow ]$
  - 1/2  $[\Delta V_{out} = 2.7 \text{ V} (9.38) = +25.3 \text{ V}]$
- 3.5 Cutoff Q2:
  - 1.5  $[I_{DQ} = 1 \text{ mA}, \Rightarrow \Delta I_{D} / \max = 1 \text{ mA} \text{ (decrease)}]$
  - 2  $[\Delta V_{out} = 1 \text{ mA} \cdot R_{Eq} = 1 \text{ mA} \cdot 1.88 \text{ k}\Omega = +1.88 \text{ V} \uparrow]$
- 3.5 Knee Q2:
  - 1  $[V_{DSQ} = 2.5 \text{ V}]$
  - 1  $[V_{DS\text{knee}} = 0.1 \text{ V} \text{ (voltage-saturated)}]$
  - 1/2  $[\Delta V_{DS} = 2.4 \text{ V} \downarrow]$

**Problem 2, 30 points**  
nodal analysis



You will be working on the circuit to the left.

Ignore DC bias analysis. You don't need it.

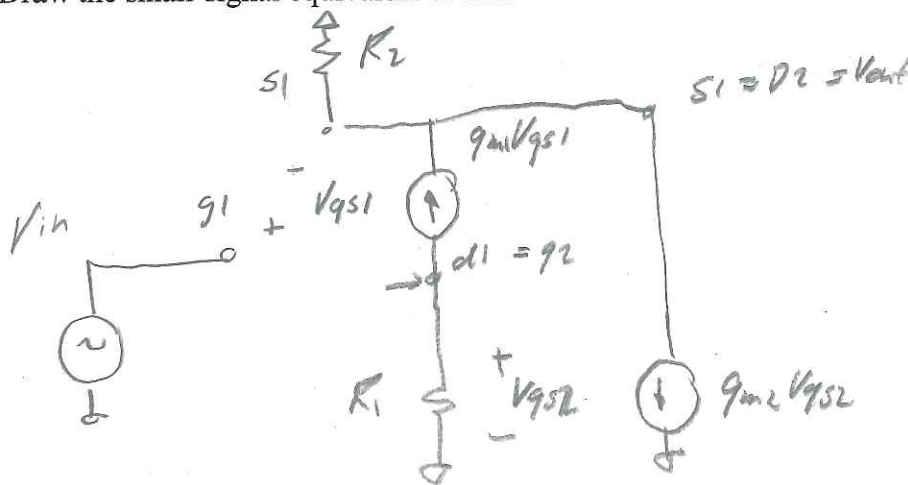
Transistor 1 has transconductance  $g_{m1}$ .

Transistor 2 has transconductance  $g_{m2}$ .

The drain-source resistances  $R_{ds}$  of both transistors are infinity (so you don't need to draw it!)

Part a, 12 points

Draw the small-signal equivalent circuit



*points off if  $g_m$  generators not labelled or controlling voltages not identified.*

Part b, 13 points

Find, by nodal analysis, a small-signal expression for  $V_{out}/V_{in}$ .

$V_{out}/V_{in} =$  \_\_\_\_\_

there are two unknown node voltages:  $V_{out}$ ,  $V_{g2}$

$\Sigma I = 0$  @  $V_{g2} = V_{d1}$ ; (+ = current leaving)

$$4. \left[ 0 = g_{m1} V_{gs1} + V_{g2} / R_1 \right]$$

$$0 = g_{m1} (V_{in} - V_{out}) + V_{g2} / R_1 = 0$$

$$\boxed{V_{g2} (1/R_1) + V_{out} (-g_{m1}) = -g_{m1} V_{in}}$$

$\Sigma I = 0$  @  $V_{out}$  (+ = current leaving)

$$4. \left[ g_{m2} V_{gs2} - g_{m1} V_{gs1} + V_{out} / R_2 = 0 \right]$$

$$g_{m2} V_{g2} - g_{m1} (V_{in} - V_{out}) + V_{out} / R_2 = 0$$

$$\boxed{V_{g2} (g_{m2}) + V_{out} (g_{m1} + 1/R_2) = g_{m1} V_{in}}$$

$$\begin{bmatrix} 1/R_1 & -g_{m1} \\ g_{m2} & g_{m1} + 1/R_2 \end{bmatrix} \begin{bmatrix} V_{g2} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -g_{m1} \\ g_{m1} \end{bmatrix} V_{in}$$

$$V_{out} = N / D$$

$$D = \begin{vmatrix} 1/R_1 & -g_{m1} \\ g_{m2} & g_{m1} + 1/R_2 \end{vmatrix} = (1/R_1)(g_{m1} + 1/R_2) + g_{m1}g_{m2}$$

$$N = \begin{vmatrix} 1/R_1 & -g_{m1} \\ g_{m2} & g_{m1} \end{vmatrix} V_{in} = (g_{m1}/R_1 + g_{m1}g_{m2}) V_{in}$$

$$\frac{V_{out}}{V_{in}} = g_{m1} \frac{(g_{m2} + 1/R_1)}{(1/R_1)(g_{m1} + 1/R_2) + g_{m1}g_{m2}}$$

← 5

(ignore)

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{g_{m1}(1 + g_{m2}R_1)}{g_{m1} + 1/R_2 + g_{m1}g_{m2}R_1} \\ &= \frac{g_{m1}(1 + g_{m2}R_1)}{g_{m1}(1 + g_{m2}R_1) + 1/R_2} = \frac{R_2}{\frac{1}{g_{m1}(1 + g_{m2}R_1)} + R_2} \end{aligned}$$

other form

12A

$$1/R_1 = 1 \text{ mS}$$

$$1/R_2 = 0.5 \text{ mS}$$

Part c, 5 points

$g_{m1} = 1 \text{ mS}$   $g_{m2} = 10 \text{ mS}$ ,  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 2 \text{ k}\Omega$

Give a numerical value for  $V_{out}/V_{in}$ .

$V_{out}/V_{in} = \underline{\hspace{2cm}}$

$$\frac{V_{out}}{V_{in}} = \frac{1 \text{ mS} (10 \text{ mS} + 1 \text{ mS})}{(1 \text{ mS}) (1 \text{ mS} + 0.5 \text{ mS}) + (1 \text{ mS} \cdot 10 \text{ mS})}$$

$$= \frac{1 (11)}{(1.5) + 10} = \frac{11}{11.5} \quad \checkmark$$

$$= \underline{\underline{0.956}}$$