Final Exam, ECE 137A  

Wednesday March 19, 2014    7:30-10:30 PM

Name: Solution A

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted. Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), after stating & justifying them. Show your work: Full credit will not be given for correct answers if supporting work is missing.

Good luck

<table>
<thead>
<tr>
<th>Time function</th>
<th>LaPlace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(t)$ impulse</td>
<td>1</td>
</tr>
<tr>
<td>$U(t)$ unit step-function</td>
<td>$1/s$</td>
</tr>
<tr>
<td>$e^{-\alpha}U(t)$</td>
<td>$\frac{1}{s + \alpha}$</td>
</tr>
<tr>
<td>$e^{-\alpha}\cos(\omega t)U(t)$</td>
<td>$\frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$</td>
</tr>
<tr>
<td>$e^{-\alpha}\sin(\omega t)U(t)$</td>
<td>$\frac{\omega}{(s + \alpha)^2 + \omega^2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Points Received</th>
<th>Points Possible</th>
<th>Part</th>
<th>Points Received</th>
<th>Points Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>5</td>
<td>15</td>
<td>2c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td>6</td>
<td>10</td>
<td>2d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td>4</td>
<td>7</td>
<td>3a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1d</td>
<td>10</td>
<td>8</td>
<td>3b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1e</td>
<td>10</td>
<td>7</td>
<td>3c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a</td>
<td>10</td>
<td>8</td>
<td>3d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2b</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1, 35 points

**This is an NOT an Op-Amp:** Analyze under the assumption that the differential and common mode input voltages are at zero volts.

All the transistors have the same (matched) $I_S$, have $\beta = \infty$, and $V_A = \infty$ Volts.

$V_{CE(sat)} = 0.5$V. $V_{be}$ is roughly $0.7$V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary and appropriate. The supplies are +3.3 Volts and -3.3 Volts. The DC voltage drops across Ree3 and Ree11 are both 300mV.

The DC collector currents of Q3,4,5,6,7,11,12 are all 1.0 mA. $R_L = 500\Omega$
Part a, 5 points
DC bias:

On the circuit diagram above, label the DC voltages at ALL nodes, the DC currents through ALL resistors, and the DC drain currents of all transistors.

Note that $V_{ee9} = V_{ee10} = V_{ee11}$

Note that $V_{ee8} = V_{ee11} + V_T \ln (2mA/1mA) = 18mV + 0.7V$

$\Rightarrow$ VEE drop across $R_{ee8} = 800 \times 0.7mV = 282mV$.

Note that $V_{ee1} = V_{ee2} = V_{ee3} + V_T \ln (2mA/1mA) = 18mV + 0.7V$.

$\Rightarrow$ drops across $R_{ee1}, R_{ee2} = 282mV.$
Part b. 6 points
DC bias:

Find the value of all resistors.
\[ R_{\text{bias}} = 3.9k \Omega \quad R_{\text{e1}} = \frac{1410}{2} \Omega \quad R_{\text{e2}} = \frac{1410}{2} \Omega \quad R_{\text{e3}} = 300 \Omega \]
\[ R_{\text{e8}} = \frac{1410}{2} \quad R_{\text{e9}} = \frac{300}{2} \Omega \quad R_{\text{e10}} = \frac{300}{2} \Omega \quad R_{\text{e11}} = 300 \Omega \]

\[ R_{\text{R6,7}} = 3.9 \ V/1mA = 3.4 \ k\Omega \]
\[ R_{\text{R6,8}} = R_{\text{R6,9}} = 252mV/12mA = 141 \Omega \]
\[ R_{\text{R6,3}} = 300mV/11mA = 300 \Omega \]
\[ R_{\text{R6,5}} = 252mV/12mA = 141 \Omega \]
\[ R_{\text{R6,10}} = 200mV/11mA = 200 \Omega \]
Part c. 4 points

Find the transconductance of the transistors below:
\[ g_m4 = g_m5 = g_m6 = g_m7 = \]

\[ 38.5 \text{ mS} \] for all

\[ = \frac{1}{9_m7} = \frac{1}{9_m6} = \frac{1}{9_m4} = \frac{1}{9_m5} = \frac{26 \mu V}{1 \text{mA}} \rightarrow 9_m4 = 9_m5 = 38.5 \text{ mS} = 9_m6 = 9_m7 \]
Part d, 10 points.

The circuit is fully differential. Assuming a differential input signal, \( V_{\text{in, diff}} = V_{+} - V_{-} \), and defining a differential output signal \( V_{\text{out, diff}} = V_{0+} - V_{0-} \), compute the differential gain

\[
A_{d} = \frac{V_{\text{out, diff}}}{V_{\text{in, diff}}} = +19 \frac{A}{V}
\]

\[
\frac{V_{+}}{V_{-}} = \frac{R_{L} / g_{m}}{R_{m, \text{in}}} = \frac{g_{m} R_{L}}{g_{m} R_{m, \text{in}}} = \frac{g_{m} R_{L}}{g_{m} R_{m, \text{in}}} = 19 \frac{A}{V}
\]

\[
\frac{V_{-}}{V_{+}} = \frac{-g_{m} R_{m, \text{in}}}{g_{m}} = -1
\]

\[
\frac{V_{+}}{V_{-}} = -19 \frac{A}{V} \Rightarrow A_{d} = +19 \frac{A}{V}
\]
Part e. 10 points

Maximum peak-peak output voltage at the positive output Vo+ (*show all your work*)

<table>
<thead>
<tr>
<th>Transistor</th>
<th>magnitude and sign of maximum output signal swing due to cutoff</th>
<th>magnitude and sign of maximum output signal swing due to saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transistor Q1</td>
<td>( \frac{V}{R} ) (within minimal) ( 1 \text{ pt} )</td>
<td>(-3.64 \text{ V} ) ( 2 \text{ pts} )</td>
</tr>
<tr>
<td>Transistor Q4</td>
<td>(-500 \text{ mV} ) ( 2 \text{ pts} )</td>
<td>(+4.8 \text{ V} ) ( 1 \text{ pt} )</td>
</tr>
<tr>
<td>Transistor Q6</td>
<td>(+500 \text{ mV} ) ( 1 \text{ pt} )</td>
<td>(-1.8 \text{ V} ) ( 1 \text{ pt} )</td>
</tr>
<tr>
<td>Transistor Q9</td>
<td>( \frac{V}{R} ) (within minimal) ( 1 \text{ pt} )</td>
<td>(+2.5 \text{ V} ) ( 1 \text{ pt} )</td>
</tr>
</tbody>
</table>

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant.
Cutoff Q1: \( \rightarrow \) not relevant; \( I_c \) does not change

Set Q1: \( V_C E_0 = 0.7V, \ V_C E_s t = 0.6V \rightarrow \Delta V_C E = 0.2V (\uparrow) \)
\[ \Delta V_C = 0.2V \cdot \mu \ \text{SEC} = 0.2V \cdot (-19.2) = -3.84V (\downarrow) \]

Set Q1: \( I_C = 1mA \)
\[ \Delta V_C = 1mA \cdot R_C = 26mV (\uparrow) \]
\[ \Delta V_C = 26mV \cdot \text{SEC} = 26mV \cdot (-19.2) = -500mV (\downarrow) \]

Set Q4: \( V_C E_0 = 3.0V, \ V_C E_{st} = 1.2V \)
\[ \Delta V_C = 2.5V \cdot \text{SEC} = 2.5V \cdot (-19.2) = 48V (\uparrow) \]

Set Q6: \( \Delta I_C = 1mA \rightarrow \Delta V_C = 1mA \cdot 500 \Omega = 500mV (\uparrow) \)

Set Q6: \( V_C E_0 = 2.3V, \ V_C E_{st} = \frac{1}{2}V \rightarrow \Delta V_{out} = -1.8V (\downarrow) \)

Cutoff Q6 \( \rightarrow \) not relevant; \( I_c \) does not change

Set Q9: \( V_C E_0 = 3V, \ V_C E_{st} = 1.4V \)
\[ \Delta V_C = 2.5V (\uparrow) \]
Problem 2, 35 points

*This is an Op-Amp—analyze the bias under the assumption* that DC output voltage is zero volts, that the positive input \( V_{i+} \) is zero volts, and that we must determine the DC value of the negative input voltage (\( V_{i-} \)) necessary to obtain this.

![Diagram of a circuit with transistors](image)

All NMOS: \( I_D = 1(\text{mA/V})(W_g / 1\mu\text{m})(V_{gs} - V_{th} - \Delta V)(1 + \lambda V_{DS}) \)

\( \Delta V = 0.1\text{V} \), \( V_{th} = 0.2\text{V} \), \( 1/\lambda = 5\text{V} \)

All PMOS: Also velocity-limited, with \( g_m = 0.5(\text{mA/V})(W_g / 1\mu\text{m}) \)

\( \Delta V = -0.1\text{V} \), \( V_{th} = -0.2\text{V} \), \( 1/\lambda = 5\text{V} \)

\( V_{DD} = +1\text{ V}, \ -V_{SS} = -1\text{ V} \), The load resistor is \( R_L = 10 \text{ k}\Omega \)
Part a, 10 points

DC bias.

Approximation: ignore the term \((1 + \lambda V_{ds})\) in DC bias analysis.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input \(V_{i+}\) is zero volts, and that we must determine the DC value of the negative input voltage \(V_{i-}\) necessary to obtain this.

Q1 is to be biased at 0.1 mA drain current.

The transistor gate widths are as follows

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Q7</th>
<th>Q8</th>
<th>Q9</th>
<th>Q10</th>
<th>Q11</th>
<th>Q12</th>
</tr>
</thead>
<tbody>
<tr>
<td>2μm</td>
<td>4μm</td>
<td>4μm</td>
<td>1μm</td>
<td>1μm</td>
<td>1μm</td>
<td>1μm</td>
<td>2μm</td>
<td>4μm</td>
<td>2μm</td>
<td>10μm</td>
<td>20μm</td>
</tr>
</tbody>
</table>

Find:

ID1 = 0.1 mA  ID2 = 0.2 mA  ID3 = 0.2 mA  ID4 = 0.3 mA  ID5 = 0.1 mA  ID6 = 0.1 mA
ID7 = 0.2 mA  ID8 = 0.2 mA  ID9 = 0.2 mA  ID10 = 0.2 mA  ID11 = 0.2 mA  ID12 = 0.2 mA
R1 = 16 kΩ

\[
I_{D1} = 0.1 \text{ mA} = \left(0.5 \text{ mA/μm}\right) \left(2 \frac{\mu\text{m}}{\mu\text{m}}\right) \left(\left|V_{gs}\right| - 0.2 \text{ V} - 0.1 \text{ V}\right) \quad \Delta +1
\]

\[
\left|V_{gs1}\right| = 0.1 \text{ V} + 0.2 \text{ V} + 0.1 = 0.4 \text{ V}
\]

\[
I_{D2} = \left(0.5 \text{ mA/μm}\right) \left(\frac{4 \mu\text{m}}{1 \mu\text{m}}\right) \left(\left|V_{gs2}\right| - 0.2 \text{ V} - 0.2 \text{ V}\right) \quad \Delta \left|V_{gs2}\right| = \left|V_{gs1}\right|\]

\[
I_{D2} = 2 \text{ mA/μm} \left(0.4 \text{ V} - 0.3 \text{ V}\right) = 0.2 \text{ mA} \quad \boxed{+2}
\]

\[
I_{D3} = I_{D2} = 0.2 \text{ mA} \quad \left(\because W_{g3} = W_{g2} \right) \quad \left|V_{gs3}\right| = \left|V_{gs1}\right|
\]

\[
I_{D3}, R_{1} = 1.6 \text{ V} \Rightarrow R_{1} = \frac{1.6 \text{ V}}{0.1 \text{ mA}} = 16 \text{ kΩ} \quad \boxed{+1}
\]

\[
I_{D4} = I_{D5} = I_{D6} = I_{D7} = I_{D2}/2 = 0.1 \text{ mA} \quad \boxed{+1.5}
\]

\[
I_{D8} = I_{D3} = I_{D10} = I_{D9} = 0.2 \text{ mA} \quad \boxed{+1.5}
\]
\[ I_{D_{11}} = I_{D_{12}} \quad (V_{out} = 0\; V) \quad 2W_{g_{11}} = 2W_{g_{12}} \]

\[ \left( \frac{1 mA}{V} \right) \left( \frac{W_{g_{11}}}{1 \mu m} \right) \left( |V_{G_{S_{11}}}| - |V_{th}| - |\Delta V| \right) = \left( 0.5 mA/V \right) \left( \frac{W_{g_{12}}}{1 \mu m} \right) \left( |V_{G_{S_{12}}}| - |V_{th}| - |\Delta V| \right) \]

\[ \therefore V_{G_{S_{11}}} = V_{G_{S_{12}}} \]

\[ I_{D_2} = I_{D_{10}} = \left( \frac{1 mA}{V} \right) \left( \frac{W_{g}}{1 \mu m} \right) \left( |V_{G_{S_{10}}}| - |V_{th}| - |\Delta V| \right) \]

\[ 0.2 mA = \left( \frac{1 mA}{V} \right) \left( 2 \right) \left( |V_{G_{S_{10}}}| - 0.2 V - 0.1 V \right) \]

\[ |V_{G_{S_{10}}}| = 0.1 V + 0.2 V + 0.1 V = 0.4 V \]

Since, \[ I_{D_2} = I_{D_{10}} \quad 2W_{g_{10}} = W_g \quad 0.5 = 2 \times 0.25 \]

\[ |V_{G_{S_{10}}}| = |V_{G_{S_{10}}} - 0.4 V| = 0.4 V \]

\[ I_{D_4} = 0.1 mA = \left( \frac{0.5 mA}{V} \right) \left( \frac{W_{g}}{1 \mu m} \right) \left( |V_{G_{S_{4}}}| - |V_{th}| - |\Delta V| \right) \]

\[ |V_{G_{S_{4}}}| = 0.2 V + 0.3 V = 0.5 V = |V_{G_{S_{5}}}| \]

\[ I_{D_5} = \left( \frac{1 mA}{V} \right) \left( \frac{W_{g}}{1 \mu m} \right) \left( |V_{G_{S_{5}}}| - |V_{th}| - |\Delta V| \right) = 0.2 mA \quad ; \quad W_g = 2 \mu m \]

\[ |V_{G_{S_{5}}}| = 0.1 V + 0.2 + 0.1 = 0.4 V \]

From circuit, \[ |V_{G_{S_{9}}}| = |V_{G_{S_{12}}}| = 1 V \quad ; \quad |V_{G_{S_{11}}}| = |V_{G_{S_{12}}}| = 0.4 V \]

\[ I_{D_{11}} = \left( \frac{1 mA}{V} \right) \left( \frac{W_{g_{11}}}{1 \mu m} \right) \left( |V_{G_{S_{11}}}| - |V_{th}| - |\Delta V| \right) \]

\[ = \frac{1 mA}{V} \left( 10 \right) \left( 0.4 V - 0.2 V - 0.1 V \right) \]

\[ I_{D_{11}} = 1 mA = I_{D_{12}} \]

\[ I_{D_6} = \left( \frac{1 mA}{V} \right) \left( \frac{W_{g_6}}{1 \mu m} \right) \left( |V_{G_{S_{6}}}| - V_{th} - |\Delta V| \right) = 0.1 mA \quad ; \quad W_{g_6} = 4 \mu m \]

\[ |N_{S_{6}}| = 0.1 V + 0.2 V + 0.1 V = 0.4 \neq |V_{G_{S_{6}}}| \]
Part b. 10 points

DC bias

On the circuit diagram above, label the DC voltages at ALL nodes and the drain currents of ALL transistors.

\[ V_{G3} = \frac{I_{D3}}{0.5\,mA} + 0.3\,V = 0.1\,V + 0.3\,V = 0.4\,V \]
Part c, 15 points.

To compute the op-amp differential gain, we will ground the positive input and apply a signal to the negative input. Assume that the DC bias conditions do not change when we do this.

Find the following

<table>
<thead>
<tr>
<th></th>
<th>Voltage Gain</th>
<th>Input impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transistor combination</td>
<td>±15</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Q4,5,6,7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transistor Q8</td>
<td>−28</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Transistor combination</td>
<td>9.494</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Q11,12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall differential</td>
<td>±207.48</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Vout/Vin</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\( V_X = 5 \text{V} \)

\[
R_{DS_2} = R_{DS_5} = \frac{1}{\frac{5V}{0.2\text{mA}}} = 50 \text{k} \Omega \]

\[
R_{Leg} = R_{DS_2} || R_{DS_5} = 25 \text{k} \Omega
\]

\[
A_{V_{45.6.7}} = \pm 1.5 \quad [+1.5]
\]

\[
A_{V_{45.6.7}} = \pm \Gamma m_4 \quad R_{Leg} = \pm 15
\]

\[
\Gamma m_4 = 0.5 \text{mS} \left(1 + \frac{1V}{0.5V}\right) = 0.5 \text{mS} \left(1 + \frac{5V}{5V}\right) = 0.5 (1 + 0.2)
\]

\[
\Gamma m_4 = 0.6 \text{mS}
\]

\[
A_{V_{6.8}}
\]

\[
R_{DS_2} = \frac{1}{\frac{5V}{0.2\text{mA}}} = 25 \text{k} \Omega = R_{DS_2}
\]

\[
R_{DS_{10}} = R_{DS_{11}} = 25 \text{k} \Omega
\]

\[
R_{Leg} = R_{DS_2} || R_{DS_5}
\]

\[
R_{Leg} = 25 \text{k} \Omega || 25 \text{k} \Omega = 12.5 \text{k} \Omega
\]

\[
A_{V_{de}} = -\Gamma m_8 \quad R_{Leg}
\]

\[
\Gamma m_8 = 2 \text{mS} \left(1 + \frac{0.6V}{5V}\right) = 2 \text{mS} \left(1 + 0.12\right)
\]

\[
\Gamma m_8 = 2.24 \text{mS}
\]

\[
A_{V_{de}} = -28
\]

\[
A_{V_{de}} = +1.5
\]
\[ R_{eq} = R_1 \parallel \left( \frac{1}{g_{m1}} \right) \parallel R_{DS1} \parallel R_{DS2} \]

\[ g_{m2} = 10 \text{mS} \left(1 + 2 \times V_{BE1}\right) = 10 \text{mS} \left(1 + \frac{1}{5}\right) = 12 \text{mS} \]

\[ R_{DS1} = \frac{1}{\alpha I_D} = \frac{5 \text{V}}{1 \text{mA}} = 5 \text{k}\Omega = R_{DS2} \]

\[ R_{eq} = \frac{10 \text{k}\Omega \parallel (8.3 \text{k}\Omega \parallel 2.5 \text{k}\Omega)} = (22.64 \text{k}\Omega \parallel 5.5 \text{k}\Omega) \]

\[ R_{eq} = 8.13 \Omega \]

\[ A_{v,11,12} = \frac{R_{eq}}{R_{eq} + \frac{1}{g_{m2}}} = \frac{8.13 \Omega}{8.13 \Omega + 8.3 \Omega} = 0.494 \]

\[ \text{Overall gain: } (0.494) (15\%) (-28) = -207.48 \]

\[ \text{The difference in the value of } A_{v,11,12} \text{ comes from assuming both } 11 \text{ and } 12 \text{ as being 'ON' or either of them being 'ON'. BOTH ARE OK} \]

\[ \text{Ambiguity in the signs of the overall gain comes from the definition of the gains for the first stage (i) wrt \ Vin \ signal input @ Vin} \]

\[ (\text{ii) wrt } (Vin - \text{Vin})} \]
Part d, 10 points

Maximum peak-peak output voltage at the positive output $V_{o+}$ (show all your work).

Recall that the FETs are velocity-limited, hence $V_{DS,\text{ave}} = \Delta V = 0.1\text{V}$.

<table>
<thead>
<tr>
<th>Transistor</th>
<th>magnitude and sign of maximum output signal swing due to cutoff</th>
<th>magnitude and sign of maximum output signal swing due to: knee voltage (saturation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3</td>
<td>$\frac{N_A}{C} \text{ output signal}$</td>
<td>$(+0.5\uparrow)$ $(\text{in})$ $(+0.25\downarrow)$</td>
</tr>
<tr>
<td>Q8</td>
<td>$\left(\begin{array}{c} 1.235V \ 2.475V \end{array}\right)$</td>
<td>$(0.5\downarrow)$ $(\text{in})$ $(+0.25\downarrow)$</td>
</tr>
<tr>
<td>Q11</td>
<td>$(N_A)$ $\text{push-pull}$</td>
<td>$+0.9\text{V} \uparrow$</td>
</tr>
<tr>
<td>Q12</td>
<td>$(N_A)$ $\text{push-pull}$</td>
<td>$-0.9\text{V} \downarrow$</td>
</tr>
</tbody>
</table>

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

$Q_{11}: (\text{knee})$  
$V_{DS,0} = 1\text{V}$  
$V_{DS,\text{ave}} = 0.1\text{V}$  
$\Delta V_{\text{out}} = 1\text{V} - 0.1\text{V} = +0.9\text{V} \uparrow$  

$Q_{12}: (\text{knee})$  
Since $N \geq N$,  
$\Delta V_{\text{out}} = -0.9\text{V} \downarrow$  

$Q_{8}: (\text{off})$  
$I_{DS} = 0.2\text{mA}$  
$R_{LEQ} = 12.5\text{k}\Omega$  
$\Delta V_{g} = 0.2\text{mA} \times 12.5\text{k}\Omega = 2.5\text{V}$  
$\Delta V_{\text{out}} = 2.5\text{V} \times 0.494 = +1.235\text{V}$  
$2.5\text{V} \times 0.49 = +2.475\text{V} \left(\text{corrected}\right)$  

$Q_{8}: (\text{knee})$  
$V_{DS,0} = 0.6\text{V}$  
$V_{DS,\text{ave}} = 0.1\text{V}$  
$\Delta V_{g} = 0.6\text{V} - 0.1\text{V} = -0.5\text{V} \downarrow$  
$\Delta V_{\text{out}} = +0.5\text{V} \uparrow - 0.5\text{V} = -0.25\text{V} \downarrow$  

[Mark: +2]
0.3, \( V_{3p} = 0.6 \text{ V} \)

\[ \Delta V_{out} = (0.6 - 0.1) = 0.5 \text{ V} \]

\( \theta \)

\( T+0.25 \text{ V} \)
Problem 3, 30 points

You will be working on the circuit to the left.

Ignore DC bias analysis. You don’t need it.

The transistor has transconductance $g_m$.

Its output resistance $R_d$ is infinity...so you don't need to include this element in the circuit diagram!

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.

---

Check controlling voltage of $g_m V_{gs}$!
Part b. 8 points

\( \text{gm} = 9 \ \text{mS}, L = 1 \ \mu\text{H}, R = 1000 \ \text{Ohms} \)

Find, by nodal analysis, a small-signal expression for \( V_{\text{out}}/V_{\text{in}} \). Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = K \cdot \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots}
\]

or (as appropriate) \( \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = K \cdot (s\tau)^n \cdot \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots} \)

Note that an expression like

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{1 + (3 \cdot 10^{-6}) s}
\]

is dimensionally wrong; \( \frac{1}{1 + (3 \cdot 10^{-6} \text{ seconds}) s} \) is dimensionally correct

\[ \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{\text{?}}{\text{?}} \]

\( V_{\text{out}}(s)/V_{\text{in}}(s) = \) ____________

---

\[ @ \text{ \textit{Vout} = 0. } \]

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\[ 25 \times \]
\[
\frac{V_{ol}}{V_{in}} = \frac{\frac{q_m}{g_{in} + 1/N}}{1 + \Delta C(g_m + 1/N)} = \frac{\Delta L \cdot \frac{q_m}{1 + \Delta C(g_m + 1/N)}}{1 + \Delta C(g_m + 1/N)} = \frac{\Delta L \cdot \frac{q_m}{1 + \Delta C(g_m + 1/N)}}{1 + \Delta C(g_m + 1/N)} = \frac{\frac{q_{ms}}{10m^2}}{1 + \Delta T} = \frac{\text{colours} \cdot 14c}{1 + \Delta C(10m^2)} = \frac{16c}{10m^2} = 0.9 \times \frac{\Delta C(10m^2)}{1 + \Delta C(10m^2)}
\]

\[
F_0 = \frac{1}{2\pi \tau} = \frac{15c}{14c} = 1.06 M \text{ Hz}
\]
Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes

\[ 20 \log_{10}(0.9) = -0.9 \text{dB} \approx -1.0 \text{dB} \]
\( N(A) = 0.9 \cdot \frac{AT}{1 + 0.7} \)

\( V_{in}(t) = 0.1V \)

Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t) = \( 9\text{mV} \cdot N(C) \cdot e^{-t/\text{time constant}} \)

Plot it below. Label axes, show initial and final values, show time constants.
\[ V_{av}(t) = 0.09V \cdot \frac{e^{-t/(\tau)}}{1 + \Delta} = 0.09V \cdot \frac{1}{\theta + 1/\tau} \]

\[ V_{av}(t) = V_{av}(\theta)e^{-t/\theta} \cdot 0.09V \]

- El/ins.

\[ = 90 \text{ mV} \cdot u(t) \cdot e \]