Final Exam, ECE 137A

Wednesday March 19, 2014 7:30-10:30 PM

Name: Solution A

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them.***
Show your work:
Full credit will not be given for correct answers if supporting work is missing.

Good luck

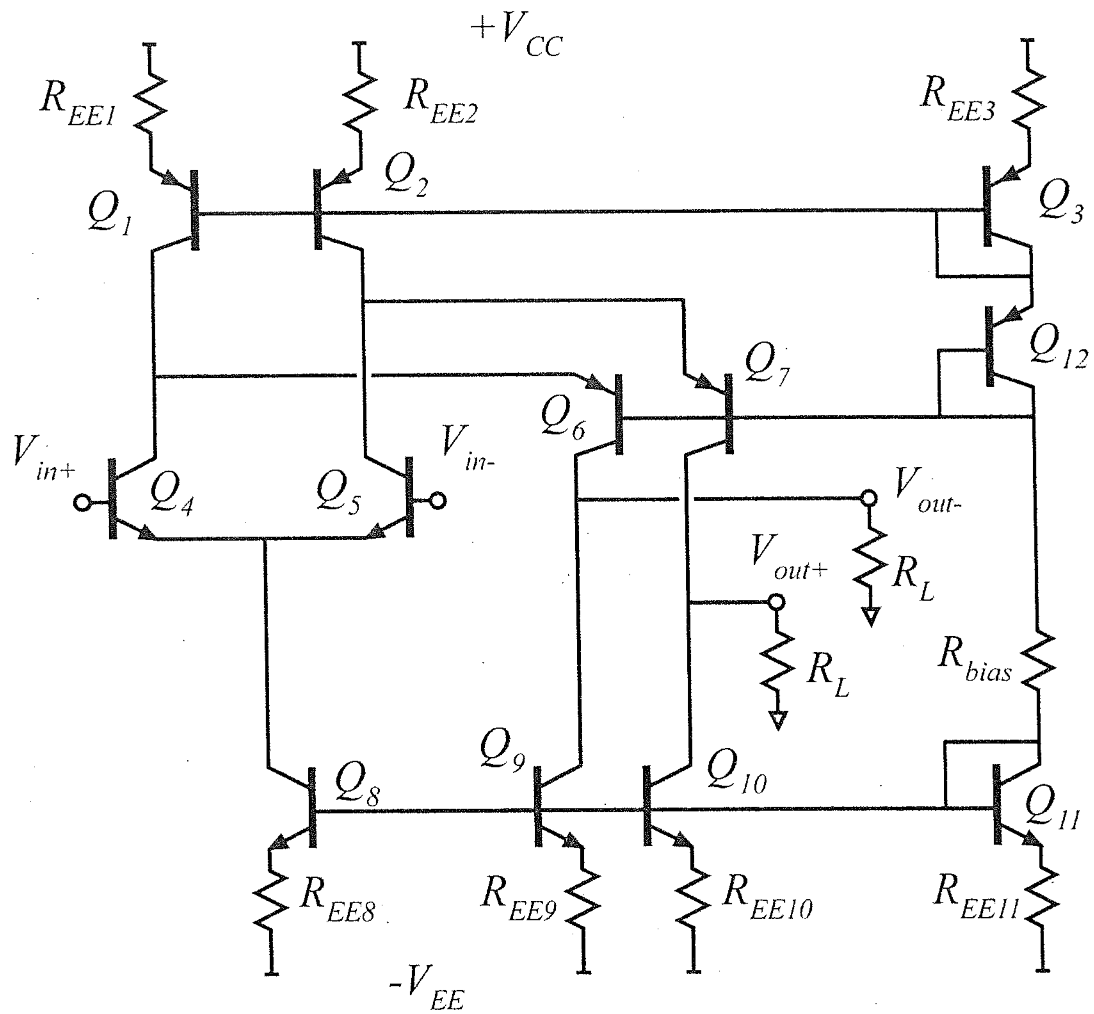
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

a

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



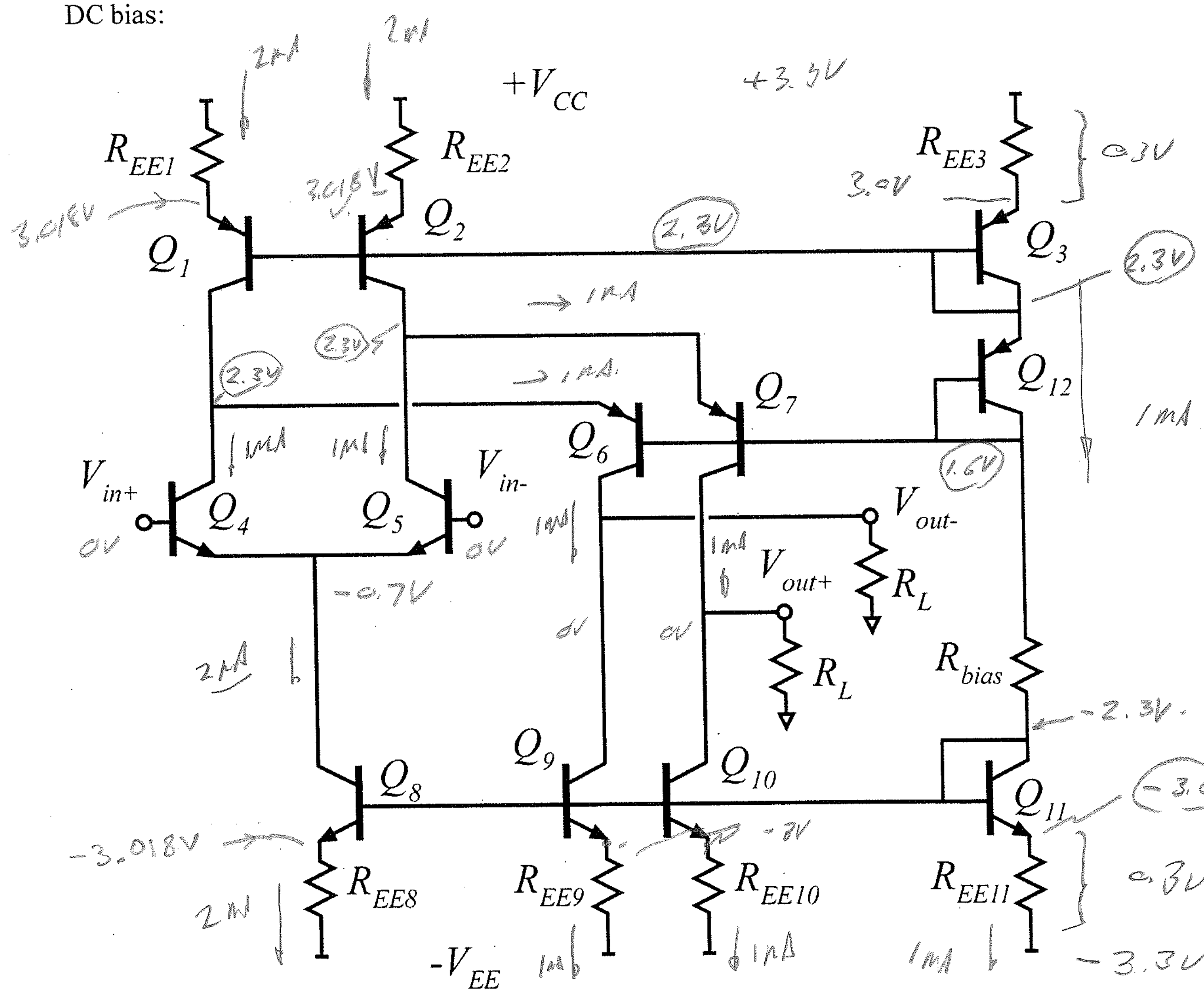
All the transistors have the same (matched) I_S, have $\beta = \infty$, and $V_A = \infty$ Volts. $V_{CE(sat)} = 0.5$V. V_{be} is roughly 0.7 V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary and appropriate. The supplies are +3.3 Volts and -3.3 Volts. The DC voltage drops across Ree3 and Ree11 are both 300mV.

The DC collector currents of Q3,4,5,6,7,11,12 are all 1.0 mA. $R_L = 500\Omega$

a

Part a, 5 points

DC bias:



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors**, and the DC drain currents of **all transistors**.

Note that $V_{BE9} = V_{BE10} = V_{BE11}$

Note that $V_{BE8} = V_{BE11} + V_T \ln(2mA/1mA) = 18mV + 0.7V$

$\Rightarrow$ V_{EE} drop across R_{EE8} = 300 - 18mV = 282mV.

Note that $V_{BE1} = V_{BE2} = V_{BE3} + V_T \ln(2mA/1mA) = 18mV + 0.7V$.

$\Rightarrow$ drops across R_{EE1}, R_{EE2} = 282mV.

a

Part b, 6 points
DC bias:

Find the value of all resistors.
Rbias=_3.9kΩ_ Ree1=_141Ω_ Ree2=_141Ω_ Ree3=_300Ω_
Ree8=_141Ω_ Ree9=_300Ω_ Ree10=_300Ω_ Ree11=_300Ω_

$R_{bias} = 3.9V/1mA = 3.9k\Omega$

$R_{EE1} = R_{EE2} = 282mV/2mA = 141\Omega$

$R_{EE3} = 300mV/1mA = 300\Omega$

$R_{EE8} = 282mV/2mA = 141\Omega$

$R_{EE11} = R_{EE9} = R_{EE10} = 300mV/1mA = 300\Omega$

a

Part c, 4 points

Find the transconductance of the transistors below:

gm4=_____ gm5=_____ gm6=_____ gm7=_____] 38.5 mS for all

$$\frac{1}{g_{m7}} = \frac{1}{g_{m6}} = \frac{1}{g_{m4}} = \frac{1}{g_{m5}} = \frac{26mV}{1mA} \longrightarrow g_{m4} = g_{m5} = 38.5mS = g_{m6} = g_{m7}$$

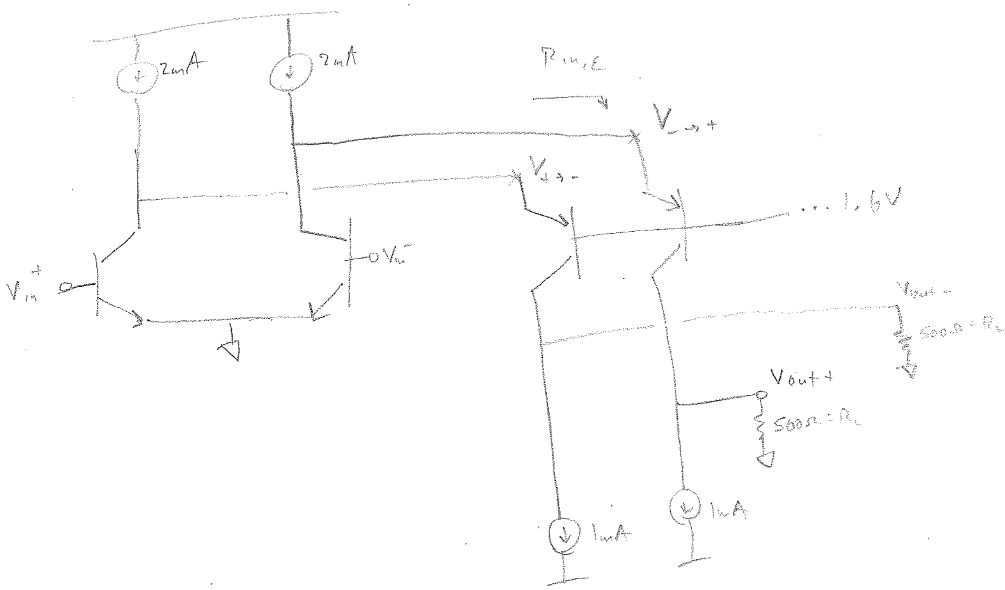
a

Exam A

Part d, 10 points.

The circuit is fully differential. Assuming a differential input signal, $V_{in,diff} = V_{i+} - V_{i-}$, and defining a differential output signal $V_{out,diff} = V_{0+} - V_{0-}$, compute the differential gain $A_d = V_{out,diff} / V_{in,diff}$

$A_d =$ ____ +19.2 ____



$$\frac{V_{out}^+}{V_{- \to +}} = \frac{R_{L,ee}}{R_{in,emitter}} = \frac{R_L}{1/g_m} = g_m R_L = \frac{500\Omega}{26\Omega} = 19.2$$

$$\frac{V_{- \to +}}{V_{in}^-} = -g_m \cdot R_{in,E} = -\frac{g_m}{g_m} = -1$$

$$V_{out}^+ / V_{in}^- = -19.2 \Rightarrow A_d = +19.2$$

a

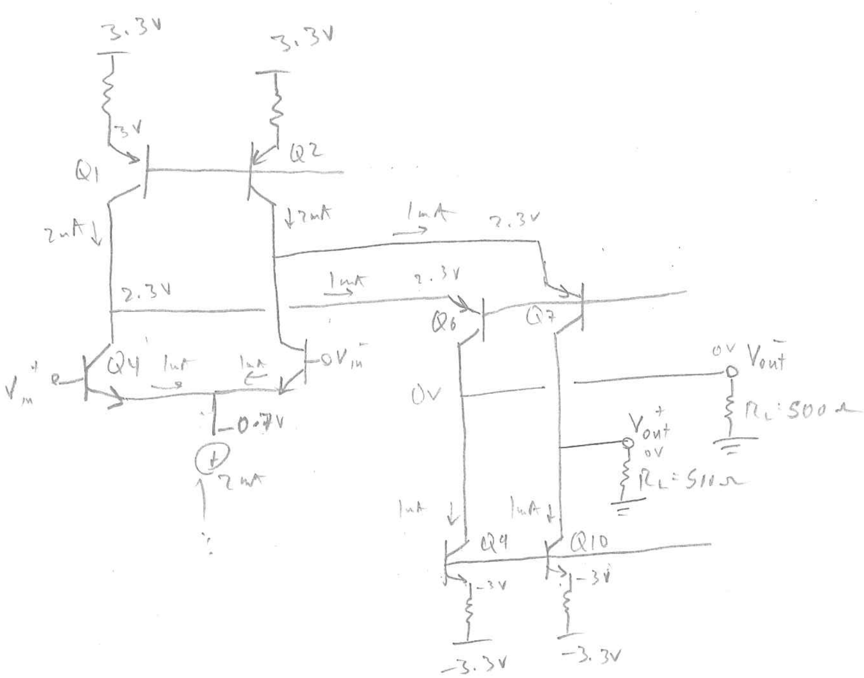
Exam A

Part e, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q1	N/R (current Mirror) 1pt	-3.84 V 2pts
Transistor Q4	-500mV 2pts	+48V 1pT
Transistor Q6	+500mV 1pt	-1.8V 1pt
Transistor Q9	N/R (current Mirror) 1pt	+2.5V 1pt

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant.



a

Cutoff Q1: → not relevant, I_C does not change

sat Q1: $V_{CEQ} = 0.7V$, $V_{CEsat} = 0.5V$ → $\Delta V_{CE} = 0.2V$ (↑)

$\Delta V_{out} = 0.2V \cdot A_{V6} = 0.2V \cdot (-19.2) = -3.84V$ (↓)

cutoff Q4: $\Delta I_C = 1mA$ $\Delta V_{CE4} = 1mA \cdot R_{eq} = 26mV$ (↑)

$\Delta V_{out} = 26mV \cdot A_{V6} = 26mV \cdot (-19.2) = -500mV$ (↓)

sat Q4: $V_{CEQ} = 3.0V$, $V_{CEsat} = 1/2V$ $\Delta V_{CE} = 2.5V$

$\Delta V_{out} = -2.5V \cdot A_{V6} = -2.5V \cdot (-19.2) = 48V$ (↑)

cutoff Q6: $\Delta I_C = 1mA$, → $\Delta V_{out} = 1mA \cdot 500\Omega = 500mV$ (↑)

sat Q6: $V_{CEQ} = 2.3V$, $V_{ce,sat} = \frac{1}{2}V$ ⇒ $\Delta V_{out} = -1.8V$ (↓)

cutoff Q9 → N/A, I_C does not change

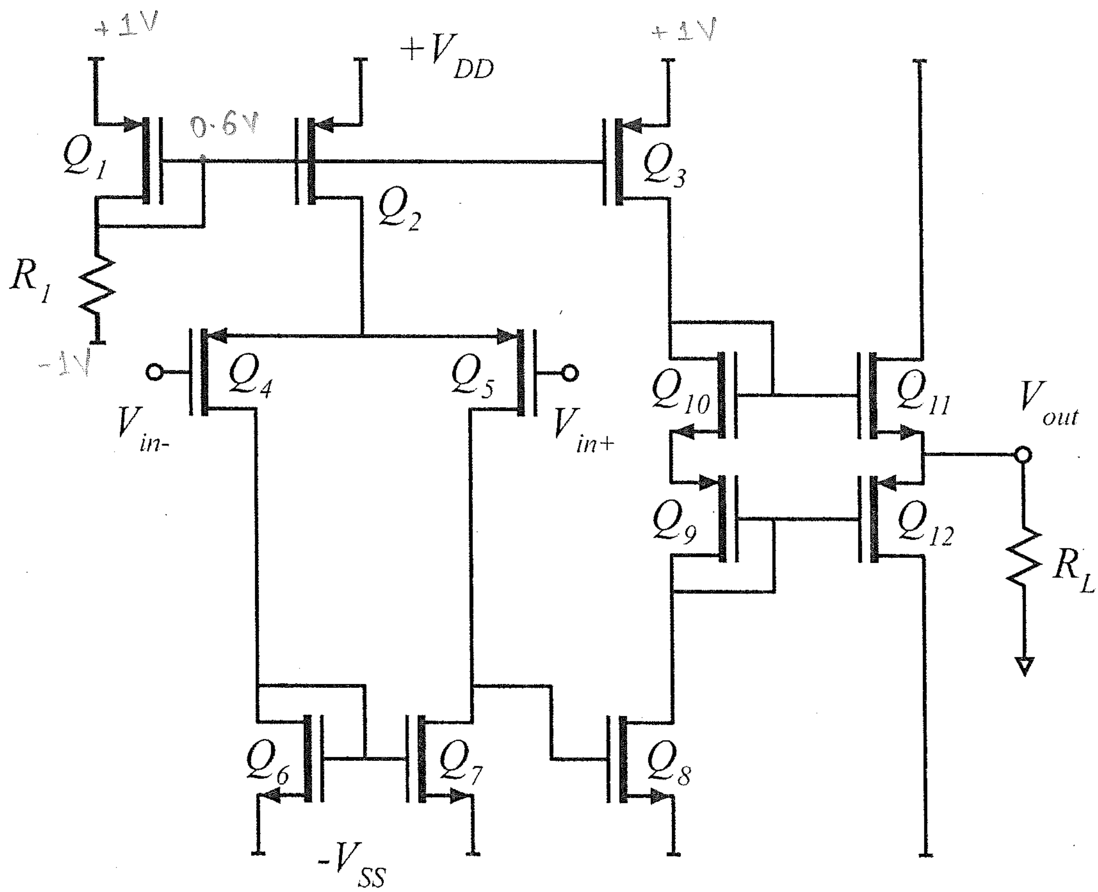
sat Q9: $V_{CEQ} = 3V$, $V_{CEsat} = 1/2V$, $\Delta V_{out} = 2.5V$ (↑)

(50)

a

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



All NMOS: $I_D = 1(\text{mA/V})(W_g/1\mu\text{m})(V_{gs} - V_{th} - \Delta V)(1 + \lambda V_{DS})$

$\Delta V = 0.1\text{V}$, $V_{th} = 0.2\text{V}$, $1/\lambda = 5\text{V}$

All PMOS: Also velocity-limited, with $g_m = 0.5(\text{mA/V})(W_g/1\mu\text{m})$

$\Delta V = -0.1\text{V}$, $V_{th} = -0.2\text{V}$, $1/\lambda = 5\text{V}$

V_{DD}=+1 V, $-V_{SS}$= -1 V, The load resistor is R_L=10 kΩ

a

Part a, 10 points
DC bias.
Approximation: ignore the term $(1+\lambda V_{DS})$ in DC bias analysis.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

Q1 is to be biased at 0.1 mA drain current.
The transistor gate widths are as follows

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
2μm	4μm	4μm	1μm	1μm	1μm	1μm	2μm	4μm	2μm	10μm	20μm

Find:
ID1= 0.1 mA ID2= 0.2 mA ID3= 0.2 mA ID4= 0.1 mA ID5= 0.1 mA ID6= 0.1 mA
ID7= 0.1 mA ID8= 0.2 mA ID9= 0.2 mA ID10= 0.2 mA ID11= 1 mA ID12= 1 mA
R1= 16 kΩ

$$I_{D_1} = 0.1\ mA = (0.5\ mA/V)\left(\frac{2\ \mu m}{1\ \mu m}\right)(|V_{GS_1}| - 0.2V - 0.1V)$$

$$|V_{GS_1}| = 0.1\ V + 0.2\ V + 0.1 = 0.4\ V$$ (+1)

$$I_{D_2} = (0.5mA/V)\left(\frac{4\mu m}{1\mu m}\right)(|V_{GS_2}| - 0.2V - 0.1) \quad ; \quad |V_{GS_2}| = |V_{GS_1}|$$

$$I_{D_2} = 2mA/V\ (0.4V - 0.3V) = 0.2\ mA$$ (+2)

(+1) $$I_{D_3} = I_{D_2} = 0.2\ mA \quad (\because W_{g_3} = W_{g_2}\ ;\ |V_{GS_3}| = |V_{GS_2}|)$$

$$I_{D_1} R_1 = 1.6\ V \Rightarrow R_1 = \frac{1.6\ V}{0.1\ mA} = 16\ k\Omega$$ (+1)

$$I_{D_4} = I_{D_5} = I_{D_6} = I_{D_7} = I_{D_2}/2 = 0.1\ mA$$ (+1.5)

$$I_{D_8} = I_{D_3} = I_{D_{10}} = I_{D_9} = 0.2\ mA$$ (+1.5)

a

$I_{D_{11}} = I_{D_{12}}$ $\quad (\because V_{out} = 0\,V)$ $\quad 2W_{g_{11}} = W_{g_{12}}$

$$\cancel{(1mA/V)(W_{g_{11}}/1\mu m)}(|V_{gs_{11}}| - |V_{th}| - |\Delta V|) = \cancel{(0.5mA/V)(W_{g_{12}}/1\mu m)}(|V_{gs_{12}}| - |V_{th}| - |\Delta V|)$$

$$\therefore V_{GS_{11}} = V_{GS_{12}}$$

$$I_{D_3} = I_{D_{10}} = (1mA/V)(W_g/1\mu m)(|V_{GS_{10}}| - |V_{th}| - |\Delta V|)$$

$$0.2mA = (1\,mA/V)(2)(|V_{GS_{10}}| - 0.2V - 0.1V)$$

$$|V_{GS_{10}}| = 0.1V + 0.2V + 0.1V = 0.4V$$

Since, $I_{D_9} = I_{D_{10}}$; $2W_{g_{10}} = W_{g_9}$; $g_{m_{10}} = 2g_{m_9}$

$$|V_{GS_9}| = |V_{GS_{10}}| = 0.4V$$

$$I_{D_4} = 0.1mA = (0.5mA/V)(W_g/1\mu m)(|V_{GS_4}| - |V_{th}| - |\Delta V|)$$

$$\therefore |V_{GS_4}| = 0.2V + 0.3V = 0.5V = |V_{GS_5}|$$

$I_{D_8} = (1mA/V)(W_g/1\mu m)(|V_{GS_8}| - |V_{TH}| - |\Delta V|) = 0.2mA$; $W_{g_8} = 2\mu m$.

$$|V_{GS_8}| = 0.1V + 0.2 + 0.1 = 0.4V$$

From circuit, $|V_{DS_{11}}| = |V_{DS_{12}}| = 1V$; $\boxed{|V_{GS_{11}}| = |V_{GS_{12}}| = 0.4V}$

$$I_{D_{11}} = (1mA/V)(W_{g_{11}}/1\mu m)(|V_{GS_{11}}| - |V_{TH}| - |\Delta V|)$$
$$= 1\,mA/V\,(10)\,(0.4V - 0.2V - 0.1V)$$

$$\boxed{I_{D_{11}} = 1\,mA = I_{D_{12}}}$$ (+2)

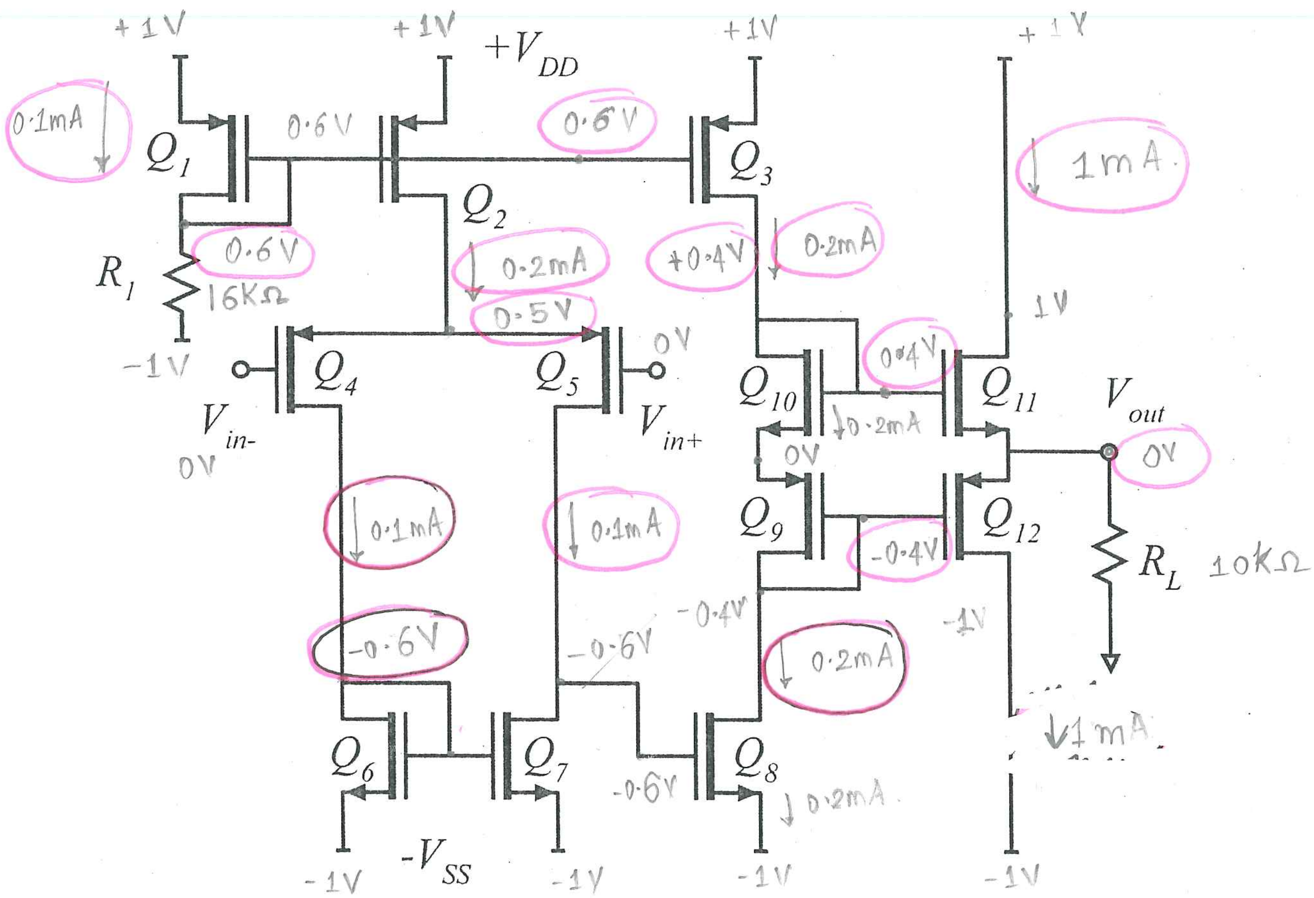
$I_{D_6} = (1mA/V)(W_{g_6}/1\mu m)(|V_{GS_6}| - V_{TH} - \Delta V) = 0.1mA$; $W_{g_6} = 1\mu m$

$$|V_{GS_6}| = 0.1V + 0.2V + 0.1V = 0.4 \neq |V_{GS_7}|$$

2/3 pt each

Part b, 10 points

DC bias



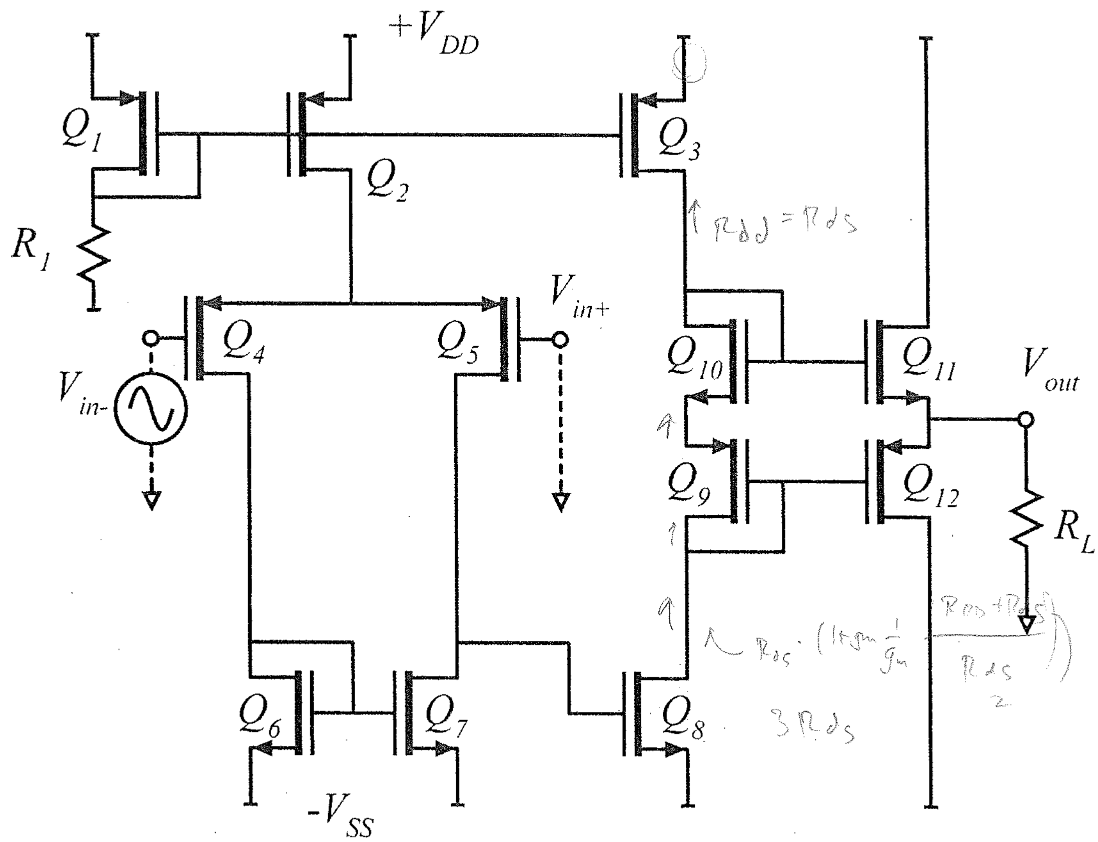
On the circuit diagram above, label the DC voltages at **ALL nodes** and the drain currents of **ALL transistors**

$$|V_{GS_3}| = \frac{I_{D3}}{(0.5\,mA/V)(4)} + 0.3\,V = 0.1\,V + 0.3\,V = 0.4\,V$$

a

Part c, 15 points.

To compute the op-amp differential gain, we will ground the positive input and apply a signal to the negative input. Assume that the DC bias conditions do not change when we do this.



Find the following

	Voltage Gain	Input impedance
Transistor combination Q4,5,6,7	±15	∞ Ω
Transistor Q8	-28	∞ Ω
Transistor combination Q11,12	0.494	∞ Ω
Overall differential Vout/Vin	∓ 207.48	∞ Ω

a

$1/\lambda = 5\text{V}$

$A_{v\ 4,5,6,7}$

$R_{DS_7} = R_{DS_5} = \frac{1}{\lambda I_D} = \frac{5\text{V}}{0.2\text{mA}} = 50\text{k}\Omega$

$R_{Leq} = R_{DS_7} \| R_{DS_5} = 25\text{k}\Omega$] (+1.5)

$A_{v\ 4,5,6,7} = \pm g_{m_4} R_{Leq} = \pm 15$] (+1.5)

$g_{m_4} = 0.5\text{mS}(1+\lambda V_{DS_4}) = 0.5\text{mS}\left(1 + \frac{1\text{V}}{5\text{V}}\right) = 0.5(1+0.2)$

$g_{m_4} = 0.6\text{mS}$] (+1.5)

$A_{v,Q8}$

$R_{DS_8} = \frac{1}{\lambda I_D} = \frac{5\text{V}}{0.2\text{mA}} = 25\text{k}\Omega = R_{DS_3}$] (+1.5)

$R_{DS_{10}} = R_{DS_{11}} = 25\text{k}\Omega$

$R_{Leq} = R_{DS_2} \| R_{DS_3}$

$= 25\text{k}\Omega \| 25\text{k}\Omega$

$R_{Leq} = 12.5\text{k}\Omega$ (+1.5)

$A_{v,Q8} = -g_{m_8} R_{Leq}$

$g_{m_8} = 2\text{mS}\left(1 + \frac{0.6\text{V}}{5\text{V}}\right) = 2\text{mS}(1+0.12\text{V})$

$g_{m_8} = 2.24\text{mS}$

$A_{v,Q_8} = -28$ (+1.5)

a

$\underline{A_{V,11,12}}$ (Assuming both Q_{11}; Q_{12} are 'ON')

$$R_{Leq} = R_L \parallel \left(\frac{1}{g_{m_{11}}}\right) \parallel R_{DS_{11}} \parallel R_{DS_{12}}$$

(+1.5)

$$g_{m_{12}} = 10mS\,(1 + \lambda V_{DS_{11}}) = 10mS\left(1 + \frac{1}{5}\right) = 12mS$$

$$R_{DS_{11}} = \frac{1}{\lambda I_D} = \frac{5\,V}{1mA} = 5\,k\Omega = R_{DS_{12}}$$

$$R_{Leq} = (10k\Omega) \parallel (83.3\Omega) \parallel (2.5k\Omega) = (82.64\Omega) \parallel (2.5k\Omega)$$

$$R_{Leq} = 81.3\,\Omega$$ (+1.5)

$$A_{V,11,12} = \frac{R_{Leq}}{R_{Leq} + 1/g_{m_{12}}} = \frac{81.3\Omega}{81.3\Omega + 83.3\Omega} = 0.494$$ (+1.5)

Overall gain: $(0.494)(\pm 15)(-28) = \mp 207.48$ (+1.5)

= The difference in the value of $A_{V,11,12}$ comes from assuming both 11 & 12 as being 'ON' (or) either of them being 'ON'. BOTH ARE OK

Ambiguity in the signs of the overall gain comes from the definition of the gain for the first stage (i) w.r.t V_{in} signal input @ V_{in}
(ii) w.r.t $(V_{in}^+ - V_{in}^-)$

a

Part d, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)
Recall that the FETs are velocity-limited, hence $V_{DS,knee} = \Delta V = 0.1V$.

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q3	N/A current mirror.	(+0.5 ↑) (or) (-0.25 ↓)
Transistor Q8	↑(+1.235V) (or) (2.475V)	(-0.5 ↓) (or) (-0.25 ↓)
Transistor Q11	(+1) [N/A push-pull	+ 0.9V ↑
Transistor Q12	(+1) [N/A push-pull	-0.9V ↓

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

Q11 : (knee) $V_{DS_Q} = 1V$ $V_{DS_{knee}} = 0.1V$

$\Delta V_{out} = 1V - 0.1V = +0.9V$ ↑ (+1)

Q12 : (knee) Same as Q11, $\Delta V_{out} = -0.9V$ ↓ (+1)

Q8 : (cut off) $I_{D_Q} = 0.2mA$, $R_{Leq} = 12.5 k\Omega$

$\Delta V_8 = 0.2mA \times 12.5k\Omega = +2.5V$

$\Delta V_{out} = 2.5V \times 0.494 = +1.235V$

$(2.5V \times 0.99 = +2.475V)$ (+2)

Q8 : knee $V_{DS_Q} = 0.6V$, $V_{DS_{knee}} = 0.1V$

$\Delta V_8 = 0.6V - 0.1V = -0.5V$ ↓

$\Delta V_{out} = -0.5V \times 0.99 \approx -0.5V$ ↓

(or) $-0.5V \times 0.494 = -0.25V$ ↓ (+2)

Q3, Krre

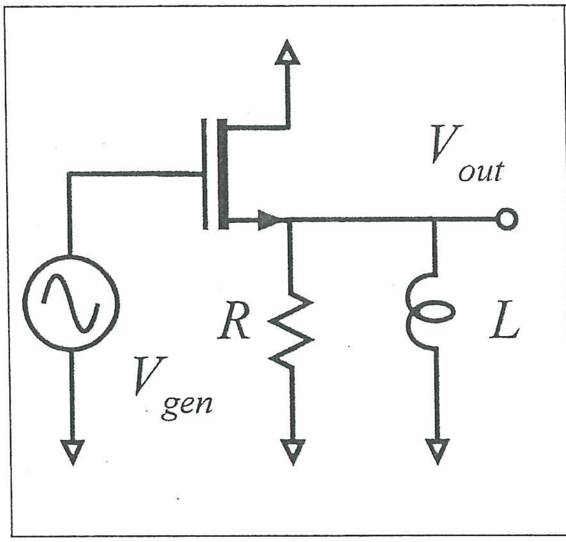
$V_{DS_Q} = 0.6\ V$

$\Delta V_{out} = (0.6 - 0.1V) = \uparrow +0.5\ V$ (+2)

(or) $\uparrow +0.25\ V$

a

Problem 3, 30 points



You will be working on the circuit to the left

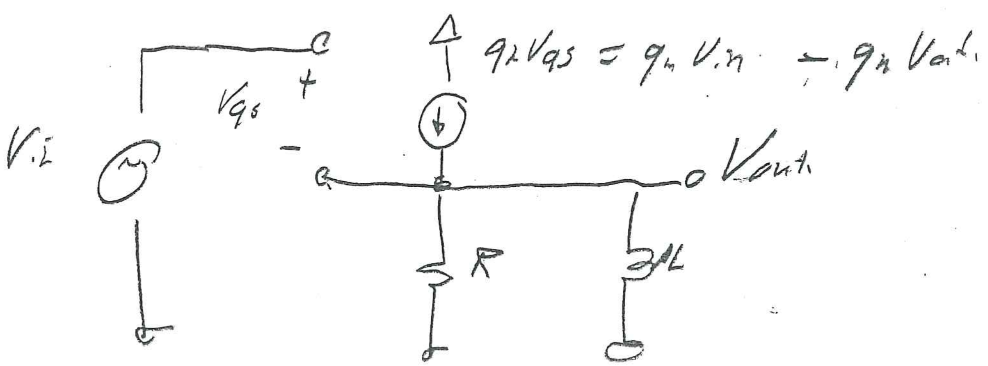
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.



Check controlling voltage of gm Vgs !

a

Part b, 8 points

gm=9 mS. L=1 μH. R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3 \cdot 10^{-6})s} \text{ is dimensionally wrong, } \frac{1}{1+(3 \cdot 10^{-6}\text{ seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ________________

ΣI @ Vout = 0.

4 [$-g_m V_{in} + g_m V_{out} + V_{out}/R + V_{out}/sL = 0$

$g_m V_{in} = V_{out}\left[g_m + 1/R + 1/sL\right]$

$$\frac{V_{out}}{V_{in}} = \frac{g_m}{g_m + 1/R + 1/sL}$$

a

$$\frac{V_{out}}{V_i} = \frac{g_m}{g_m + 1/R + 1/sL} = \frac{sL \cdot g_m}{1 + sL(g_m + 1/R)}$$

$$= \frac{g_m}{g_m + 1/R} \cdot \frac{sL(g_m + 1/R)}{1 + sL(g_m + 1/R)}$$

$$= \frac{9\,mS}{10\,mS} \cdot \frac{s\tau}{1 + s\tau}$$

where τ
$\tau = L(g_m + 1/R)$
$= 1\mu H(10\,mS)$
$= 10\,ns$

4

$$= 0.9 \cdot \frac{s(10\,ns)}{1 + s(10\,ns)}$$

$$f_0 = \frac{1}{2\pi\tau} = \overset{15.9}{16.0}\ MHz =$$

$1\,ns \rightarrow$

a

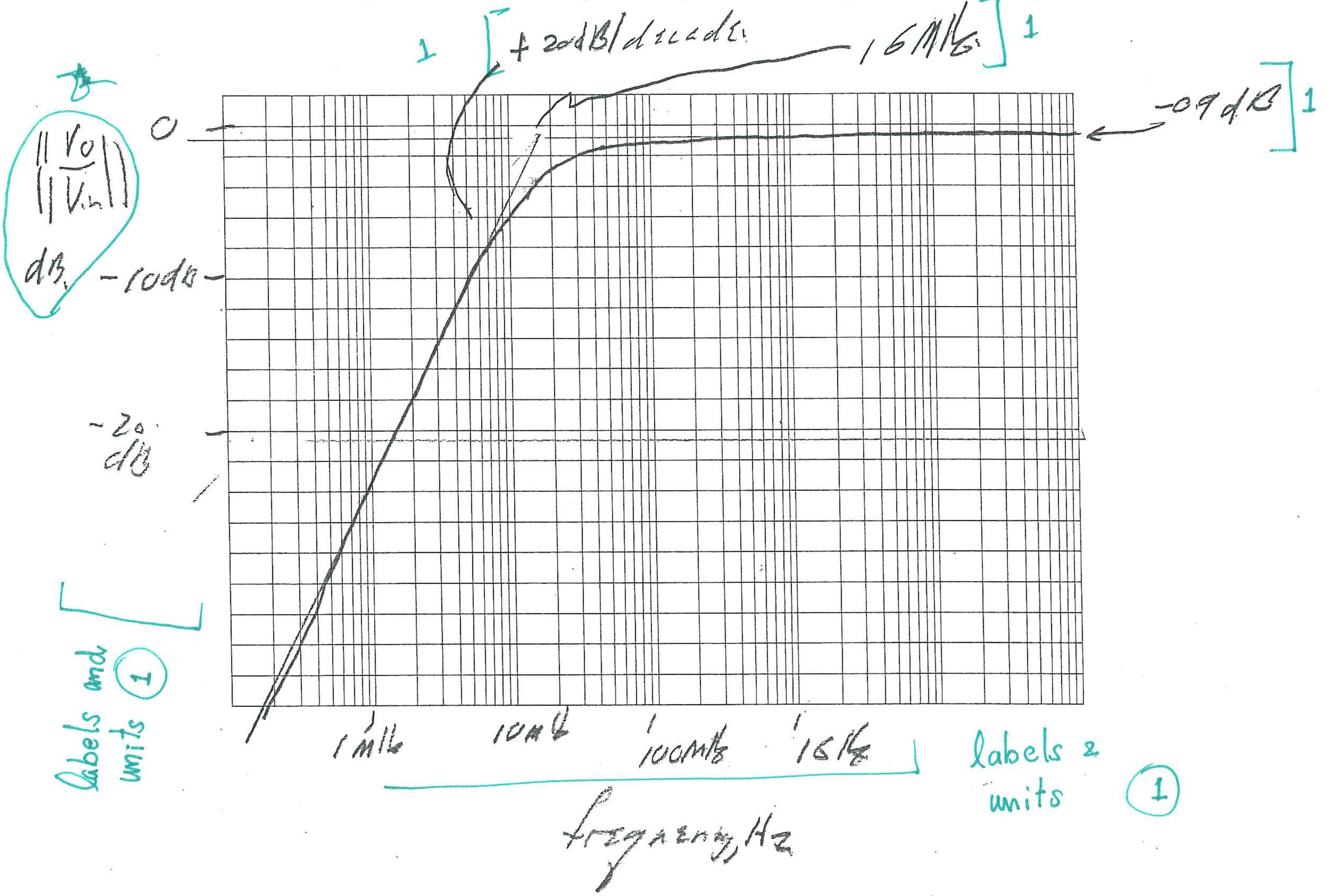
zero @ dc , pole @ 16 MHz 2

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

______________, ______________, ______________, ______________

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



$20 \cdot \log_{10}(0.9) = -0.9 dB \cong -1.0 dB$.

a

$T = 10ms$.

$$H(s) = 0.9 \cdot \frac{sT}{1+sT} \qquad V_{in}(s) = \frac{0.1V}{s}$$

Part d, 8 points

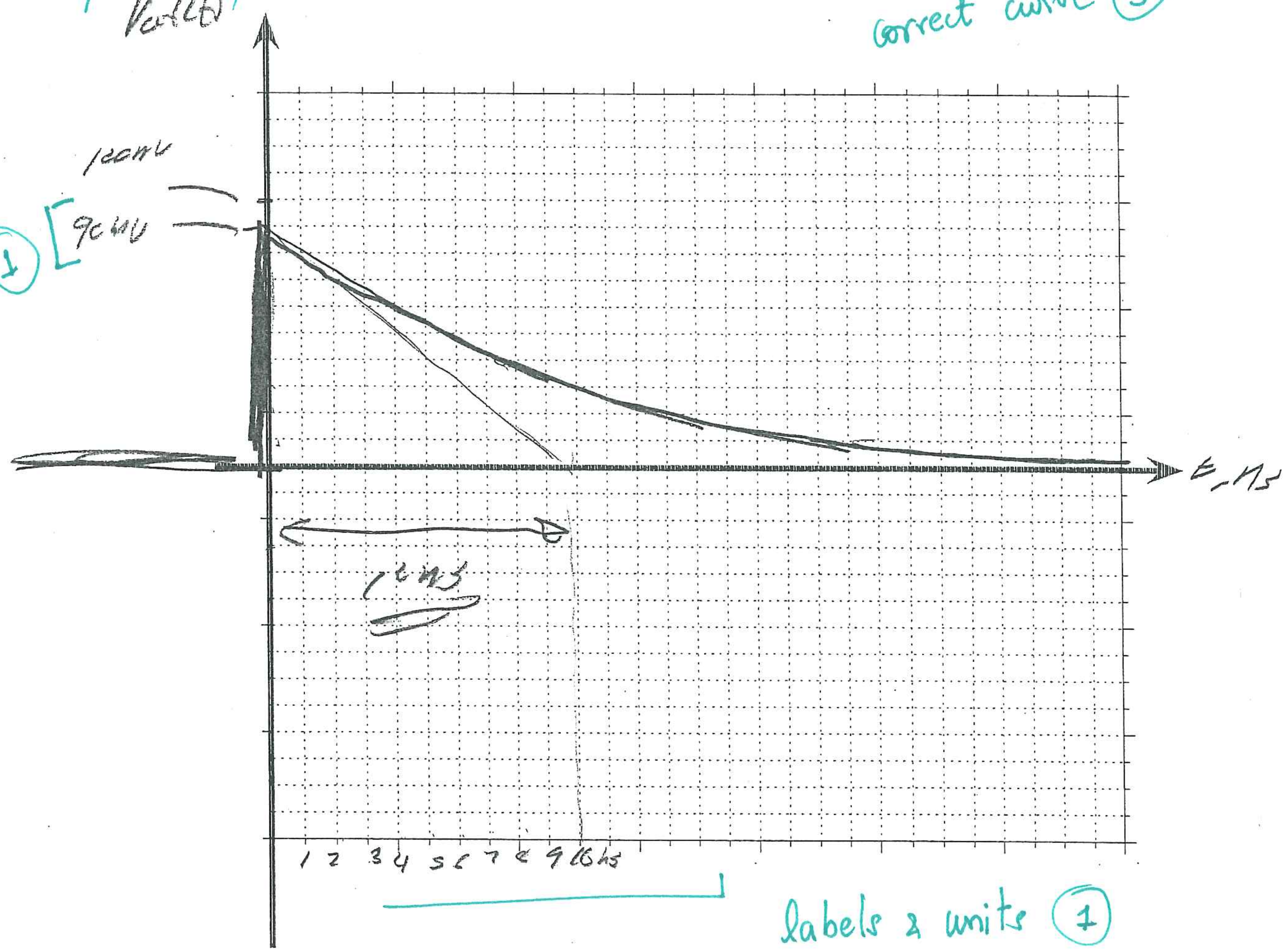
Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)= $90mV \cdot u(t) \cdot e^{-t/10ms}$ (2)

Plot it below. Label axes, show initial and final values, show time constants.

labels + units (1)

correct curve (3)



(1) labels & units (1)

Vout

a

8 25

$$V_{out}(s) = 0.09V \cdot \frac{\tau}{1+s\tau} = 0.09V \cdot \frac{1}{s + 1/\tau}$$

$$V_{out}(t) = u(t)\, e^{-t/\tau} \cdot 0.09V$$

$$= 90mV \cdot u(t) \cdot e^{-t/10ns.}$$