ECE 137 A Mid-Term Exam

Thursday February 5, 2015

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 3 problems on this exam, and you have 75 minutes.

Use any and all reasonable approximations (5% accuracy is fine.),
AFTER STATING and approximately Justifying them.

Name: Solution B

<table>
<thead>
<tr>
<th>Part</th>
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Problem 1, 30 points

You will be working on the circuit below:

The transistor has:
\[ L_g = 45\text{nm}, \quad \mu = 400 \text{ cm}^2/\text{V} \cdot \text{s}, \quad \varepsilon_{r,ox} = 3.8, \quad T_{ox} = 1\text{nm}, \quad v_{sat} = 10^7\text{cm/s}, \quad V_{th} = 0.284V, \]
\[ 1/\lambda = 10\text{V}, \]
From which we calculate:
\[ c_{ox} v_{sat} = 3.36 \text{ mA/V/\mu m}, \quad \mu c_{ox} / 2L_g = 15 \text{ mA/V}^2/\mu \text{m}, \quad \Delta V = L_g v_{th} / \mu = 0.113 \text{V}, \]

The supplies are +1V and -1V
You are to bias the transistor at 1mA drain current,
with 0.5V DC drain voltage, and with -0.35 V DC source voltage.
\[ R_{SAC} = 10\Omega, \quad R_G = 1\ \text{M}\Omega, \quad R_{gen} = 100\ \text{k}\Omega, \quad R_I = 10\mu\Omega. \]

\( C_S \) and \( C_{out} \) are very large (AC short-circuit)
Part a, 10 points
DC bias.
Use this approximation: Ignore (i.e. set to zero) the FET $\lambda$ parameter in the DC bias calculation.
Find the following:
FET gate width $W_g = \underline{15.3 \mu m}$, $R_{ss} = \underline{640 \Omega}$, $R_D = \underline{500 \Omega}$.

\[ V_g = 0 \text{V} \quad \text{so} \quad V_{gs} = 0.35 \text{V} \]

1. Note $V_{dd} + \Delta V = 0.284 \text{V} + 0.113 \text{V} = 0.397 \text{V}$.
2. So, we are mobility-limited.

\[ I_D = \frac{15 \text{mA}}{V_g^2 \mu m} \cdot W_g \cdot (V_{gs} - V_{dd})^2 \]

\[ \rightarrow W_g = \frac{I_D}{\frac{15 \text{mA}}{V_g^2 \mu m} \cdot (V_{gs} - V_{dd})^2} = 15.3 \mu m. \]

1. $R_{ss} + R_{ac} = \frac{1 \text{V} - 0.35 \text{V}}{1 \text{mA}} = \frac{0.65 \text{V}}{1 \text{mA}} = 650 \Omega$.
2. $R_{ss} = 650 \Omega - R_{ac} = 640 \Omega$.

\[ R_D = \frac{V_{DD} - V_D}{I_D} = \frac{1 \text{V} - 0.35 \text{V}}{1 \text{mA}} = 500 \Omega. \]
Part b, 5 points

DC bias

On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors.
Part c, 5 points

Using the actual (nonzero) FET $\lambda$ parameter, find the FET small signal parameters

$\text{gm} = \quad \text{Rds} = \quad$

\[ G_m = \frac{w}{L} \mu C_{ox} (V_{gs} - V_{th}) (1 + \lambda V_{ds}) \]

\[ = \frac{2 \cdot 15 mA}{V^2 \cdot \mu m} \cdot 15.3 \mu m \cdot (0.35V - 0.25V) (1 + \frac{0.85V}{10V}) \]

\[ = 30 \mu A \cdot (1 + \lambda V_{ds}) = 32.9 \mu A \]

\[ \text{Ok to om. (1+}\lambda V_{ds} \text{ term)} \]

\[ R_{ds} = \frac{V_{ds} + 1/\lambda}{I_D} = \frac{1V}{1mA} = 10k\Omega. \]
Part d, 10 points.

Find the small signal voltage gain $V_{out}/V_{in}$ and the amplifier small-signal input resistance.

$V_{out}/V_{in} = \frac{9m}{1 + 9m R_{sc}}$ 

$R_{in, amplifier} = \frac{1}{9m R_{sc}}$

\[ R_{ds} = R_{ds} (1 + 9m R_{sc}) = 10 \Omega \left(1 + \frac{10 \Omega}{9 \Omega}ight) = 12.5 \Omega \]

\[ R_{eq} = R_{ds} || R_{b} || R_{L} = 12.5 \Omega || 50 \Omega || 10 \Omega \]

\[ = 459 \Omega \]

\[ V_{out} = -9m R_{eq} = \frac{-459 \Omega}{10 \Omega} = -45.9 \]

\[ R_{in, amplifier} = R_{g} = 10 \Omega \]
Problem 2, 50 points

You will be working on the circuit below:

Q1: $\beta = 100$, $V_A = \infty$ V
The supplies are +7.5 V and -7.5 V.
You will bias the transistor with 1 mA collector current.
The DC collector bias voltage is -4 V.
$R_L$ is 10 kΩ, $R_{gen}$ is 75 Ω
Part a. 10 points
DC bias.
Find the following:
\[ R_{EE} = \quad R_c = \quad \]

\[ 5 \left[ R_{EE} = \frac{7.5V - 0.7V}{1mA} = \frac{6.8V}{1mA} = 6.8k\Omega \right] \]

\[ 5 \left[ R_c = \frac{3.5V}{1mA} = 3.5k\Omega \right. \]
Part b, 5 points

DC bias

On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors
Part c. 5 points

Find the small signal parameters of Q1.

\[ gm = \frac{1mA}{26mV} = \frac{1}{26mv} = 38mS \]

\[ R_{ce} = 219m + 100 \times 26m = 2.66k\Omega \]

\[ R_{be} = \frac{V_{CE} + V_{B}}{I_{C}} \to \infty \]
Part d, 10 points.

Find the small signal voltage gain \((V_{out}/V_{in})\) of Q1 and the amplifier small-signal input resistance.

\[ V_{out}/V_{in} = \] 

\[ R_{in, amp} = \]
Part e, 5 points

Find \((V_{\text{in}}/V_{\text{gen}})\) and \((V_{\text{out}}/V_{\text{gen}})\)

\[
\frac{V_{\text{in}}}{V_{\text{gen}}} = \frac{26}{26.2 + 75.0} = \frac{26}{101} = 0.257
\]

\[
\frac{V_{\text{out}}}{V_{\text{gen}}} = 0.257 \times 100 = 25.7
\]
Part f. 15 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. *Give the sign (+ or -) in your answers below.*

Cutoff of Q1: Maximum $\Delta V_{out}$ resulting = __________

Saturation of Q1: Maximum $\Delta V_{out}$ resulting = __________

\[ I_C = 1mA \]
\[ I_C = 0mA \]
\[ \Delta I_C = 1mA_{max} \]
\[ R_{eq} = 2.6 \Omega \]
\[ \Delta V_{out} = 2.6 \text{V}. \text{max} = 2.6 \text{V}, \text{negative} \]

\[ V_{CEQ} = 4.7 \text{V} \]
\[ V_{CEQ, mi} = V_{CEQ, sat} = 4.2 \text{V}. \]
\[ \Delta V_{out} = 4.2 \text{V}, \text{positive} \]
Problem 3, 20 points
nodal analysis

You will be working on the circuit to the left.

Ignore DC bias analysis. You don’t need it.

Transistor 1 has transconductance $gm_1$.
Transistor 2 has transconductance $gm_2$.

The drain-source resistances $R_{ds}$ of both transistors are infinity (so you don’t need to draw it!)

Part a. 8 points

Draw the small-signal equivalent circuit

- 3 points for each missing control voltage
- Take off points if controlling voltages not shown
Part b. 8 points

Find, by nodal analysis, a small-signal expression for $V_{out}/V_{in}$.

$V_{out}/V_{in} =$ ________________

\[
\begin{align*}
\text{4.} & \quad E_I = 0 \quad \text{at} \quad V_{in}.
\end{align*}
\]

\[
\begin{align*}
q_{m1} V_{gs1} + q_{m2} V_{gs2} &= 0 \\
\text{Let} & \quad V_{gs1} = V_{in} - V_{at} \\
V_{gs2} &= -V_{at}.
\end{align*}
\]

\[
\begin{align*}
q_{m1} (V_{in} - V_{at}) + q_{m2} (-V_{at}) &= 0. \\
q_{m1} V_{in} &= (q_{m1} - q_{m2}) V_{at} +
\end{align*}
\]

\[
\begin{align*}
4. & \quad \frac{V_{at} +}{V_{at} -} = \frac{q_{m1}}{q_{m1} + q_{m2}}.
\end{align*}
\]
Part c, 4 points

$g_m1 = 1 \text{ mS}$  $g_m2 = 2 \text{ mS}$

Give a numerical value for $V_{out}/V_{in}$.

$V_{out}/V_{in} =$

\[
\left( \frac{V_o}{V_i} \right) = \frac{g_m1}{g_m1 + g_m2} = \frac{1}{3}.
\]