Final Exam, ECE 137A

Wednesday March 18, 2015    7:30-10:30 PM

Name: Solution A

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable
approximations (5% accuracy), after stating & justifying them.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.

Good luck

<table>
<thead>
<tr>
<th>Time function</th>
<th>Laplace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ(t) impulse</td>
<td>1</td>
</tr>
<tr>
<td>U(t) unit step-function</td>
<td>1/s</td>
</tr>
<tr>
<td>e⁻ᵃᵗU(t)</td>
<td>1/(s + a)</td>
</tr>
<tr>
<td>e⁻ᵃᵗcos(ωₐt)U(t)</td>
<td>(s + a) / [(s + a)² + ωₐ²]</td>
</tr>
<tr>
<td>e⁻ᵃᵗsin(ωₐt)U(t)</td>
<td>ωₐ / [(s + a)² + ωₐ²]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Points Received</th>
<th>Points Possible</th>
<th>Part</th>
<th>Points Received</th>
<th>Points Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>5</td>
<td>10</td>
<td>2c</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>1b</td>
<td>6</td>
<td>10</td>
<td>2d</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1c</td>
<td>4</td>
<td>7</td>
<td>3a</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>1d</td>
<td>10</td>
<td>8</td>
<td>3b</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>1e</td>
<td>10</td>
<td>7</td>
<td>3c</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>2a</td>
<td>10</td>
<td>8</td>
<td>3d</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>2b</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts.

All the transistors have the same (matched) $I_s$, have $\beta = 100$, and $V_A = \infty$ Volts.

$V_{CE(sat)} = 0.5V$. $V_{be}$ is roughly 0.7 V, but use $V_{be} = (kT/q) \ln(I_H/I_S)$ when necessary and appropriate. The supplies are +2 Volts and -2 Volts.

Q1ab,2ab,4a,5a,6a are to be biased at 250 $\mu$A collector current.
Q4B,5B,6B are to be biased at 1 mA collector current.
Q8ab are to be biased at 500 $\mu$A collector current.
Q9ab are to be biased at 250 $\mu$A collector current.
$R_L = 250\Omega$ R1a=R1b=500 Ohms
Part a. 5 points
DC bias—to simplify, assume $\beta = \infty$ for the DC analysis only.

On the circuit diagram above, label the DC voltages at ALL nodes, the DC currents through ALL resistors, and the DC collector currents of all transistors.

- $V_{be6b} - V_{be6a} = \frac{kT}{q} \ln \left[\frac{1mA}{250uA}\right] = 36mV \rightarrow R_{ea} = \frac{36mV}{250uA} = 144\, \Omega$
- $V_{be6b} - V_{be8b} = \frac{kT}{q} \ln \left[\frac{1mA}{500uA}\right] = 18mV \rightarrow R_{eb} = \frac{18mV}{500uA} = 36\, \Omega$
- $V_{be7b} - V_{be9b} = \frac{kT}{q} \ln \left[\frac{500uA}{200uA}\right] = 18mV \rightarrow R_{eb} = \frac{18mV}{200uA} = 72\, \Omega$

by some arguments, $R_{ea} = 36\, \Omega$, $R_{9a} = 12\, \Omega$
\[ V_{be_a} - V_{be_4} = \frac{kT}{q} \ln \left( \frac{1mA}{250uA} \right) = 36mV \rightarrow R_{4a} = \frac{36mV}{250uA} = 144\Omega \]

\[ V_{be_3} - V_{be_1} = \frac{kT}{q} \ln \left( \frac{1mA}{500uA} \right) = 18mV \rightarrow R_3 = \frac{18mV}{500uA} = 36\Omega \]

\[ R_{ref} = \frac{1.3V - (-0.6V)}{1mA} = 1900\Omega \]
Part b. 6 points
DC bias:

Find the value of all resistors.
R3 = 2.6 kΩ  R4ba = 2.0 kΩ  R6a = 1.4 kΩ  Rref = 1.4 kΩ  R8a = 2.6 kΩ  R8b = 2.6 kΩ
R9a = 77 Ω  R9b = 77 Ω.
Part c. 4 points

Find the transconductance of the transistors below:

\( g_{m1a} = 9.6 \text{mS} \)  \( g_{m1b} = 9.6 \text{mS} \)  \( g_{m5a} = 9.6 \text{mS} \)  \( g_{m7a} = 19.2 \text{mS} \)

\( g_{m7b} = 19.2 \text{mS} \)  \( g_{m9a} = 9.6 \text{mS} \)  \( g_{m9b} = 9.6 \text{mS} \)

\( Q_{1a,b}: I_c = 2.50 \text{mA}, \quad \frac{1}{g_m} = 10^4 \Omega \to 9.6 \text{mS} \)

\( Q_{5a}: I_c = 2.50 \text{mA}, \quad \frac{1}{g_m} = 10^4 \Omega \to 9.6 \text{mS} \)

\( Q_{7a,b}: I_c = 5.00 \text{mA}, \quad \frac{1}{g_m} = 52 \Omega \to 19.2 \text{mS} \)

\( Q_{9a,b}: I_c = 2.50 \text{mA}, \quad \frac{1}{g_m} = 10^4 \Omega \to 9.6 \text{mS} \)
Part d. 10 points.

Find the following, using the actual value of $\beta$, i.e. $\beta = 100$

<table>
<thead>
<tr>
<th>Transistor combination</th>
<th>Voltage Gain</th>
<th>Input impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2a,2b,1a,1b</td>
<td>0.172</td>
<td>70.8kΩ</td>
</tr>
<tr>
<td>Q5a</td>
<td>95.67</td>
<td>104.5kΩ</td>
</tr>
<tr>
<td>Q7a or 7b</td>
<td>1.0</td>
<td>4.26MΩ</td>
</tr>
<tr>
<td>Q9a or 9b</td>
<td>0.587</td>
<td>42.6kΩ</td>
</tr>
<tr>
<td>Overall differential</td>
<td>9.66</td>
<td>70.18kΩ</td>
</tr>
<tr>
<td>Vout/Vin</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Push-pull → analyze with one off and one on.

Q9a:

Voltage Divider gain:
\[
\frac{250}{250 + 72} = 0.776
\]

Emitter-Follower gain:
\[
R_{eq} = 250Ω + 12Ω = 322Ω
\]
\[
A_V = \frac{322Ω}{322Ω + 104Ω} = 0.756
\]

Overall: $(0.776)(0.756) = 0.587$

\[
R_{ina} = \beta (104Ω + 72Ω + 250Ω) = 42.6kΩ
\]
Q7b: \( R_{\text{req}} = R_{\text{in b}} = 4.26 \, \text{k}\Omega \) (reject \( R_{\text{in c, sb}} \))

\[
\frac{1}{g_{m b}} = 52.0 \\
A_V = \frac{R_{\text{eq}}}{R_{\text{eq}} + g_m} = 1.0 \\
R_{\text{in b}} = \beta [42.6 \, \text{k}\Omega] = 14.26 \, \text{M}\Omega
\]

Q5a: \( R_{\text{req sa}} = 10 \, \text{k}\Omega \parallel R_{\text{in a}} = 10 \, \text{k}\Omega \)

Impedance presented to base of Q5a is that of diodes \( Q_{b2Q_{6b}} \); each \( 26.5 \Omega \) for a total of \( 53.0 \, \text{R}_\text{base} \)

\[
R_{\text{in sa}} = \frac{1}{g_m \, sa} + \frac{R_{\text{base, sa}}}{\beta} = 104.0 + \frac{52.0}{100} = 104.52 \, \text{ohms}
\]

\[
A_V \, sa = \frac{10 \, \text{k}\Omega}{104.52 \, \text{ohms}} = 0.956755
\]
Overall transconductance = \( \frac{1}{g_{m1} + R_{1a}} \) = (60+Ω)^{-1}

Gain = \( \frac{104Ω}{60Ω} \) = 1.73

Input impedance per side is \( \beta \frac{60Ω + 104Ω}{10.8Ω} \) = 10.8Ω
Part e. 10 points

Maximum peak-peak output voltage (*show all your work*)

<table>
<thead>
<tr>
<th>Transistor</th>
<th>magnitude and sign of maximum output signal swing due to <em>cutoff</em></th>
<th>magnitude and sign of maximum output signal swing due to <em>saturation</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q4a</td>
<td>+0.88V</td>
<td>Not Relevant</td>
</tr>
<tr>
<td>Q5a</td>
<td>-0.47V</td>
<td>+1.47V</td>
</tr>
<tr>
<td>Q7a</td>
<td>+12.5V</td>
<td>-1.29V</td>
</tr>
<tr>
<td>Q7b</td>
<td>-12.5V</td>
<td>+1.29V</td>
</tr>
<tr>
<td>Q9a</td>
<td>Not Relevant</td>
<td>+1.16V</td>
</tr>
<tr>
<td>Q9b</td>
<td>Not Relevant</td>
<td>-1.16V</td>
</tr>
</tbody>
</table>

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q7ab and Q8ab form a push pull stage, so be careful about your answers here.

1. 

\[ Q_{ab}: \begin{cases} 
0V & \text{Sat.} \\
+2V & \text{Sat.} \\
-2V & \text{Cutoff}
\end{cases} \]

\[ \Delta V_{emitter} = -1.5V \]

\[ \Delta V_{out} = \left( 1.5V \times 0.776 \right) = -1.16V \]

2. 

\[ Q_{aa}: \text{analysis exactly same but signs are reversed.} \]
\[ Q_{7a} \]

\[
0.7V \quad + \quad 2.7V \quad \rightarrow \quad -2V
\]

\[ \Delta V_{emitt} = 2.2V \]

\[ \Delta V_{a} = (-2.2V)(A_{va}) = (-2.2V)(0.587) = -1.29V \]

Cutoff: \[ R_{eq} = R_{in} = 42.6k\Omega \]

\[ I_{c_{0}} = 0.5mA \]

\[ \Delta V_{emitt} = +2.13V \]

\[ \Delta V_{a} = 2.13V \times A_{va} = 12.5V \text{ (relevant)} \]

\[ Q_{7b} \]

Mirror-symmetric w/ \( Q_{7a} \), so answers the same w/ sign reversed

\[ Q_{5a} : \]

\[
0V \quad + \quad \text{sat.} -0.8V \quad \rightarrow \quad +0.5V \quad -1.3V
\]

\[ \Delta V_{colletor} = -0.8V \]

\[ \Delta V_{a} = -0.8V \cdot A_{v} \cdot A_{va} = -0.47V \]

Cutoff: \[ \Delta V_{colletor} = 2.50mA \cdot 10k\Omega = +2.5V \]

\[ \Delta V_{a} = +2.5V \cdot A_{v} \cdot A_{va} = +1.47V \]

\[ Q_{4a} \]

\[
2V \quad + \quad \text{sat.} \quad \rightarrow \quad 2V + 0.5V \quad \rightarrow \quad -1.5V
\]

\[ \Delta V_{colletor} = +1.5V \]

\[ \Delta V_{a} = 1.5V \cdot A_{v} \cdot A_{va} = +0.88V \]
Problem 2, 35 points

This is an Op-Amp—analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

The NMOSFETs and the PMOSFETs have a 0.20 V threshold, a 22 nm gate length, 300 cm²/Vs mobility, a 10⁷ cm/s saturation drift velocity, and 1/\lambda=3 Volts. The gate oxide thickness is 1.0 nm and the dielectric constant is 3.8. This gives

\[ \mu C_{ox} W_g / L_g = 15 \text{ mA/V}^2 \cdot (W_g / 1 \mu \text{m}) \]

and

\[ v_{sat} C_{ox} W_g = 3.36 \text{ mA/V} \cdot (W_g / 1 \mu \text{m}) \]  

(both are a bit unrealistic for a real technology).

and \( v_{sat} L_g / \mu = 0.113 \text{ V} \)

\[ V_{DD} = +1 \text{ V}, \quad -V_{SS} = -1 \text{ V}, \]
Part a, 10 points
DC bias.

*Approximation: ignore the term \((1 + \lambda V_{DS})\) in DC bias analysis.*

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input \(V_i^+\) is zero volts, and that we must determine the DC value of the negative input voltage \(V_i^-\) necessary to obtain this.

Q1ab,2ab are to be biased at 50 \(\mu A\) drain current.
Q4ab,5ab,6ab,7ab are to be biased at 200 \(\mu A\) drain current.
All transistors are to operate with \(|V_{gs}| = 0.30V\).

Find the gate widths of all transistors.

Find:
- \(W_{g1a} = \quad W_{g1b} = \quad W_{g2a} = \quad W_{g2b} = \quad W_{g3} = \quad W_{g4a} = \quad W_{g4b} = \quad W_{g5a} = \quad W_{g5b} = \quad W_{g6b} = \quad W_{g6b} = \quad W_{g7a} = \quad W_{g7b} = \quad R_{ref} =\)

2. \(V_{gs} = 0.3V\) so \(V_{gs} - V_{th} = 0.1V < \frac{V_{sat}L_g}{W_m}\) so mobility limited

2. \(I_d = \frac{15mA}{V_{gs}^2}\) \(\frac{W_{g4}}{L_{um}}\) \((V_{gs} - V_{th})^2\)

2. \(I_d = 150mA \left(\frac{W}{L_{um}}\right) \rightarrow W_g = \frac{I_d}{150mA}\)

1.33 \(Q_{1a,2a,1b,2b}: I_d = 50mA \rightarrow W_g = 0.333um\)

1.33 \(Q_3: I_d = 100mA \rightarrow W_g = 0.666um\)

1.33 \(Q_{4ab,5ab,6ab,7ab}: I_d = 200mA \rightarrow W_g = 1.33um\)
Part b. 10 points

DC bias

On the circuit diagram above, label the DC voltages at ALL nodes, the drain currents of ALL transistors, and the gate widths of ALL transistors.
Part e, 15 points.

You will now compute the op-amp differential gain. **You must consider the \((1 + \lambda V_{DS})\) term in the FET IV characteristics when you do this.**

The capacitors C1-C4 are all zero Ohms AC impedance. (They would not be present in a real design; they are added here to simplify the exam).

Find the following

<table>
<thead>
<tr>
<th>Transistor combination</th>
<th>Voltage Gain</th>
<th>Input impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2a,2b,1a,1b</td>
<td>10.22</td>
<td>∞</td>
</tr>
<tr>
<td>Q5a</td>
<td>59</td>
<td>15.5 kΩ</td>
</tr>
<tr>
<td>Overall differential</td>
<td>60.3</td>
<td>∞</td>
</tr>
<tr>
<td>Vout/Vin</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Alternative----if you very skilled, you might be able to compute the combined gain of Q2a,2b,1a,1b and Q5a, all together, in a single step using Norton or Thévenin methods. If you do so, first, don't ask for hints on how to do this and, second, do please also calculate the input impedance of Q5a.)

\[
\lambda = \frac{1}{\sqrt{V_{GS}-V_{TH}}} \sqrt{\frac{W}{L}}
\]

\[
g_m = 3.2 mS, \frac{W}{L} \frac{V_{GS}-V_{TH}}{\mu m}
\]

\[
Q_2a,2b,1a,1b: g_m = 1 mS
\]

\[
Q_3: g_m = 2 mS
\]

\[
Q_4a,5a,b,6a,b: g_m = 4 mS
\]

\[
R_{DS} = \frac{1}{\lambda I_0}, \text{ so } R_{DS} = \frac{3V}{20mA} = 15 k\Omega
\]

\[
R_{out, drain} = R_{DS}(1 + \lambda g_m R_{DS}) = 915 k\Omega
\]

\[
R_{L, eq} = R_{out, drain} = \frac{915 k\Omega}{15 k\Omega}
\]

\[
R_{in, sa} = \frac{1}{g_{msa}(R_{out, drain} + R_{DS})} = 15 k\Omega
\]

\[
A_{rs, a} = \frac{R_{L, eq}}{R_{in, sa}} = 15.9
\]
\[ Q_{1ab} = Q_{2ab} \]

0. Call \( g_m = 1 \text{MS} \).

0. Call \( R_S = \frac{3V}{3\text{mA}} = 60 \text{k}\Omega \).

2. Stage transconductance: \( g_{m\text{ab}} = 1 \text{MS} \).

0. \[ R_{\text{eq}} = R_s \parallel R_{\text{osb}} \parallel R_{\text{osa}} = 15.5k\Omega \parallel 60k\Omega \parallel 60k\Omega = 10.22k\Omega \]

2. \[ \text{Gain} = g_{m\text{ab}} \times 10.22k\Omega = 10.22 \]
Part d. 10 points

Maximum peak-peak output voltage at the positive output $V_{o+}$ (show all your work)

<table>
<thead>
<tr>
<th>Transistor</th>
<th>magnitude and sign of maximum output signal swing due to cutoff</th>
<th>magnitude and sign of maximum output signal swing due to: knee voltage (saturation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5a</td>
<td>-183 V</td>
<td>+0.6 V</td>
</tr>
<tr>
<td>Q6a</td>
<td>Cannot be ruled out</td>
<td>-0.6 V</td>
</tr>
<tr>
<td>Q2a</td>
<td>+ huge</td>
<td>Cannot be ruled out</td>
</tr>
<tr>
<td>Q2b</td>
<td>- huge</td>
<td>- irrelevant</td>
</tr>
</tbody>
</table>

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

1. **Q5a**
   - $V_{DS, sat} = V_{GS} - V_{th} = 0.1 V$ for all $f$.
   - $\Delta V_{out} = +0.6 V$

2. **Q6a**
   - $0.6 V$
   - $0.7 V$
   - $0.1 V$
   - $0.3 V$
   - $\Delta V_{out} = +0.6 V$

3. By symmetry, knee voltage of Q6a limits $\Delta V_{out}$ to -0.3V

   - $\Delta I_d = 200 \mu A$
   - $R_{eq} = 91.5 k \Omega$
   - $\Delta V_{out} = -183 V \rightarrow$ irrelevant
As $2a$ & $2b$ are driven, one
full-on and the other cut-off
\[ I_{out} \text{ varies by } \pm 100 \text{ mA} \]
This drives $100 \text{ mA}$ into
an effective load of $915 \text{k} \Omega$
\[ = \pm 91.5 \text{ V} \]
Huge $\rightarrow$ irrelevant

Knee voltage of $Q_{2a}$ cannot be reached unless past $Q_{2b}$ cutoff

Knee voltage of $Q_{2b}$
\[ V_{in} = \begin{cases} 
0.1 \text{ V} & \text{ from knee} \\
-0.2 \text{ V} & \text{ to knee} \\
0.4 \text{ V} & \text{ from knee} \\
-0.3 \text{ V} & \text{ to knee}
\end{cases} \]
\[ V_{out} = -0.9 \text{ V} \]
\[ A_{ Vox } = -0.9 \text{ V} \]
\[ A_{ Vox } = \text{ large number} \]
Problem 3, 30 points

You will be working on the circuit to the left.

Ignore DC bias analysis. You don’t need it.

The transistor has transconductance $g_m$.

Its output resistance $R_{ds}$ is infinity... so you don’t need to include this element in the circuit diagram!

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.
Part b, 8 points

\( \text{gm} = 20 \, \text{mS}, \, C = 1 \, \mu \text{F}, \, R = 1000 \, \text{Ohms} \)

Find, by nodal analysis, a small-signal expression for \( V_{\text{out}}/V_{\text{in}} \). Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

\[
\frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} = K \cdot \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots}
\]

or (as appropriate)

\[
\frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} = K \cdot (s \tau)^n \cdot \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots}
\]

Note that an expression like

\[
\frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} = \frac{1}{1 + (3 \cdot 10^{-6}) s}
\]

is dimensionally wrong; \( \frac{1}{1 + (3 \cdot 10^{-6} \, \text{seconds}) s} \) is dimensionally correct

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \ldots
\]

\[
\sum \text{I @ } V_{\text{out}} = 0 \Rightarrow V_{\text{out}} \left( \Delta C + \frac{1}{R} \right) + V_{\text{in}} \left( g_m - \frac{1}{R} \right) = 0
\]

4 pts

\[
\frac{V_o}{V_m} = \frac{-\left( g_m - \frac{1}{R} \right)}{\Delta C + \frac{1}{R}} = -\left[ g_m R - 1 \right] \frac{1}{1 + \Delta C R}
\]

\( R C = 1 \, \text{mS} = 10^{-3} \, \text{sec} \)

4 pts

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = -19 \left[ 1 + 0.1 \cdot \text{msec} \right]^{-3}
\]

2 pts

\( 1 \, \text{mS} = 1 \Rightarrow f_{\text{pole}} = 159 \, \text{Hz} \)
Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

pole at: 1.59 Hz

Draw a clean Bode Plot of $V_{out}/V_{in}$,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes

2 points for axes labelled sensibly
Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t) = 

Plot it below. Label axes, show initial and final values, show time constants.