

# ECE 137 A Mid-Term Exam

Thursday February 9, 2017

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 3 problems on this exam, and you have 75 minutes.

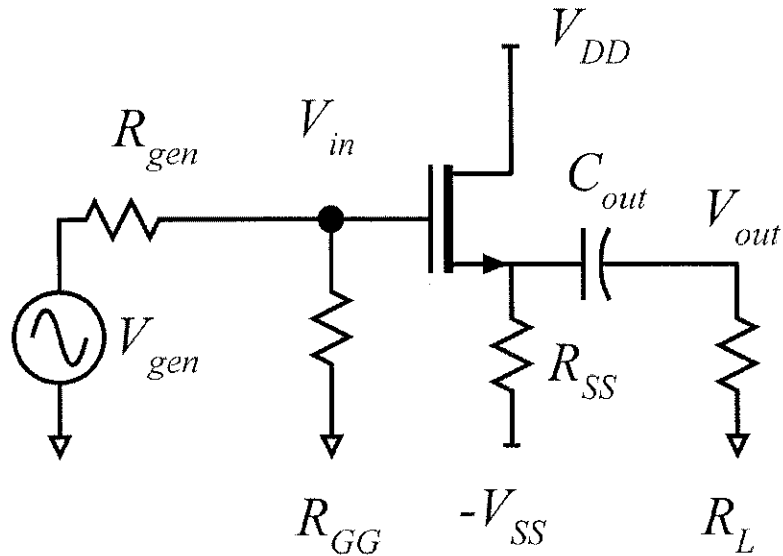
Use any and all reasonable approximations (5% accuracy is fine. ),  
*AFTER STATING and approximately Justifying them.*

Name: Solution A.

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		10	2f		15
1b		5	3a		8
1c		5	3b		8
1d		10	3c		4
1e		15			
2a		10			
2b		5			
2c		5			
2d		10			
2e		5			
TOTAL					100

**Problem 1, 30 points**

You will be working on the circuit below:



The transistor has

$$L_g = 22 \text{ nm}, \mu = 180 \text{ cm}^2/\text{V}\cdot\text{s}, \varepsilon_{r,ox} = 3.8, T_{ox} = 1 \text{ nm}, v_{sat} = 10^7 \text{ cm/s}, V_{th} = 0.3 \text{ V}, 1/\lambda = 10 \text{ V},$$

From which we calculate:

$$c_{ox} v_{sat} = 3.36 \text{ mA/V}/\mu\text{m}, \mu c_{ox} / 2L_g = 13.8 \text{ mA/V}^2/\mu\text{m}, \Delta V = L_g v_{th} / \mu = 0.122 \text{ V},$$

The supplies are +1V and -1 V

You are to bias the transistor at 1.5mA drain current, and with -0.40 V DC source voltage.

$$R_{GG} = 10 \text{ M}\Omega, R_{gen} = 100 \text{ k}\Omega, R_L = 1 \text{ k}\Omega$$

$C_{out}$  are is very large (AC short-circuit)

Part a, 10 points

DC bias.

Use this approximation: Ignore (i.e. set to zero) the FET  $\lambda$  parameter in the DC bias calculation.

Find the following:

FET gate width  $W_g = 10.9 \mu\text{m}$   $R_{ss} = \frac{1060 \Omega}{(400 \Omega)}$

Transistor is biased with  $V_g = 0\text{V}$  and  $V_c = -0.4\text{V}$   
 $\rightarrow V_{gs} = 0.4\text{V}$

$$V_{EL} + \Delta V = 0.2\text{V} + 0.122\text{V}$$

$\rightarrow$  FET is mobility limited.

$$I_D = \frac{\mu_{COV}}{2L_g} \frac{W_g}{V_{th}} \cdot \underbrace{(1 + \lambda V_{DS})}_{\text{ignore for DC}} (V_{GS} - V_{th})^2$$

$$1.5\text{mA} = 13.8 \frac{\text{mA}}{\text{V}^2} \cdot \frac{W_g}{1\mu\text{m}} \cdot \underbrace{(V_{GS} - V_{th})^2}_{0.1\text{V}}$$

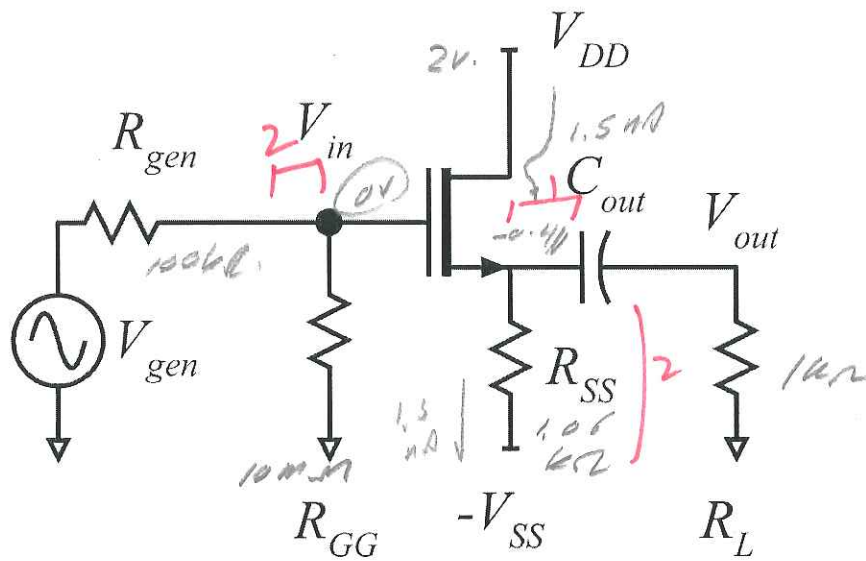
$$\Rightarrow W_g = 10.87 \mu\text{m}$$

$$R_{ss} = \frac{2\text{V} - 0.4\text{V}}{1.5\text{mA}} = 1.06\text{k}\Omega$$

$$\frac{1\text{V} - 0.4\text{V}}{1.5\text{mA}} = 400\Omega$$

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

Part c, 5 points

Using the actual (nonzero) FET  $\lambda$  parameter, find the FET small signal parameters

$g_m = \underline{37.3 \text{ mS}}$ ,  $R_{ds} = \underline{6.66 \text{ k}\Omega}$

$34.2 \text{ mS}$

3

$$g_m = \frac{\partial I_D}{\partial V_{gs}} = 2 \cdot \frac{\mu C_{ox}}{2L} \cdot W \cdot (1 + \lambda V_{DS}) (V_{gs} - V_{th})$$

$$= 2 \cdot \frac{13.8 \text{ } \mu\text{A}}{\text{V}^2} \cdot \frac{10.9 \text{ } \mu\text{m}}{1 \text{ } \mu\text{m}} \left(1 + \frac{2.4 \text{ V}}{10 \text{ V}}\right) (0.1 \text{ V})$$

$$= 37.3 \text{ mS}$$

2

$$R_{DS} = \left\{ \begin{array}{l} 1/\lambda I_D = \frac{10 \text{ V}}{1.5 \text{ mA}} = 6.66 \text{ k}\Omega \\ \text{or} \\ \frac{V_{DS} + 1/\lambda I_D}{I_D} = \frac{2.4 \text{ V} + 10 \text{ V}}{1.5 \text{ mA}} = 8.26 \text{ k}\Omega \end{array} \right.$$

either answer is ok; I will use 6.66 k $\Omega$ .

Part d, 10 points.

Find the small signal voltage gain  $V_{out}/V_{in}$  and the amplifier small-signal input resistance.

$V_{out}/V_{in} = \underline{0.947} \quad 0.904$

$R_{in, \text{ amplifier}} = \underline{\hspace{2cm}}$

3  $R_{eq} = R_{B5} \parallel R_L \parallel R_{B5} = 1.06k\Omega \parallel 1k\Omega \parallel 6.66k\Omega$   
 $= 477.6\Omega \quad 275.4\Omega$

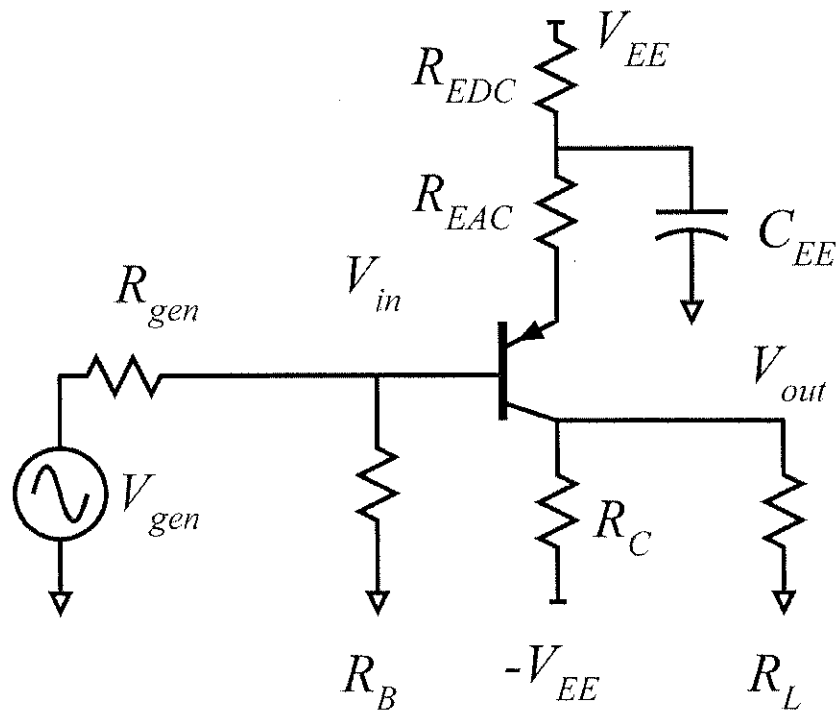
2  $1/g_m = \frac{1}{37.3A} = 26.8\Omega$

3  $A_v = \frac{R_{eq}}{R_{eq} + 1/g_m} = \frac{477.6\Omega}{477.6\Omega + 26.8\Omega}$   
 $= 0.947$

2  $R_{in, \text{ amp}} = R_{B5} = 1M\Omega$

**Problem 2, 50 points**

You will be working on the circuit below:



Q1:  $\beta = 100$ ,  $V_A = \text{infinity V}$

The supplies are +15V and -15 V.

You will bias the transistor with 2mA collector current.

The DC collector bias voltage is 0V.

$R_L$  is  $1000\Omega$ ,  $R_{gen}$  is  $100\Omega$ ,  $R_b$  is  $1\text{ k}\Omega$ ,  $R_{EAC}$  is  $25\Omega$

$C_{EE}$  is very large. Assume that it is an AC short-circuit.

Part a, 10 points

DC bias.

Find the following:

~~$R_{EE} = 1125\Omega$~~   $R_C = 7.5k\Omega$   $R_{EDC} = 7125\Omega$

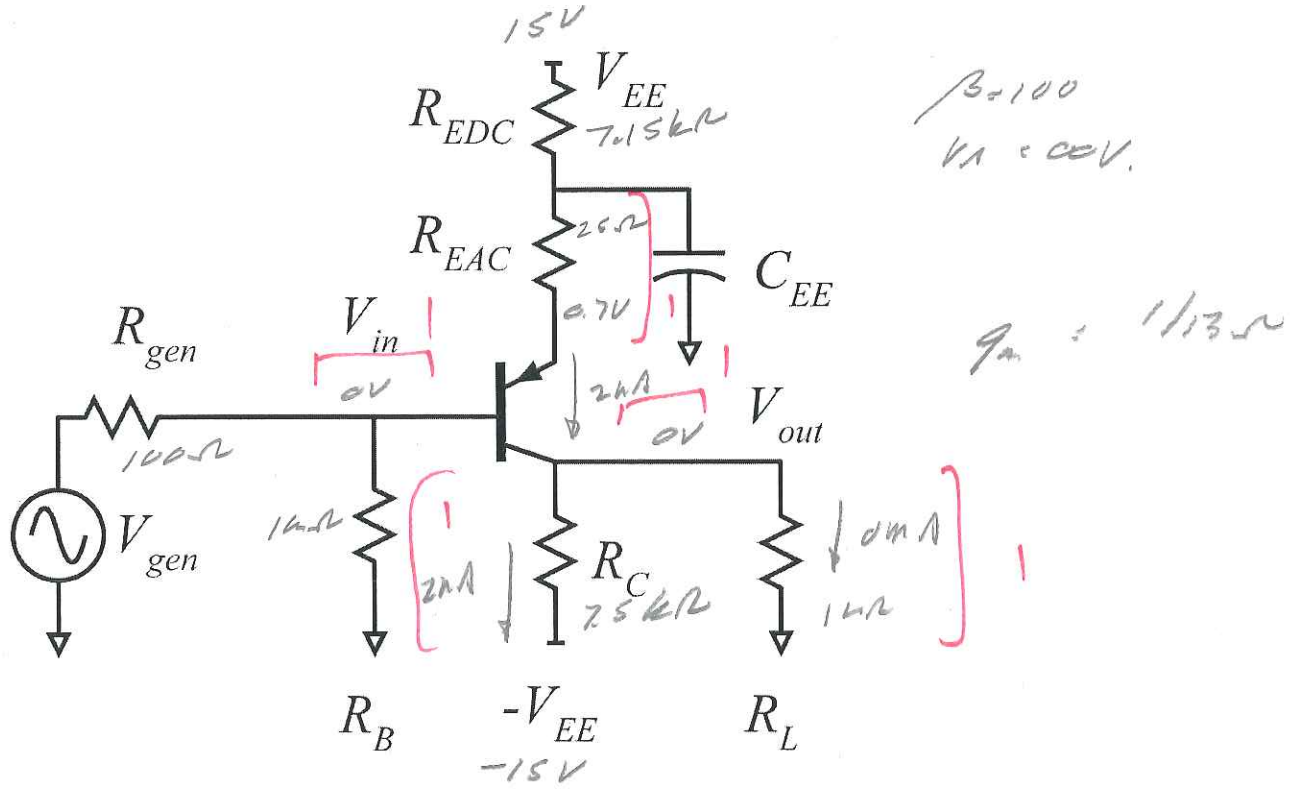
5 {  $R_{EE} = \frac{15V - 0.7V}{2mA} - R_{EBC} = 7125\Omega$   $\overset{25\Omega}{\swarrow}$

5 {  $R_C = 15V / 2mA = 7.5k\Omega$



Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors

Part c, 5 points

Find the small signal parameters of Q1.

$$g_m = \underline{76.9 \mu S} \quad R_{ce} = \underline{\quad\quad\quad} \quad R_{be} = \underline{1.3 \text{ k}\Omega}$$

$$g_m = \frac{2 \text{ mA}}{26 \text{ mV}} = \frac{1}{13 \Omega} = 76.9 \mu S \quad ] 1$$

$$R_{ce} = \frac{V_A + V_{CE}}{I_C} = \frac{0.7 \text{ V} + 100}{2 \text{ mA}} = \infty \quad ] 2$$

$$R_{be} = \beta / g_m = 100 \cdot 13 \Omega = 1.3 \text{ k}\Omega \quad ] 2.$$

Part d, 10 points.

Find the small signal voltage gain ( $V_{out}/V_{in}$ ) of Q1 and the amplifier small-signal input resistance.

$$V_{out}/V_{in} = \underline{-23.2}$$

$$R_{in,amp} = \underline{792\Omega}$$

$$2.5 \left( R_{eq} = R_B \parallel R_C = 1k\Omega \parallel 7.8k\Omega = 882\Omega \right)$$

$$2.5 \left( A_v = -\frac{R_{eq}}{R_{ECC} + 1/g_m} = \frac{882}{25\Omega + 13\Omega} = -23.2 \right)$$

$$2.5 \left( R_{in} = \beta (1/g_m + R_{ECC}) = 100 \left( \frac{25\Omega + 13\Omega}{34\Omega} \right) = 3.86\Omega \right)$$

$$2.5 \left( R_{in,amp} = 3.8k\Omega \parallel R_B = 792\Omega \right)$$

Part e, 5 points

Find  $(V_{in}/V_{gen})$  and  $(V_{out}/V_{gen})$

$$(V_{in}/V_{gen}) = \frac{0.854}{\quad}$$

$$(V_{out}/V_{gen}) = \frac{-20.6}{\quad}$$

2.5

$$\left[ \begin{aligned} V_{in}/V_{gen} &= \frac{R_{in \text{ out}}}{R_{in \text{ out}} + R_{gen}} = \frac{792 \Omega}{792 \Omega + 100 \Omega} \\ &= 0.858 \end{aligned} \right.$$

2.5

$$\left[ \begin{aligned} V_{out}/V_{gen} &= \frac{V_{in}}{V_{gen}} \cdot \frac{V_{out}}{V_{in}} = 0.858 \cdot (-23.2) \\ &= -20.6 \end{aligned} \right.$$

Part f, 15 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. **Give the sign (+ or -) in your answers below.**

Cutoff of Q1; Maximum  $\Delta V_{out}$  resulting =  $-1.76V$

Saturation of Q1; Maximum  $\Delta V_{out}$  resulting =  $+0.2V$

cutoff:  $\Delta I = 2mA$ ,  $R_{eq} = 882\Omega$

$\Delta V_{out} = 882\Omega \cdot 2mA = 1.76V$

saturation

$V_{CE sat} = 4.2V$

$V_{CE, bias} = 0.7V$

$\Delta V_{CE} = 0.2V$  ← acceptable answer.

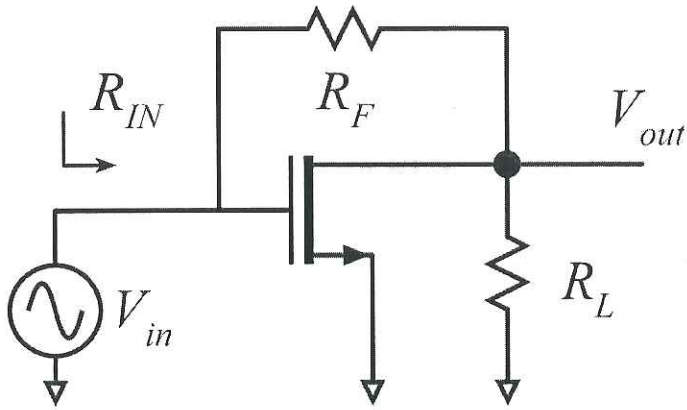
$\Delta V_{out} = \Delta V_{CE} \cdot \frac{5V_C}{5V_{CE}} = \Delta V_{CE} \cdot \frac{R_{eq}}{R_{eq} + r_{eac}}$

$= 0.2V \cdot \frac{882\Omega}{882\Omega + 25\Omega} = 0.194V$

more exact answer

either ok

**Problem 3, 20 points**  
nodal analysis



You will be working on the circuit to the left.

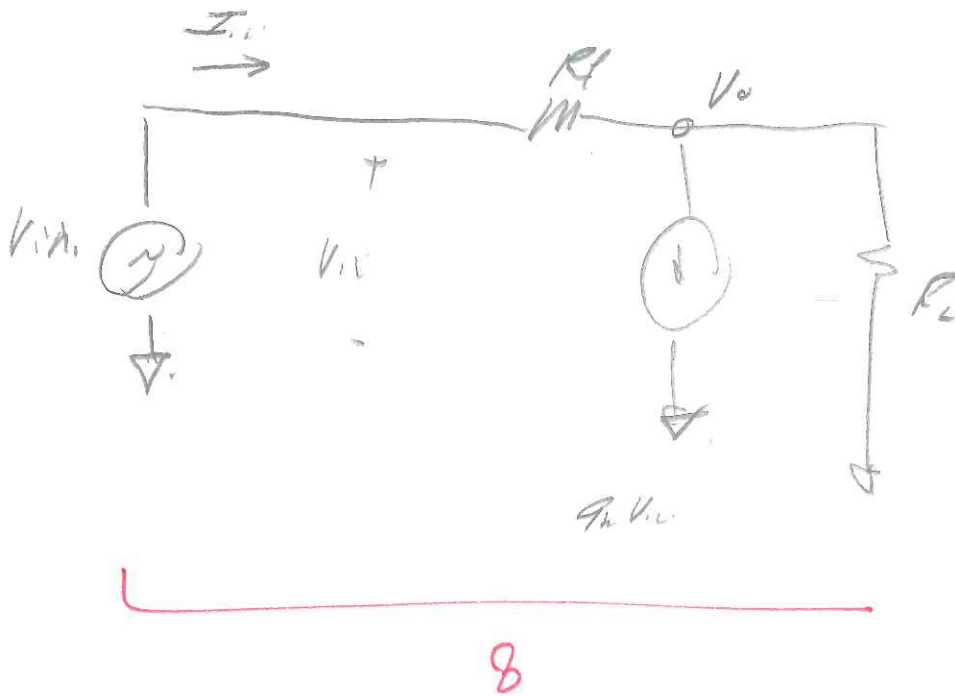
Ignore DC bias analysis. You don't need it.

The transistor has transconductance  $g_m$ .

The drain-source resistance  $R_{ds}$  of the transistor is infinity (so you don't need to draw it!)

Part a, 8 points

Draw the small-signal equivalent circuit



Part b, 8 points

Find, by nodal analysis, a small-signal expression for  $R_{in}$ .

$R_{in} =$  \_\_\_\_\_

$$G_L = 1/R_L$$

$$G_F = 1/R_F$$

Σ I @  $V_o = 0$

$$(V_o - V_{in})/R_F + V_o/R_L + g_m V_o = 0$$

$$V_{in}(g_m - G_F) + V_o(G_F + G_L) = 0$$

$$V_o = V_{in} \frac{(G_F - g_m)}{G_F + G_L}$$

$$I_{in} = (V_{in} - V_o)G_F = G_F V_{in} \left[ 1 + \frac{g_m - G_F}{G_F + G_L} \right]$$

$$\frac{I_{in}}{V_{in}} = \frac{1}{R_{in}} = G_F \left[ \frac{G_F + G_L + g_m - G_F}{G_F + G_L} \right]$$

$$R_{in} = R_F \frac{1/R_F + 1/R_L}{1/R_L + g_m} = R_F \frac{1 + R_L/R_F}{1 + g_m R_L}$$

$$R_{in} = \frac{R_F + R_L}{1 + g_m R_L}$$

Part c, 4 points

$g_m = 1 \text{ mS}$ ,  $R_L = 3 \text{ k}\Omega$ ,  $R_f = 2 \text{ k}\Omega$ .

Give a numerical value for  $R_{in}$ .

$$R_{in} = \underline{1.25 \text{ k}\Omega}$$

$$R_{in} = \frac{R_f + R_L}{1 + g_m R_L}$$

$$= \frac{5 \text{ k}\Omega}{1 + 3 \text{ k}\Omega \cdot 1 \text{ mS}} = \frac{5 \text{ k}\Omega}{4} = 1.25 \text{ k}\Omega$$

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