Final Exam, ECE 137A

Wednesday March 20, 2019, Noon - 3 p.m.

Name: Solution

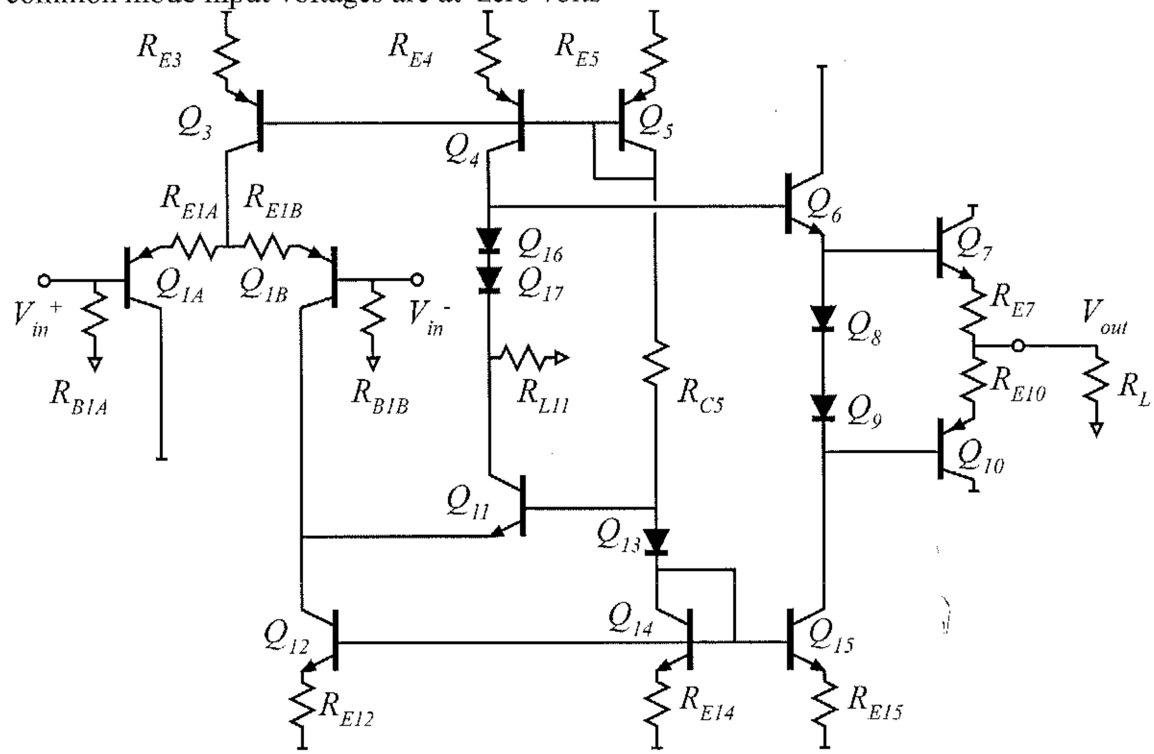
Closed Book Exam:
Class Crib-Sheet and 4 pages (4 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\dfrac{1}{s+\alpha}=\dfrac{1/\alpha}{1+s/\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\dfrac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		6	2c		15
1b		5	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



All the transistors have the same (matched) I_S, have $\beta = 100$, and $V_A = \infty$ Volts.

$V_{CE(sat)} = 0.5\text{V}$.

V_{be} is approximately 0.7 V,

but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary or appropriate.

The supplies are +3 Volts and -3 Volts.

All transistors (and diodes) have the same I_S

Q1A,1B,5,11 are biased at 5mA collector current.

Q6 is biased at 20mA collector current.

Q7 and Q10 are biased at 5mA collector current.

The DC voltage drops across RE5 and RE14 are both 300mV.

RB1A=RB1B=2kOhm. RE1A=RE1B=44.8Ohm. RL11=1.1kOhm.

RL=1kOhm.

b

Part a, 6 points
DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***

Find the value of the following resistors:

Re5=____________ Re14=____________ Re4=____________

Re15=____________ Re12=____________ Re3=____________

Re7=____________ Re10=____________

1 $R_{E3} = \frac{300mV}{10mA} = 30\Omega = R_{E12}$.

1 $R_{E4} = \frac{300mV}{5mA} = 60\Omega = R_{E5} = R_{E14}$

1 $R_{E15} = \frac{300mV}{20mA} = 15\Omega$

3 $R_{E7} = R_{E10} = \frac{V_T}{5mA} \ln\left(\frac{20mA}{5mA}\right) \cong 7.2\Omega$

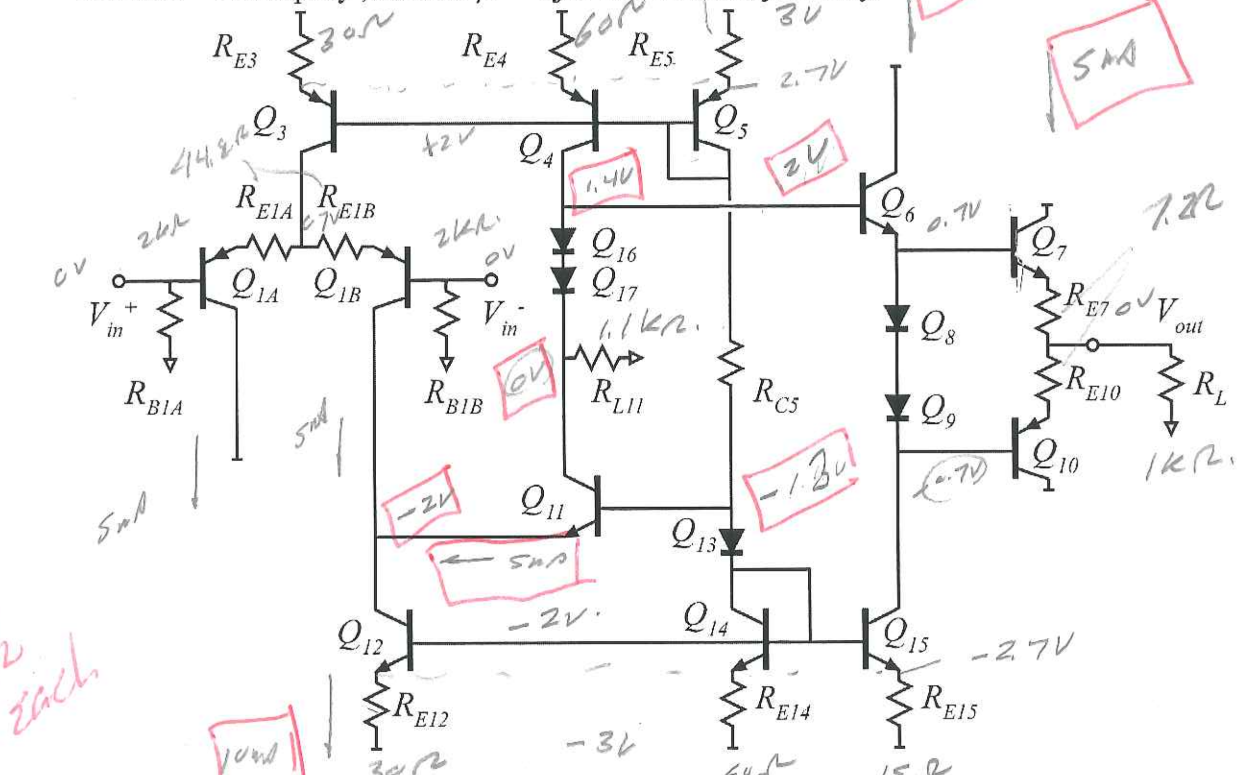
b

$\beta = 100$ $V_{CESAT} = 1/2\ V.$

$V_A = \infty V$

Part b, 5 points

DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***



On the circuit diagram above, label the DC voltages at **ALL nodes**, and the DC collector currents of **all transistors**. Label the values of all resistors (except RC5).

$$R_{C5} = \frac{2V + 1.3V}{5mA} = \frac{3.3V}{5mA} = \frac{\cancel{4V}}{5mA}$$

$$= 5\cancel{60}\Omega$$

$$= 660\ \Omega$$

b

Part c, 4 points

find the following

device	Q1AB	11	12	4	6	15	7	10
gm, mS								

Q 1A, 1B, 11, 14

$I_C = 5mA \rightarrow g_m = \frac{5mA}{26mV} = \frac{1}{5.2\Omega} = 192 mS.$

1

Q 12, 3

$I_C = 10mA \quad g_m = \frac{10mA}{26mV} = \frac{1}{2.6\Omega} = 385 mS.$

1

Q 6, 15

$I_C = 20mA \quad g_m = \frac{20mA}{26mV} = \frac{1}{1.3\Omega} = 769 mS.$

1

Q 7, 10

$I_C = 5mA \quad g_m = \frac{5mA}{26mV} = \frac{1}{5.2\Omega} = 192 mS$

1

b

Part d, 10 points.

Find the following, ***using the actual value of*** β***, i.e.*** β***=100***

	Voltage Gain	Input impedance
Q1AB	~~0~~ 0.0583	1.43 kΩ.
Q11	188.6	5.83 Ω
Q6	1	10 MΩ
Q7	0.98931	100kΩ
Overall differential Vout/Vin	11	1.43kΩ.

Note: with some insight, you can find the combined gain of Q1AB/11 in a single step. If would would like to do so, omit the separate answers for Q1AB and Q11 in the table above, and instead fill in the table below,

	Voltage Gain	Input impedance
Q1AB/ Q11 combination.	11	1.43 kΩ.

Note - $R_{CE} = \infty\ \Omega$ - Simplifies calculations.

We can treat ~~Q7,Q10~~ / Q7,10 as both on, or on one at a time.

TAs: Wide range of assumptions ok for this part.

Treat Q7 as on / 10 as off.

$$\text{Q7: EF:} \quad A_v = \frac{1k\Omega}{1k\Omega + 5.2\Omega + 7.2\Omega} = 0.988 \cong 1$$

$$R_{i7} = \beta(1k\Omega + 5.2\Omega + 7.2\Omega) \cong \beta(1k\Omega) = 100k\Omega.$$

b

Q6: EF

$R_{Leq} = R_{in7} = 100k\Omega$

$A_v = \dfrac{100k\Omega}{100k\Omega + 7.2\Omega + 5.2\Omega} \cong 1$

$R_{i6} \cong \beta(100k\Omega) = 10M\Omega$

Q11 - common base.

$R_{base} = (1/g_{m13} + 1/g_{m14} + R_{E14}) \,||\, (r_{o5} + 1/g_{m5} + R_{E5})$

$= (5.2\Omega + 5.2\Omega + 60\Omega) \,||\, (660\Omega + 5.2\Omega + 60\Omega)$

$= 70.4\Omega \,||\, 725\Omega = 64\Omega.$

(-1pt for using $R_B = 0\Omega$)

$R_{in11} = 1/g_m + R_{base}/\beta = 5.2\Omega + \dfrac{64\Omega}{100} = 5.84\Omega$

$R_{Leq11} = R_{C11} \,||\, R_{in6} = 1.1k\Omega \,||\, 10M\Omega \cong 1.1k\Omega.$

$A_{v11} = \dfrac{1.1k\Omega}{5.83\Omega} = 188.6$

or work on next pg.

Q1A / 1B

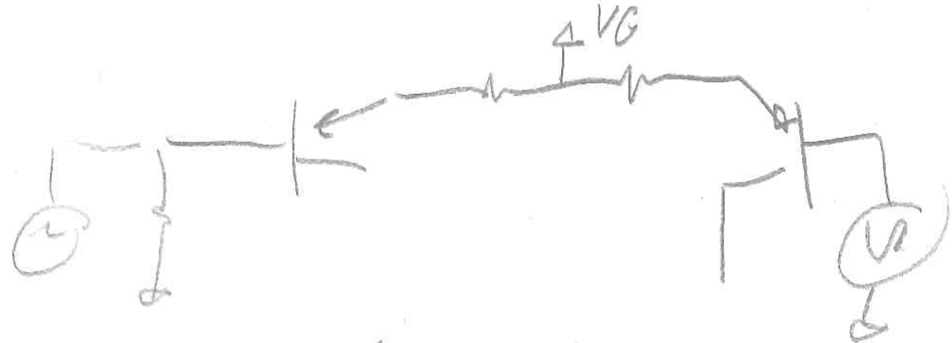
$R_{Leq} = R_{in11} = 5.83\Omega.$

$A_v = \dfrac{R_{Leq}}{2(1/g_m + R_{E1})} = \dfrac{5.83\Omega}{2(5.2\Omega + 44.8\Omega)} = \dfrac{5.83\Omega}{100\Omega}$

$= 0.0583.$

b

Input impedance depends on input configurat.



$Z_i = 2k\Omega \,//\, \beta(1/g_m + R_{emc})$
$= 2k\Omega \,//\, 100(50\Omega) = 2k\Omega \,//\, 5k\Omega$
$= 1.43k\Omega$.

Or the Q1A&B/11 combinat.

$A_v = \dfrac{R_{eq11}}{2(1/g_{m1A} + R_{em})} = \dfrac{1.1k\Omega}{100\Omega} = \underline{\underline{11}}$

or 6pt

b

Part e, 10 points

Maximum peak-peak output voltage (***show all your work***)
For this, you must use the full circuit diagram, not the half circuit diagram.

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q7	Not Relevant	2.47V
Transistor Q10	N.R.	−2.47V
Transistor Q6	−2000V	+1.78V
Transistor Q15	NR - no signal	−1.5V
Transistor Q4	NR - no signal	0.8V
Transistor Q11	+5.5V	−2.2V
Transistor Q1A	+5.5V	omit
Transistor Q1B	−5.5V	omit

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q7/10 form a push pull stage, so be careful about your answer there. .

1/2 Q7 cutoff - not relevant - push-pull

1/2 Q7 saturation: $V_{out} = (3V - V_{CE sat}) \frac{1k\Omega}{1k\Omega + 7.2\Omega + 5.2\Omega} =$

$= (3V - 1/2V)\ 0.988 = 2.47V$

1 Q10 same as Q7 - polarities reversed.

1 Q6 saturation: $V_{out} = (V_{cc} - V_{ce sat 16} - V_{CE sat 7})(0.988)$

$= (3V - 1/2V - 0.7V)\ 0.988 = (1.8V)\ 0.988 = 1.78V$

1 Q6 cutoff: $\Delta V_{out} = -I_{C6} \cdot R_{eq} \cdot A_{v7} = 2mA \cdot 1000k\Omega \cdot 0.99$

$= -2000V \cdot 0.99 \cong -2000V$

1 Q15 saturation: $V_{out} = (-2.7V + V_{ce sat})\ 0.99$

$= -2.2V\ (0.99) \cong -2.2V.$

b

Q4 saturati

$V_{out} = (+2.7V - V_{ce,sat} - V_{be6} - V_{be7})\,0.99$

$\cong 2.7V - 1.2V - 1.4V = 0.8V.$

Q11 saturati

$V_{out} = (-20V + V_{ce\,sat} + V_{be16} + V_{be17} - V_{be1} - V_{be7})\,0.99$

$\cong -20V + 0.5V = -1.5\ V.$

Q1A cutoff

If $I_{c1A} \rightarrow 0mA$ then $I_{c1B} = 10mA$.

and $I_{c11} = 0mA$

$\Rightarrow \Delta V_{c11} = 5mA \cdot 1.1k\Omega = +5.5V.$

$\Delta V_{out} = 5.5V\,(0.99) \cong 5.5V.$

Q1B cutoff - same as above, but will signs reversed.

$\Delta V_{out} = -5.5V.$

Q11 cutoff

$\Delta I_c = 5mA \rightarrow \Delta V_{c11} = 5mA \cdot 1.1k\Omega$

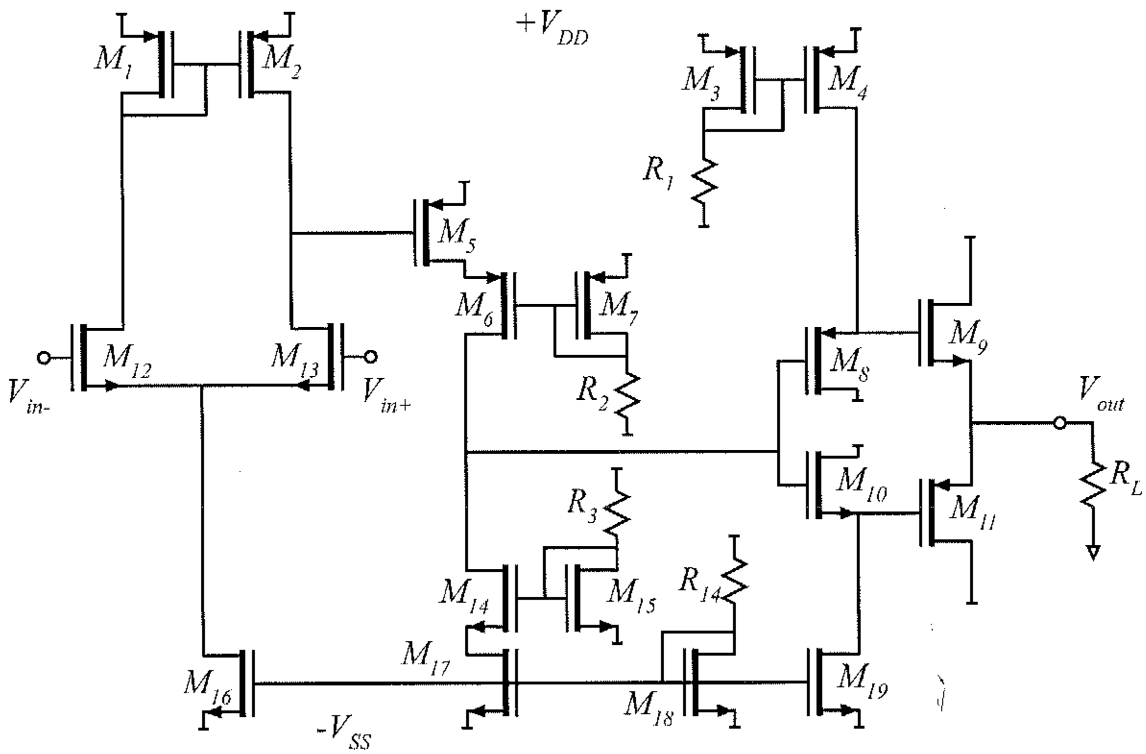
$= +5.5V$

$\Delta V_{out} = 5.5V\,(0.99) \cong 5.5V.$

b

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



The NMOSFETs have $K_\mu = \mu c_{gs} W_g / 2L_g = 0.55\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 0.69\text{mA/V} \cdot (W_g / 1\mu\text{m})$, $\Delta V = v_{inj} L_g / \mu = 0.625\text{V}$, $V_{th} = 0.3\text{V}$,

$1/\lambda = 20\text{V}$

The PMOS have identical parameters, except, of course, V_{th} is negative.

$V_{DD} = +1$ V, $-V_{SS} =$ -1 V, R_L=50 kOhm

All transistors have |Vgs|=0.4V, ***except for M7 and M15***, which have |Vgs|=0.5V, ***and except for M8,9,10,11***, which have |Vgs|=0.35V

M12,13 are biased at I_D=50 μA.

M5,7,15 are biased at I_D=35 μA.

M8,9,10,11 are biased at I_D=25 μA.

Ignore $(1+\lambda V_{DS})$ term

Part a, 10 points

DC bias.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.
(Hint, this should give Vi- = 0V)

Find the following:

Gate widths of M12 and M13 = ____________________

Gate width of M7 = ____________________

Gate width of M8 = ____________________

Gate width of M9 = ____________________

$$I_D = 0.55\,\mu A/V^2 (V_{GS}-V_{TL})^2 \cancel{(1+\lambda V_{DS})}$$

M12, 13: $$W_g = \frac{50\,\mu A}{0.55\,\mu A/V^2}\frac{1}{(0.1V)^2} = 9\,\mu m.$$

M7: $$\underline{W_g} = \frac{35\,\mu A}{0.55\,\mu A/V^2}\frac{1}{(0.2V)^2} = 1.6\,\mu m$$

M8/9: $$\underline{W_g} = \frac{25\,\mu A}{0.55\,\mu A/V^2}\frac{1}{(0.05V)^2} = 18\,\mu m$$

3.33 each

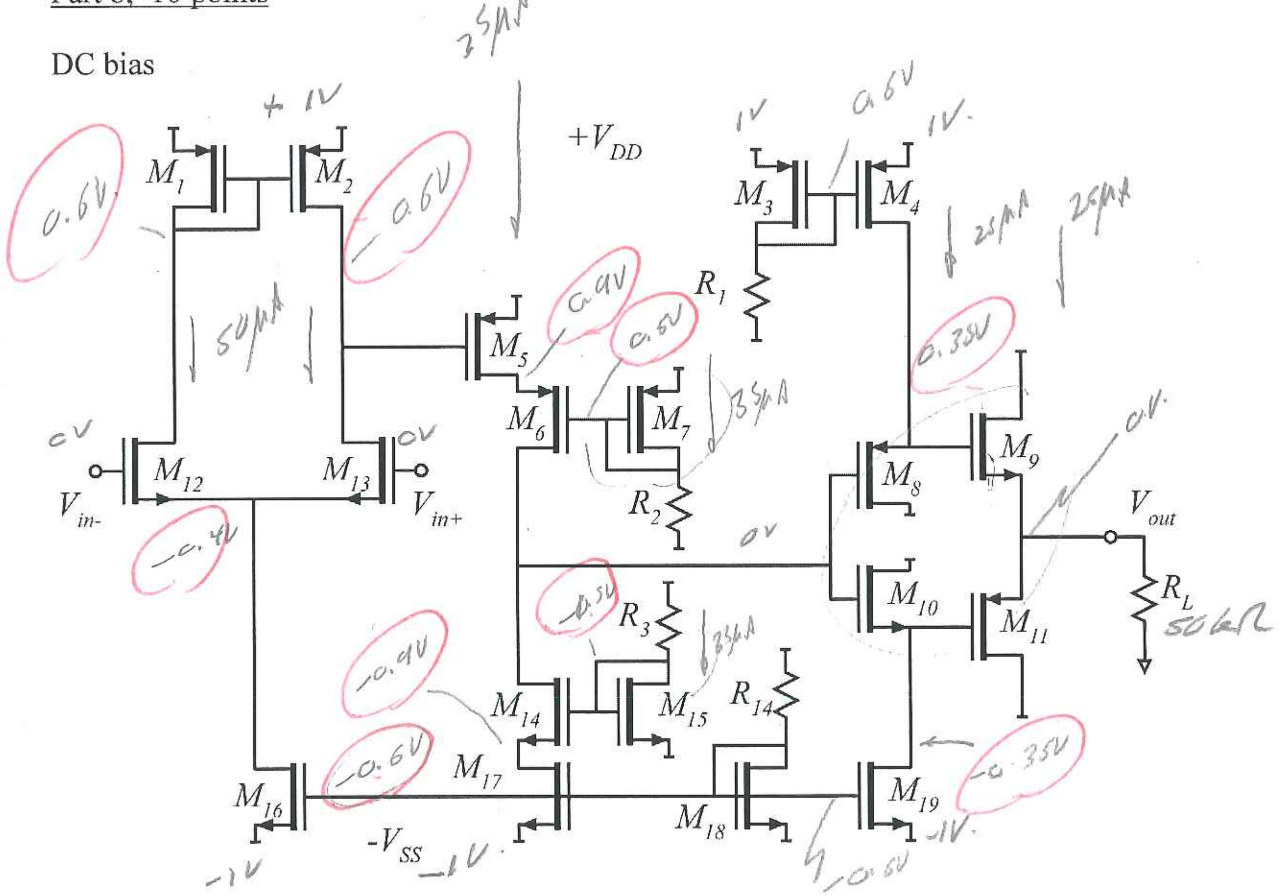
b

$K_n = 0.55mA/V^2 \cdot \frac{W_g}{\mu m}$ $1/\lambda = 20V.$

$V_{th} = 0.3V$

Part b, 10 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes,** the drain currents of **ALL transistors,** ~~and the gate widths of **ALL transistors**~~

1 each

b

Part c, 15 points.

You will now compute the op-amp differential gain. Find the following

	Voltage Gain	Input impedance
Transistor combination M1,2,13, 13	~~@~~1 200	$\infty\,\Omega$
M5,6 combination	80,000	"
~~Q9 or Q12~~: M8 or M10	~ 1	"
~~Q8 or Q15~~ M10 or M11	~ 1	"
Overall differential Vout/Vin	16 Million	$\infty\,\Omega$

Notes:

1) You can analye M5 and M6 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

2) For M8/9and for M10/11, you can assume that M8 and M9 are on for the positive signal swing and M10 and M11 are on for the negative signal swing. More accurately, you can assume, for the signal swing near zero volts, that all are on. If you take the latter approach (and do it correctly), you will receive a couple of extra credit points. One hint (don't ask for any other hints): use symmetry.

M9 & M11 take both* as on (signal near zero V)
each effectively draws $2R_2 = 100k\Omega$.

$R_{Leq9} = R_{os9} \| 100k\Omega = \frac{1}{\lambda I_D} \| 100k\Omega = \underbrace{\frac{20V}{25mA}}_{800k\Omega} \| 100k\Omega$

$\simeq 89k\Omega$.

(if we take M9 on, M10 off $\rightarrow R_{Leq} \simeq 47k\Omega$ - ok)

$A_{v9/11} = \frac{89k\Omega}{89k\Omega + 1/g_m}$; $g_m = \frac{2I_D}{V_{gs}-V_{tl}} = \frac{2(25mA)}{50mV} = 1mS$.

$\simeq 0.989$

$R_{in9} = \infty\,\Omega$

b

M8 or M10

1/2 $\left[R_{DS4} = R_{DS8} = \dfrac{1}{\lambda I_D} = \dfrac{20V}{25\mu A} = 800k\Omega \right.$

$\left[R_{Leq} = R_{DS4} \| R_{DS8} = 400k\Omega \right.$

1/2 $\left[g_m = 2 I_D / (V_{gs} - V_{th}) = 2(25\mu A) / 50mV = 1mS \right.$

1 $\left[A_V = \dfrac{400k\Omega}{400k\Omega + 1/g_m} = 0.9988 \cong 1. \right.$

M6 1/2 $\left[R_{DS14} = R_{DS17} = 1/\lambda I_D = 20V / 35\mu A = 571k\Omega. \right.$

1/2 $\left[g_{m14} = g_{m17} = g_{m5} = g_{m6} = 2(35\mu A) / (V_{gs} - V_{th}) \right.$

$\doteq 70\mu A / 0.1V = 0.7mS$

Note that M7 is biased at $V_{DS} = V_{knee} \Rightarrow$ ok. $R_{out} = R_{DS} = 1/\lambda I_D$

1 $\left[R_{out14} = R_{DS}(1 + g_m R_{DS}) \cong g_m R_{DS}^2. \right.$

1 $\left[R_{Leq6} = R_{out14} = g_m R_{DS}^2 \right.$

1 $\left[R_{in6} = \dfrac{1}{g_m}\left(1 + \dfrac{R_{Leq6}}{R_{DS}}\right) = \dfrac{1}{g_m}\left[1 + \dfrac{g_m R_{DS}^2}{R_{DS}}\right] \doteq R_{DS} \; (!) \right.$

1 $\left[A_{V6} = R_{Leq6} / R_{in6} \doteq g_m R_{DS}. \right.$

1 $\left[\right.$ M5/ $R_{Leq5} = R_{DS5} \| R_{in6} = R_{DS} \| R_{DS} = R_{DS}/2.$

1 $\left[A_{V5} = -g_m R_{Leq} = -g_m R_{DS}/2. \right.$

So:

1 $\left[A_{V5} A_{V6} \doteq (-g_m R_{DS}/2)(g_m R_{DS}) = -(g_m R_{DS})^2/2 \right.$

$= -(0.7mS \cdot 571k\Omega)^2/2 = \underline{-80,000}$

b

$M1, 2, 12, 13$

1/2 $g_{m\,12,13} = \frac{2 I_D}{(V_{GS} - V_{th})} = \frac{2(50\mu A)}{0.1V} = 1mS$

1/2 $R_{DS} = 1/\lambda I_D = 20V/50\mu A = 400k\Omega$

1 $R_{Leq} = R_{DS2} \parallel R_{DS13} = R_{DS}/2$

1 $A_v = g_m R_{Leq} = g_m R_{DS}/2$

$= \underline{200}$

b

Part d, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor M9	NR	0.95V
Transistor M11	NR	−0.95V
Transistor M8	+10V	NR
Transistor M10	−10V	NR
Transistor M4	NR	0.55V
Transistor M19	NR	−0.55V
Transistor M6	−400V	0.8V
Transistor M14	NR	−0.8V

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant".

1/2 [M9, M11 cutoff – push pull – not relevant.

1 [M9 Saturation $V_{GS}=0.35V$, $V_{th}=0.30V \Rightarrow V_{DS,sat}=0.05V$

$V_{out}=V_{DD}-V_{DSsat}=1V-0.05V=0.95V$

1/2 [M11 – same calculation, – sign.

1/2 [M8 Sat. not relevant as this limits – gate drive to M9

1/2 [M10 " " " " " " + " " " M11

1 [M8 cutoff $R_{Leq}=400k\Omega$, $\Delta I_{max}=25\mu A$

$\Delta V=\Delta I \cdot R_{Leq} \cdot A_{V9}= 400k\Omega \cdot 25\mu A = 10V$

1/2 [M10 cutoff – same calculations – sign.

1/2 [M4, M19 Cutoff – Not relevant – no signal

1 [M4 Sat: $V_{GS}=0.4V$, $V_{th}=0.1V$, $V_{DSsat}=0.3V$

$\Delta V_{out}=(1V-0.1V-0.35V)A_{V9}\cong 1V-0.1V-0.35V$

$=0.55V$.

1/2 [M19 Sat: same as above, – sign.

b

M6 Sat $V_{gs} = 0.8V$, $V_{th} = 0.3V \rightarrow V_{DSsat} = 0.1V$.

maximum drain voltage $= 0.9V - 0.1V = 0.8V$

$\Delta V_{out} = 0.8V \cdot A_{v8} A_{v9} \approx 0.8V$

1

M6 cutoff Thevenin impedance @ M6 drain:

$= R_{out6} \| R_{out14} = g_m r_{DS}^2 / 2$.

maximum $\Delta I = 35\mu A$

$\Delta V = -35\mu A \left(g_m r_{DS}^2 / 2 \right) A_{v8} A_{v9}$

$= -4000V$ (!)

1

1/2

M14 cutoff: Not relevant, no signal.

M14 Sat

1

$V_{DS,sat} = 0.1V$.

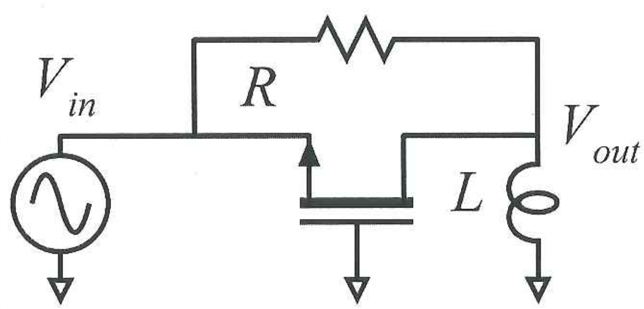
$\Delta V_{out} = (-0.9V + 0.1V) A_{v8} A_{v9} = -0.8V$

overall, can drive $\pm 0.55V$

as limited by M6, M14 saturation.

b

Problem 3, 30 points



You will be working on the circuit to the left

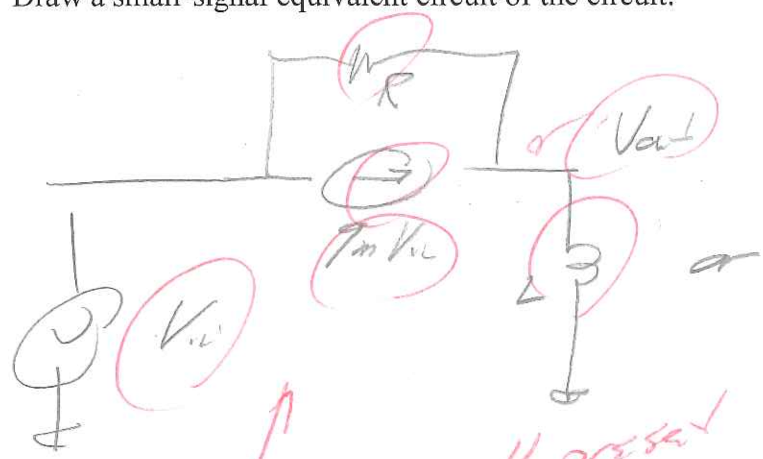
Ignore DC bias analysis. You don't need it.

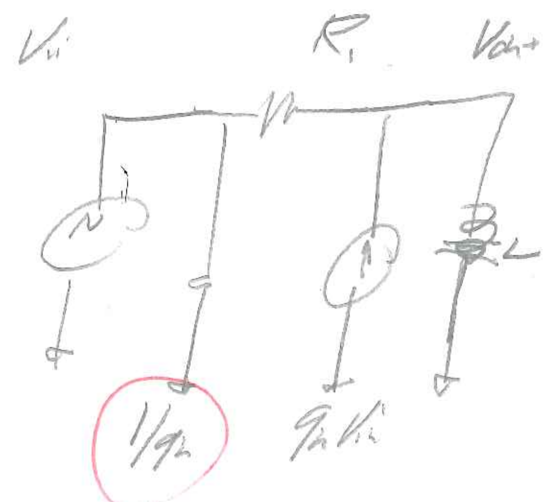
The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.





make sure all present
about 1pt each

b

Part b, 8 points

gm=10 mS. L=1nH. R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensionally wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ____________________

2 $\Sigma I = 0$ @ V_{out} $(G = 1/R)$

3 $(V_{out} - V_{in})G + V_{out}/sL - g_m V_{in} = 0.$

$$V_{out}(G + 1/sL) = V_{in}(g_m + G)$$

$$\frac{V_{out}}{V_{in}} = \frac{g_m + G}{G + 1/sL} = \frac{g_m + G}{G}\,\frac{1}{1 + R/sL}$$

$$= (1 + g_m R)\,\frac{s(L/R)}{1 + s(L/R)}$$

$L/R = 1ps$

3

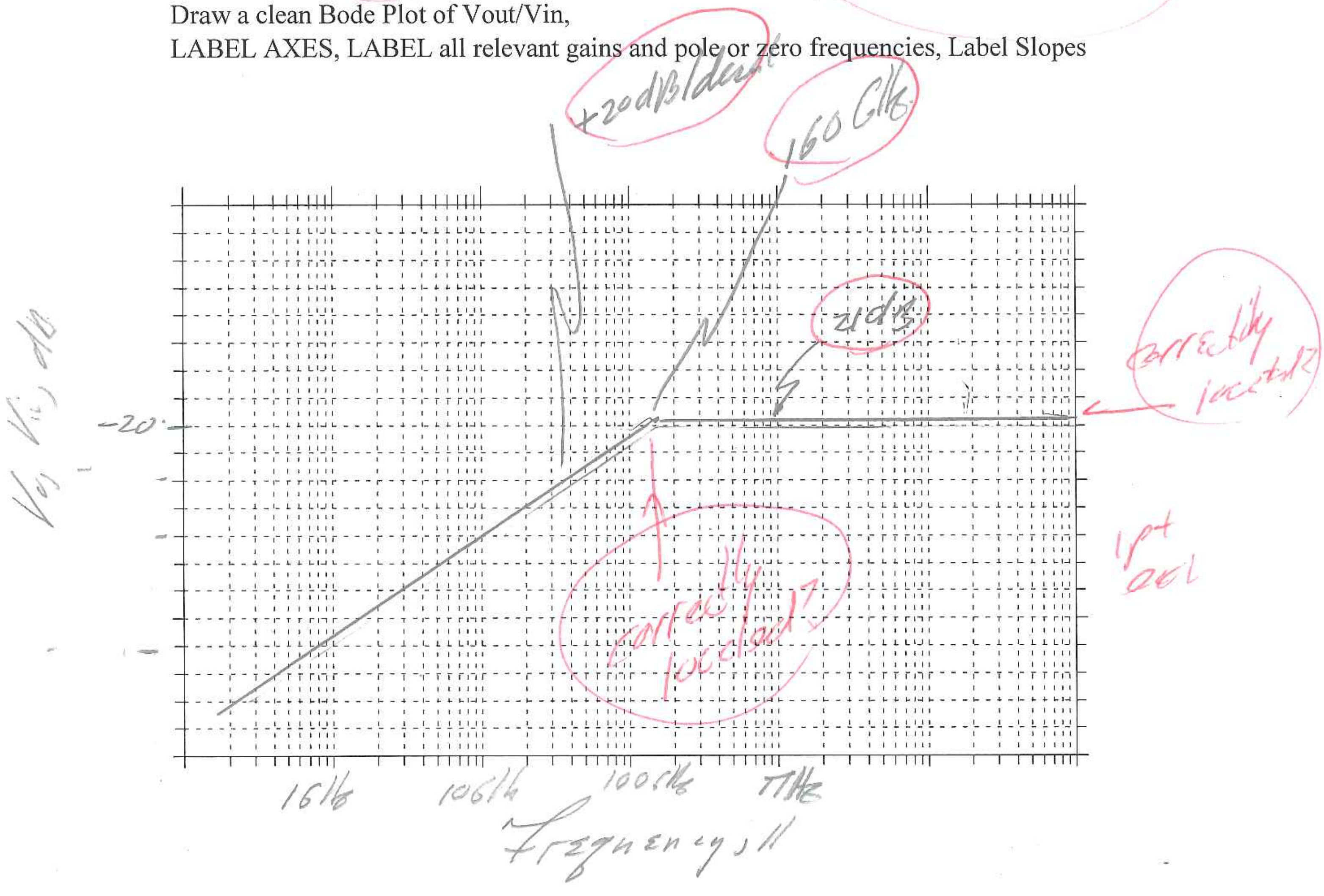
$$\boxed{\frac{V_{out}}{V_{in}} = 11 \cdot \frac{s(1ps)}{1 + s(1ps)}}$$

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

Fzero = 0 Hz , Fpole = 1/2π · 1ps = 159 GHz

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



F

b

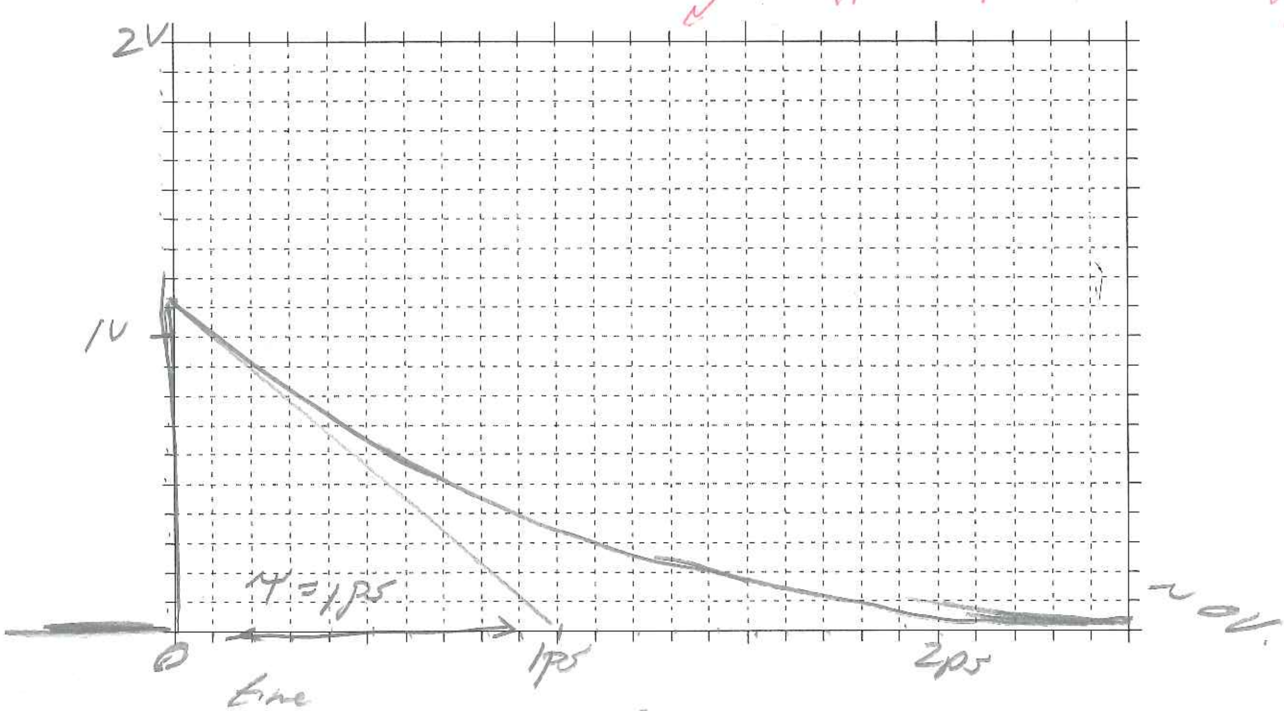
Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)=____________________________

Plot it below. Label axes, show initial and final values, show time constants.

✓ 4pts for correct & accurate graph



$$V_{out}(s) = 11 \cdot \frac{s(1ps)}{1+s(1ps)} \cdot \frac{0.1V}{s} = 1.1V \cdot \frac{1ps}{1+s(1ps)}$$

$$v_{out}(t) = 1.1V \cdot u(t) \cdot e^{-t/1ps}$$

4

b