(V4)

Final Exam, ECE 137A

Thursday March 17, 12 - 3 p.m.

Name: solution.

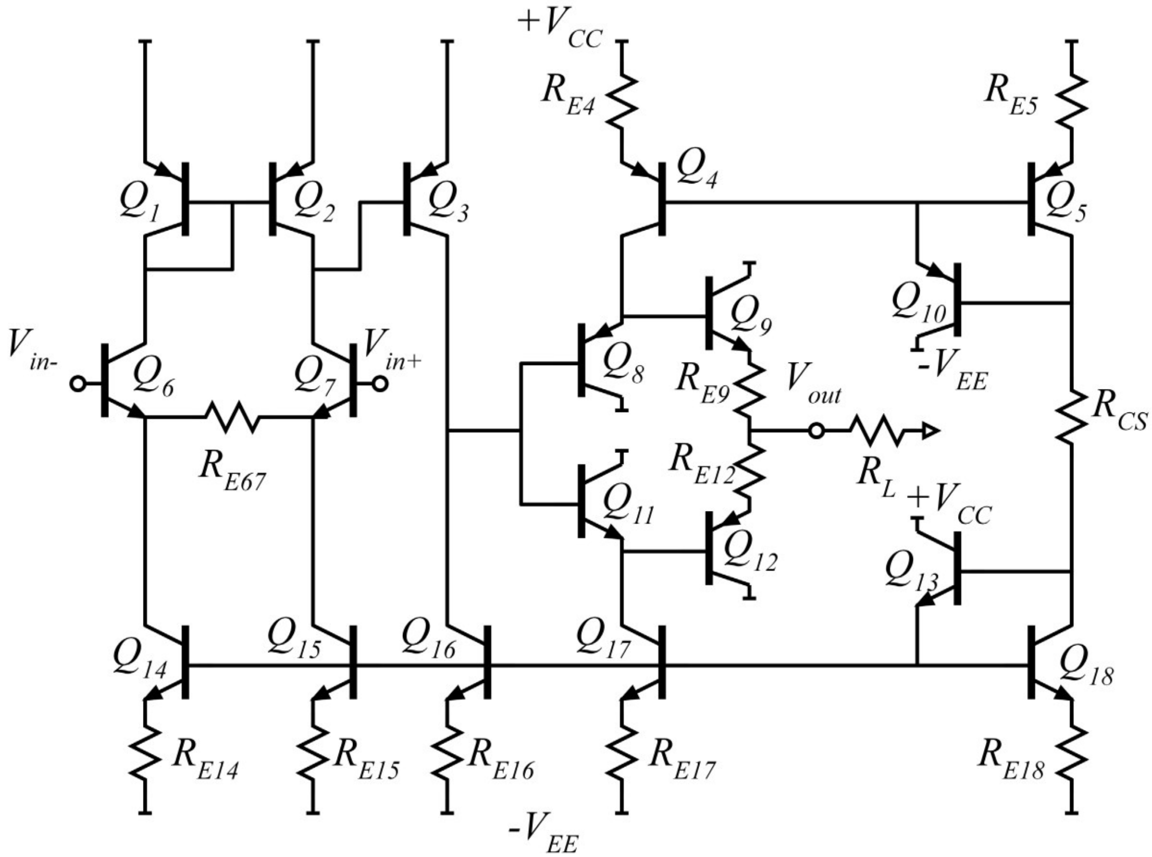
Closed Book Exam:
Class Crib-Sheet and 4 pages (4 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		6	2c		10
1b		5	2d		10
1c		4	3a		10
1d		10	3b		10
1e		10	3c		10
2a		10			
2b		5			
total		100			

b

Problem 1, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



All the transistors have the same (matched) I_S, have $\beta = 100$, and $V_A =$ *infinity* Volts . $V_{CE(sat)} = 0.5\text{V}$. V_{be} is approximately 0.7 V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary or appropriate. The supplies are +3 Volts and -3 Volts.

All transistors have the same I_S .

The resistors RE5 and RE18 have a 300mV DC voltage drop acoss them.

Re67=100 Ohms, RL=1000 Ohms.

DC bias currents: Ic6=Ic7=Ic9=Ic12=Ic18=0.1 mA. Ic3=Ic8=Ic11=0.2mA

b

Part a, 6 points

DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

(Hint, this should give Vi- = 0V)

Find the value of the following resistors:

Re4=1.4kΩ, Re5=3kΩ, Re9=180Ω, Re12=180Ω, Re14=3kΩ,
Re15=3kΩ, Re16=1.4kΩ, Re17=1.4kΩ, Re18=3kΩ, Rcs=26kΩ.

1/2 [$R_{E14} = R_{E15} = R_{E18} = \frac{0.3V}{100\mu A} = 3k\Omega$

1 [$V_{BE16} = V_{BE17} = V_{BE18} + V_T \ln \frac{200\mu A}{100\mu A}$
$= V_{BE18} + 18mV$
so the voltage drops across R_{E16} & R_{E17}
are $300mV - 18mV = 282mV$

1 [$R_{E16} = R_{E17} = \frac{282mV}{200\mu A} = 1.41k\Omega$

1 [Same calculation for $R_{E4} \rightarrow 1.41k\Omega$

1 [similar calculation for $R_{E5} = \frac{300mV}{100\mu A} = 3k\Omega$.

1 [$R_{E9} = R_{E12} = \frac{V_T}{100\mu A} \ln\left(\frac{200\mu A}{100\mu A}\right) = \frac{18mV}{0.1mA} = 180\Omega$

1/2 [$R_{cs} = \frac{2(1.3V)}{0.1mA} = \frac{2.6V}{0.1mA} = 26k\Omega$

b

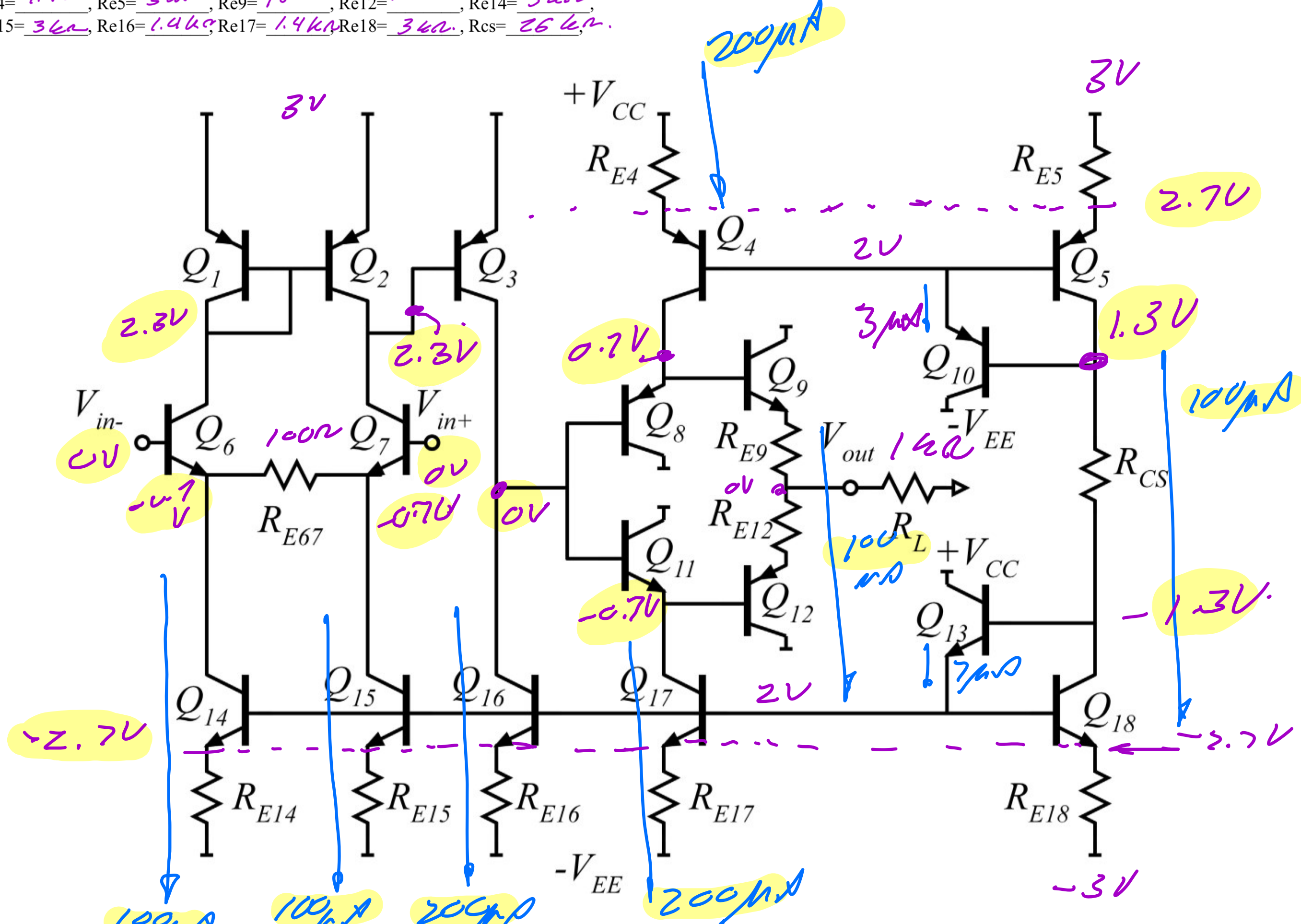
1/4 pt each for yellow

$\beta = 100$

$V_A = \infty V$

Part b, 5 points

Find the value of the following resistors:
Re4=__1.4kΩ__, Re5=__3kΩ__, Re9=__180Ω__, Re12=__180Ω__, Re14=__3kΩ__,
Re15=__3kΩ__, Re16=__1.4kΩ__, Re17=__1.4kΩ__, Re18=__3kΩ__, Rcs=__26kΩ__.



On the circuit diagram above, label the DC voltages at **ALL nodes**, and the DC collector currents of **all transistors**. Label the values of all resistors.

$$I_{B13} = \frac{I_{C14} + I_{C15} + I_{C16} + I_{C17} + I_{C18}}{\beta}$$

$$= \frac{700\mu A}{100} = 7\mu A$$

$$I_{B10} = \frac{I_{C4} + I_{C5}}{\beta} = \frac{300\mu A}{100} = 3\mu A$$

b

Part c, 4 points

find the following

device	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
gm, mS	3.85	3.85	7.7	7.7	3.85	3.85	3.85	7.7

device	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
gm, mS	3.85	don't bother*	7.7	3.85	don't bother*	3.85	3.85	7.7

device	Q17	Q18
gm, mS	7.7	3.85

*don't bother calculating these

2 [$I_C = 100\mu A$ for Q1, 2, 6, 7, 14, 15, 9, 12, 5, 18

$g_m = \frac{1}{260\Omega} = 3.85 mS$,

2 [$I_C = 200\mu A$ for Q3, 16, 4, 8, 11, 17

$g_m = \frac{1}{130\Omega} = 7.7 mS$,

Note that all R_{ce} are $\infty\,\Omega$

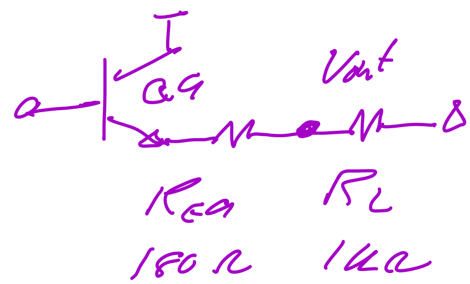
b

Part d, 10 points.

Find the following, ***using the actual value of β, i.e. β=100***

	Voltage Gain	Input impedance
Q9 or Q12	0.694	144 kΩ
Q8 or Q11	0.999 ~ 1	14.4MΩ
Q3	-111,000	13kΩ.
Q1,2,6,7 combination.	-41.9	62kΩ
Overall differential Vout/Vin	$3.2 \cdot 10^6$	62kΩ

Assume that either Q9 or Q12 is on;
here I assume Q9



Q9: EF

1/2 [$R_{LEQ9} = (R_{E9} + R_L) = 1.18k\Omega$
$1/g_{m9} = 260\Omega$

1 [$A_{v9} = \frac{1k\Omega}{1k\Omega + 180\Omega}$ $\frac{1k\Omega + 180\Omega}{1k\Omega + 180\Omega + 260\Omega} = 0.694$

1 [$R_{in9} = \beta(1k\Omega + 180\Omega + 260\Omega) = 144k\Omega.$ (β = 100)

Q8: EF

1/2 [$R_{LEQ8} = R_{in9} = 144k\Omega$

1 [$A_{v8} = 144k\Omega / (144k\Omega + 1/g_{m8}) = 0.999 \doteq 1$ ($1/g_{m8}$ = 130Ω)

1 [$R_{in8} = \beta(144k\Omega + 1/g_{m8}) \simeq \beta \cdot (144k\Omega) = 14.4M\Omega$ ($1/g_{m8}$ = 130Ω, β = 100)

b

Q3: CE

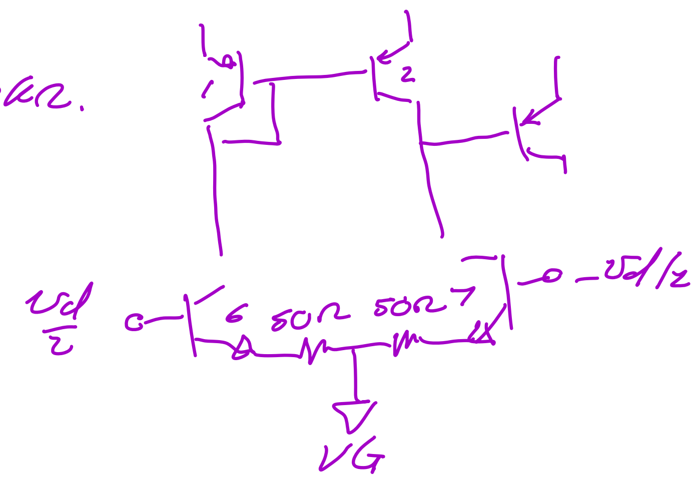
1/2 [$R_{Leq3} = R_{in8} = 14.4M\Omega$

1 [$A_{v3} = -g_{m3} R_{Leq3} = -\frac{R_{Leq3}}{r_{E3}} = \frac{14.4M\Omega}{130\Omega} = -111,000$

1 [$R_{in3} = \beta / g_{m3} = 100 \cdot 130\Omega = 13k\Omega.$

Q1, 2, 6, 7

1/2 [$R_{Leq7} = R_{in3} = 13k\Omega.$



1 [$A_v = \frac{R_{Leq7}}{50\Omega + 1/g_{m7}}$

$= \frac{13k\Omega}{50\Omega + 260\Omega} = -41.9$

1 [Differential input impedance between V^+ & V^-

$R_{in\,diff} = \beta(R_{EE67} + 1/g_{m6} + 1/g_{m7})$

$= 100(100\Omega + 260\Omega + 260\Omega)$

$= 100 \cdot 620\Omega = 62k\Omega$

b

Part e, 10 points

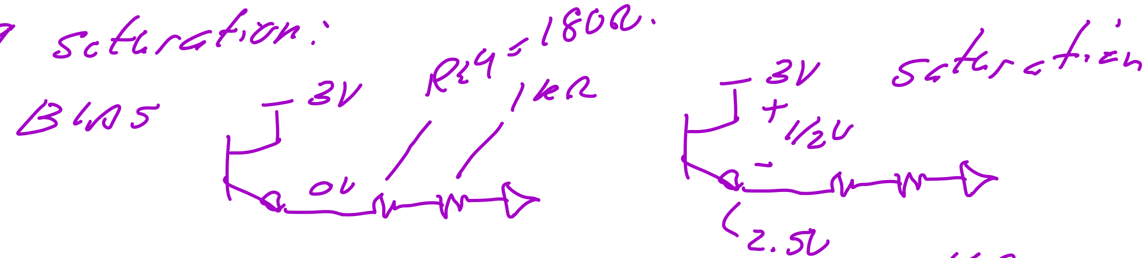
Maximum peak-peak output voltage (***show all your work***)
For this, you must use the full circuit diagram, not the half circuit diagram.

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q9	not relevant	+2.12 V ↑
Transistor Q12	not relevant	−2.12 V ↓
Transistor Q8	+20 V ↑	−2.2 V ↓
Transistor Q11	−20 V ↓	+ 2.2 V ↑
Transistor Q4	not relevant	+1.04 V ↑
Transistor Q17	not relevant	−1.04 V ↓

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q9/12 form a push pull stage, so be careful about your answer there. .

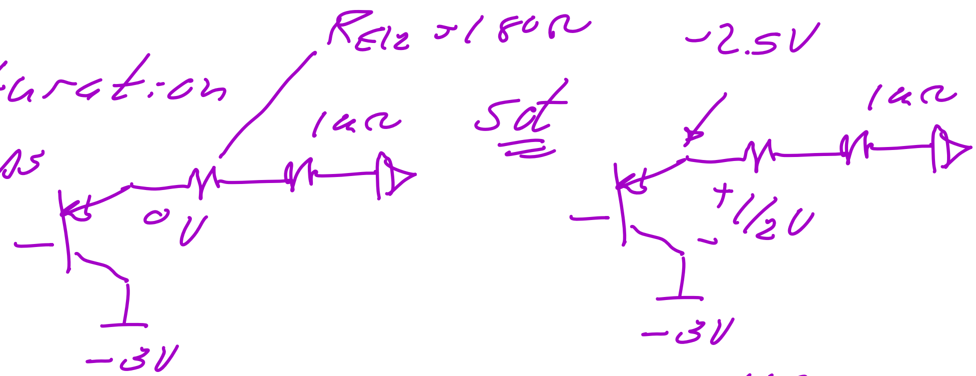
1 Q12, Q9 cutoff – not relevant – push-pull

1 Q9 saturation:



$\Delta V_{emitter} = +2.5V$ $\Delta V_{out} = 2.5V \cdot \frac{1k\Omega}{1.18k\Omega} = +2.12V$

1 Q12 saturation

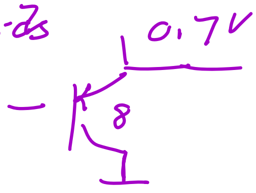


$\Delta V_{emitter} = -2.5V$ $\Delta V_{out} = -2.5V \cdot \frac{1k\Omega}{1.18k\Omega} = -2.12V$

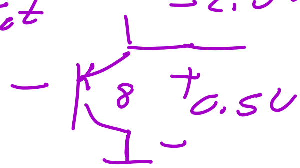
b

Q 8 saturation.

bias



sat



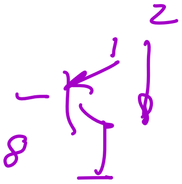
$\Delta V_{emitter} = -3.2V$ $\Delta V_{out} = -3.2V \cdot A_{vg} = -2.2V.$ (0.694)

Q 11 saturation.

This is +/− symmetric with Q 8 saturation

→ $\Delta V_{out} = +2.2V$

Q 8 cutoff   bias



cutoff

$V_{emitter} = 200\mu A \cdot R_{Leq\,8}$

$= 200\mu A \cdot 144k\Omega$

$= 28.8V$

$\Delta V_{out} = 28.8V \cdot A_{vg} = 20V$ (0.694)

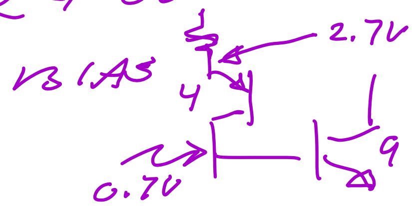
Q 11 cutoff

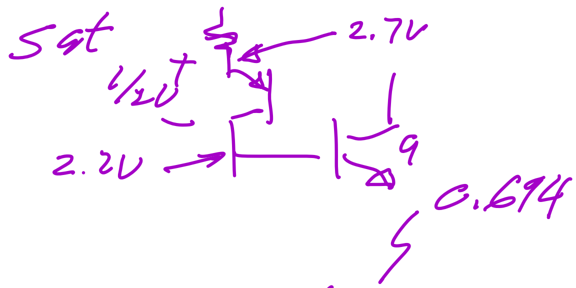
This is +/− symmetric with Q 8 cutoff

→ $\Delta V_{out} = -20V$

Q4 and Q17 cutoff – not relevant; I_c not modulated.

Q4 saturation





$\Delta V_{collector} = 1.5V$ ↑ $\Delta V_{out} = 1.5V \cdot A_{v9} = +1.04V$

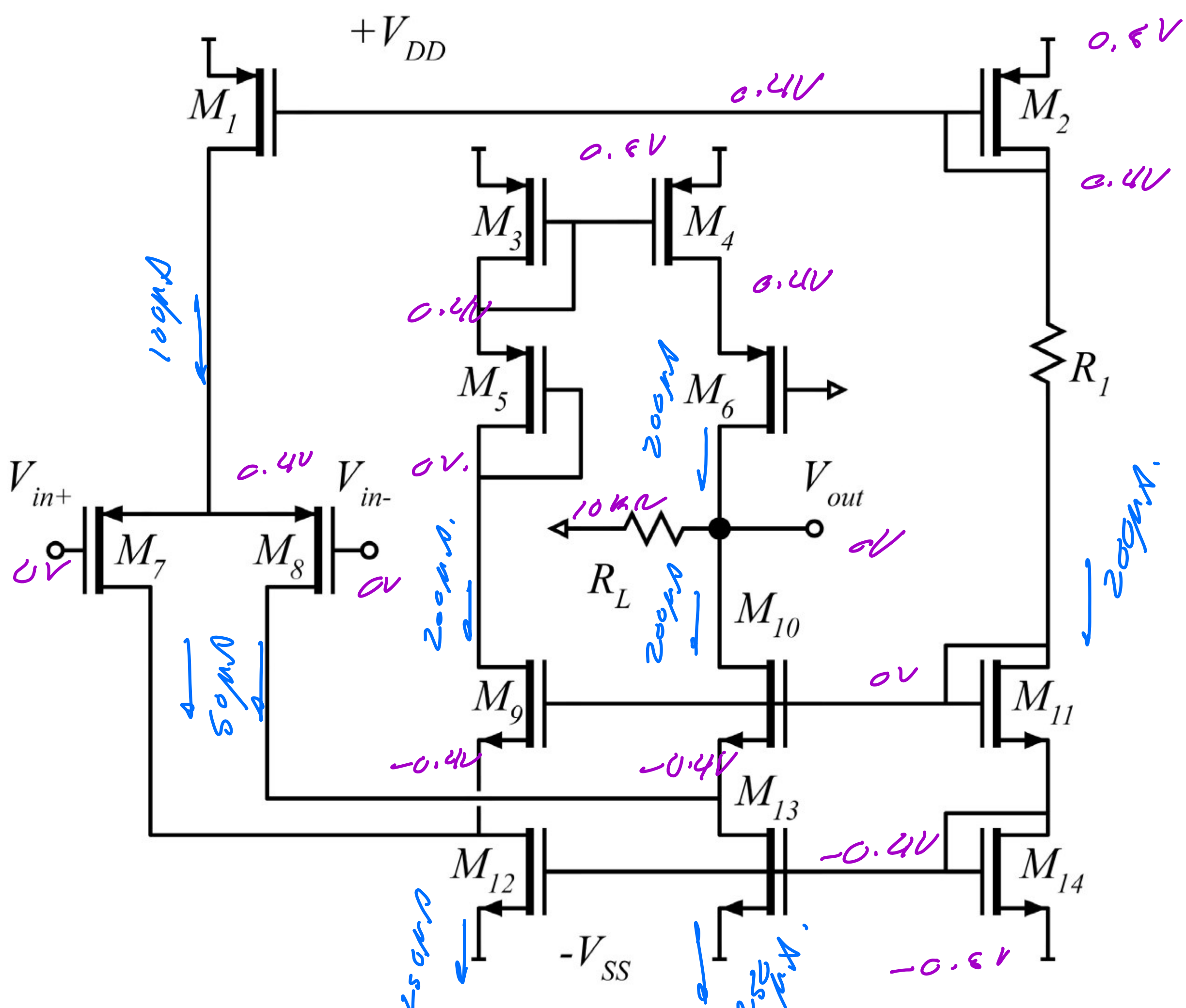
Q17 saturation.

This is +/– symmetric with Q4 saturation

→ $\Delta V_{out} = -1.04V$

Problem 2, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



The NMOSFETs have $K_\mu = \mu c_{gs} W_g / 2L_g = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 2.0\text{mA/V} \cdot (W_g / 1\mu\text{m})$, $\Delta V = v_{inj} L_g / \mu = 0.10\text{V}$, $V_{th} = 0.3\text{V}$,

$1/\lambda = 5\text{V}$

The PMOS have identical parameters, except, of course, V_{th} is negative.

V_{DD} = +0.8 V, $-V_{SS}$ = -0.8 V, R_L=10kOhm

All transistors have $|V_{gs}|$=0.4V

M7,8 are biased at I_D=50 µA.

M5,6,9,10,11 are biased at I_D=200 µA

b

Part a, 10 points
DC bias.

Find the Gate widths, in µm, of
M1 0.43 µm , M7 0.43 µm

Note that, by using the mobility-limited formula $g_m = 2I_D/(V_{gs} - V_{th})$, we can solve the exam without calculating any of the FET widths. ***So, there's no reason to spend time calculating other FET widths.***

FETS M7 & M8: [$V_{DS} = 0.8V$; $V_{gs} = 0.4V$] 2

5 [$I_D = \frac{10\,mA}{V^2} (V_{gs} - V_{th})^2 \frac{W_g}{1\,\mu m} (1 + \frac{V_{DS}}{1/\lambda})$

I_D = 50 µA, $(V_{gs} - V_{th})$ = 0.1V, $1/\lambda$ = 5

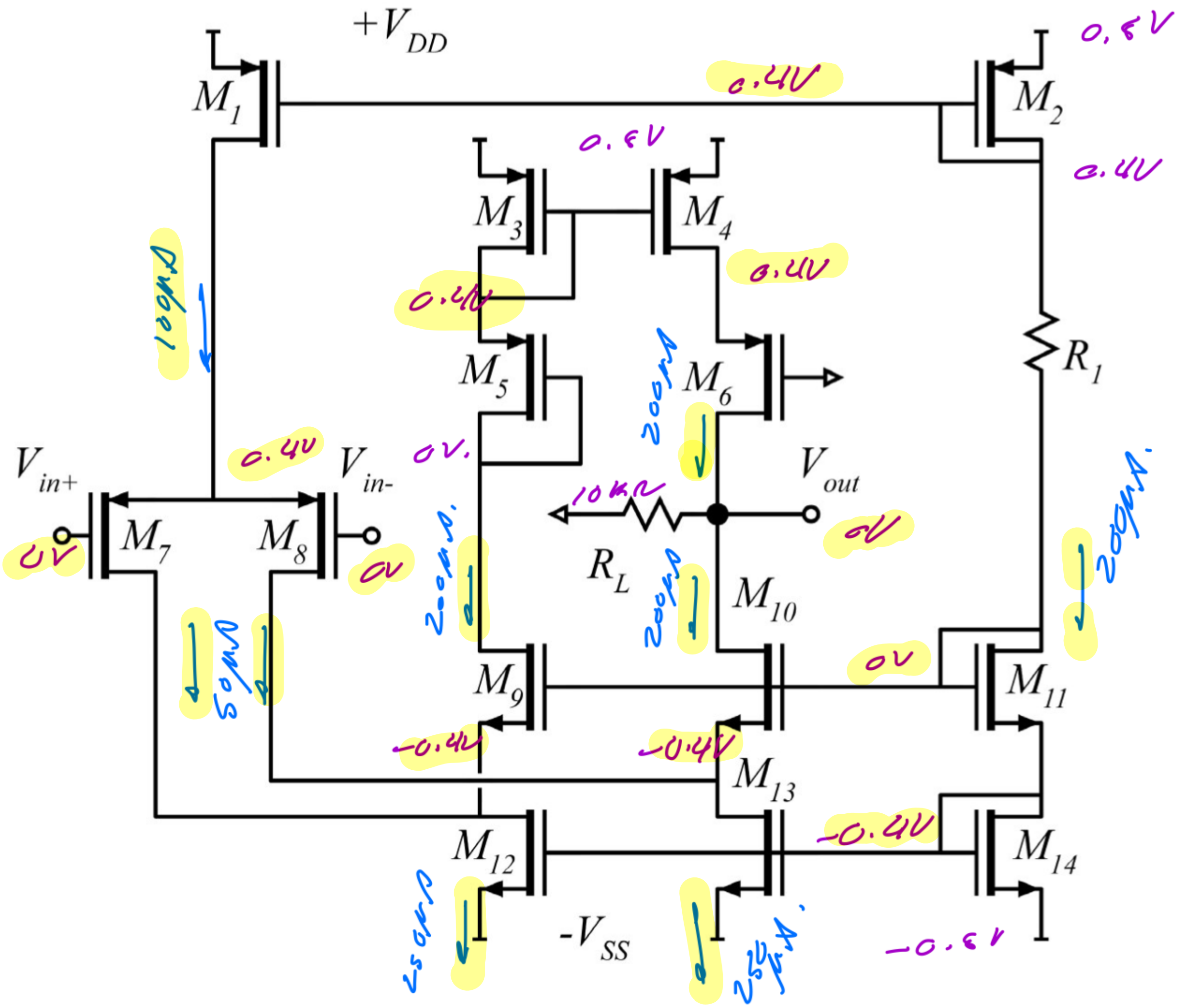
3 [→ $W_g = 0.431\,\mu m$

b

1/4 pt each

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes,** the drain currents of **ALL transistors**

b

Part c, 10 points.
This amplifier has *two* signal paths between input and output.
One is the path (M7 and M8, M9, M3, M4, M6, output).
The other is the path (M7 and M8, M10, output).

You will now compute the differential gain for the path (M7 and M8, M10, output).
Find the following

	Voltage Gain	Input impedance
Transistor M10	28.6	350Ω
M7-M8 differential pair	0.1715	∞ Ω
Overall differential Vout/Vin for this path	4.90	∞ Ω

1 $R_{out4} = R_{DS} = 1/\lambda I_D = 5V/200\mu A = 25k\Omega$

1 $g_{m6} = \frac{2 I_d}{(V_{gs}-V_{th})} = \frac{2(0.2mA)}{0.1V} = \frac{0.4mS}{0.1V} = 4mS = \frac{1}{250\Omega}$

1 $R_{out6} = R_{DS6}(1 + g_{m6} R_{out4})$
$= 25k\Omega(1 + 25k\Omega \cdot 4mS) = 25k\Omega(101)$
$= 2.52\ M\Omega$

1 $R_{Leq10} = R_L // R_{out6} = 10k\Omega // 2.5M\Omega = 9.96k\Omega \cong 10k\Omega$

1 $R_{ds10} = R_{DS6} = 25k\Omega$; $g_{m10} = g_{m6} = 4mS$

1 $R_{in10} = 1/g_{m10}(1 + R_{Leq10}/R_{DS10})$
$= 250\Omega(1 + 10k\Omega/25k\Omega) = 350\Omega$

1 $A_{v10} = R_{Leq10}/R_{in10} = \frac{10k\Omega}{350\Omega} = 28.6$

1 $R_{Leq8} = R_{DS8} // R_{DS13} // R_{in10}$
$= \frac{5V}{50\mu A} // \frac{5V}{250\mu A} // 350\Omega =$

b

$= 100k\Omega \,//\, 20k\Omega \,//\, 350\Omega = 343\Omega$

1 $g_{m8} = \dfrac{2(50\mu A)}{0.1V} = \dfrac{0.1mA}{0.1V} = 1mS$

1 $A_{v8,9} = \frac{1}{2} \cdot g_{m8} \cdot R_{Leq8} = \underline{0.1715}$

(↑ diff, ↑ 1mS, ↑ 343Ω)

$$\begin{array}{r} 28.6 \\ \times 0.1715 \\ \hline = 4.90 \end{array} \leftarrow \text{total gain}$$

Comment... In the limit of $r_{os} \gg R_L$, the overall gain is $\dfrac{g_{m7,8} \cdot R_L}{2} = 5$ for each path, giving a total differential gain of $\underline{10.0}$

b

Part d, 10 points

This amplifier has *two* signal paths between input and output.
One is the path (M7 and M8, M9, M3, M4, M6, output).
The other is the path (M7 and M8, M10, output).

You will now compute the differential gain for the path (M7 and M8, M9, M3, M4, M6, output). Find the following

	Voltage Gain	Input impedance
Transistor M6	28.6	350Ω
Transistor M4	1.38	∞ Ω
Transistor M9	1.0	250Ω
M7-M8 differential pair	0.125	∞ Ω
Overall differential Vout/Vin for this path	4.93	∞ Ω

(the overall amplifier gain is the sum of the answers for parts c and d, but you are not asked to calculate this.

1 [$R_{Leq6} = R_L \,//\, R_{out10}$ but $R_{out10} >> R_L$

$\simeq 10k\Omega$]

1/2 [$g_{m6} = \frac{2I_D}{V_{gs}-V_{th}} = \frac{2(0.2mA)}{0.1V} = 4mS$]

1/2 [$R_{DS6} = \frac{1}{\lambda I_D} = \frac{5V}{0.2mA} = 25k\Omega$]

1/2 [$R_{in6} = 1/g_{m6}\left(1+\frac{R_{Leq6}}{R_{DS6}}\right) = 250\Omega\left(1+\frac{10k\Omega}{25k\Omega}\right)$

$= 350\Omega$]

1/2 [$A_{v6} = R_{Leq6}/R_{in6} = \frac{10k\Omega}{350\Omega} = 28.6$]

1/2 [$R_{Leq4} = R_{DS4} \,//\, R_{in6} = 25k\Omega // 350\Omega = 345\Omega$]

1/2 [$A_{v4} = -g_{m4} \cdot R_{Leq4} = -4mS \cdot 345\Omega = -1.38$]

b

1/2 [For M9, the FETs M3 & M5 act as resistors = 1/gm

1/2 [There's a 2:1 voltage divider between M3 & M4) = 250Ω

1/2 [$R_{Leq4} = 1/g_{m3} + 1/g_{m5} = 500\Omega$

1/2 [$R_{DS9} = 25k\Omega$, $g_{m9} = 4mS$

1 [$R_{in9} = 1/g_{m9}\left(1 + \frac{R_{Leq9}}{R_{DS9}}\right) = 250\Omega\left(1 + \frac{500\Omega}{25k\Omega}\right) \cong 250\Omega$

1/2 [$A_{v9} = \frac{1}{2}\,\frac{R_{Leq9}}{R_{in9}} = \frac{1}{2}\,\frac{500\Omega}{250\Omega} = 1.0$

(1/2: due to M3/M5 resistive divider)

1 [$R_{Leq7,8} = R_{DS7} \,//\, R_{DS12} \,//\, R_{in9} = \frac{5V}{50\mu A} \,//\, \frac{5V}{250\mu A} \,//\, 250\Omega$

$= 100k\Omega \,//\, 20k\Omega \,//\, 250\Omega \cong 246\Omega \cong 250\Omega$

1/2 [$g_{m7} = g_{m8} = \frac{2I_D}{V_{GS}-V_{th}} = \frac{2(50\mu A)}{0.1V} = \frac{0.1mA}{0.1V} = 1mS$

1/2 [$A_{v7,8} = \frac{1}{2}\,g_{m7}\cdot R_{Leq7} = \frac{1}{2}\cdot 1mS\cdot 250\Omega = \frac{1}{8} = 0.125$

(1/2: diff)

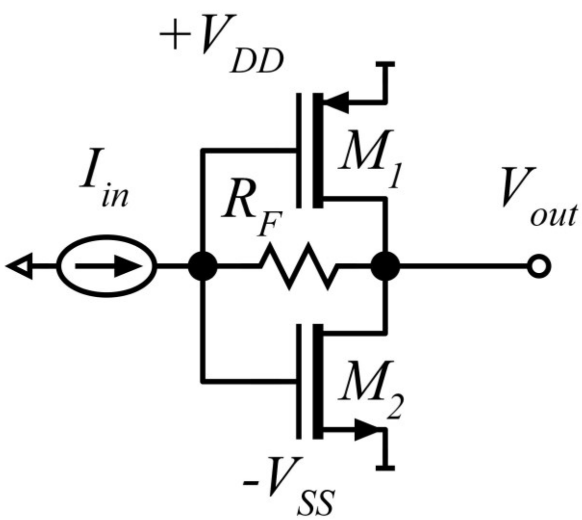
[28.6
× 1.38
× 1.0
× 0.125
= 4.93

b

Problem 3, 30 points

Nodal analysis: optical receiver preamplifier as real-world example.

Part a, 10 points



You will be working on the circuit to the left

Ignore DC bias analysis. You don't need it.

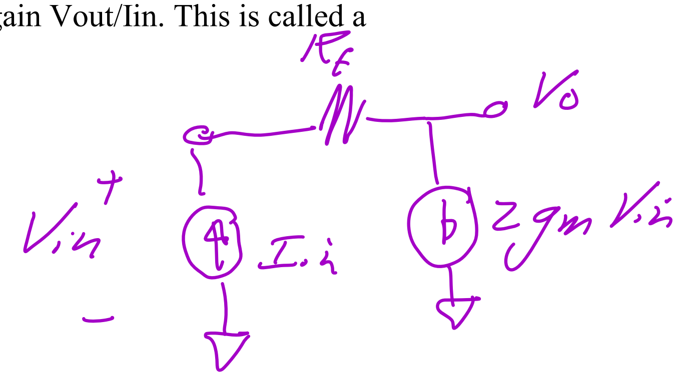
The NFET and the PFET each have transconductance gm.

The NFET and the PFET each have output resistance Rds of infinity...so you don't need to include this element in the circuit diagram !

gm=0.5 mS
Rf=11 kOhm

Compute, from nodal analysis, the small-signal gain Vout/Iin. This is called a transimpedance gain.

Vout/Iin = $-10\ k\Omega$



3 $2g_m V_{in} = I_{in}$

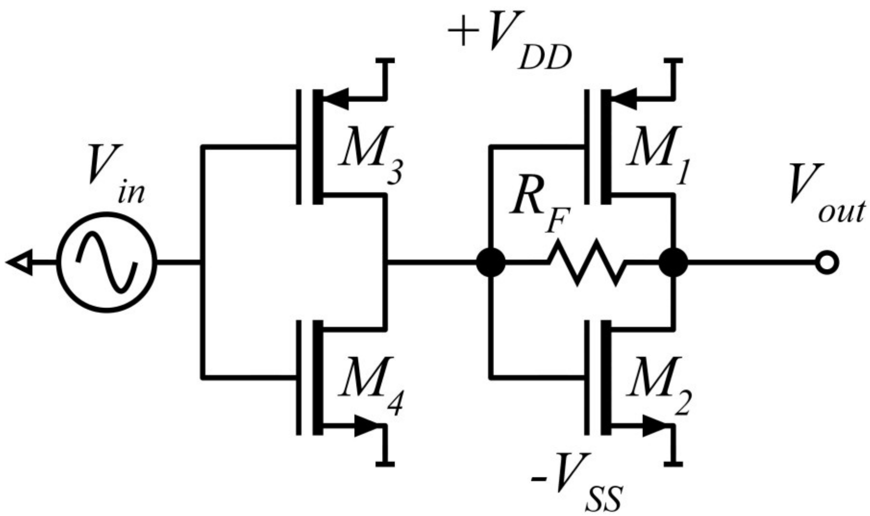
$\rightarrow V_{in} = I_{in}/2g_m$

3 Also $I_{in} R_f = V_{in} - V_{out} = I_{in}/2g_m - V_{out}$

$\rightarrow I_{in}(R_f - 1/2g_m) = -V_{out}$

4 $\rightarrow \frac{V_{out}}{I_{in}} = 1/2g_m - R_f = 1000\Omega - 11\,k\Omega = -10\,k\Omega$

Part b, 10 points

	The NFETs and the PFETs all have transconductance gm. The NFETs and the PFETs have have output resistance Rds of infinity gm=0.5 mS Rf=11 kOhm

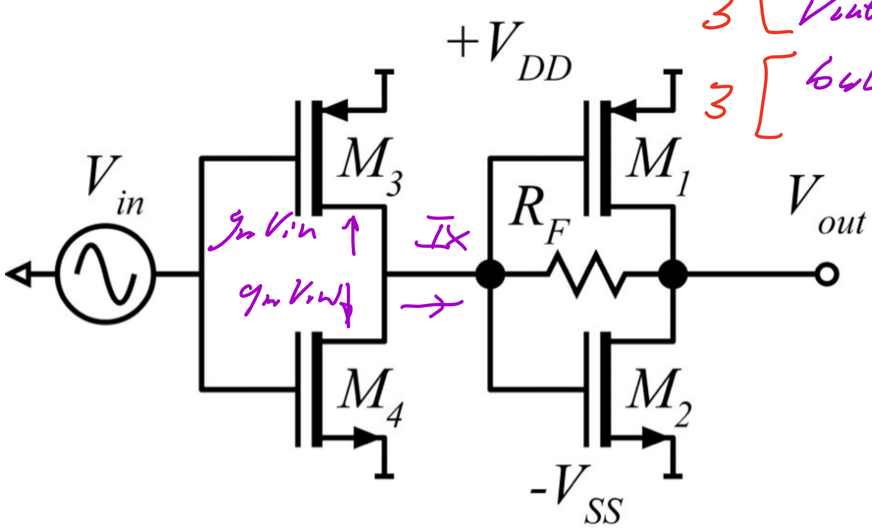
Compute, from nodal analysis, the small-signal gain Vout/Vin. This is a voltage gain.
Hint: you can save some work by using the result from part A.

Vout/Vin = + 10

from part A,

3 [$V_{out} = -10k\Omega \cdot I_x$

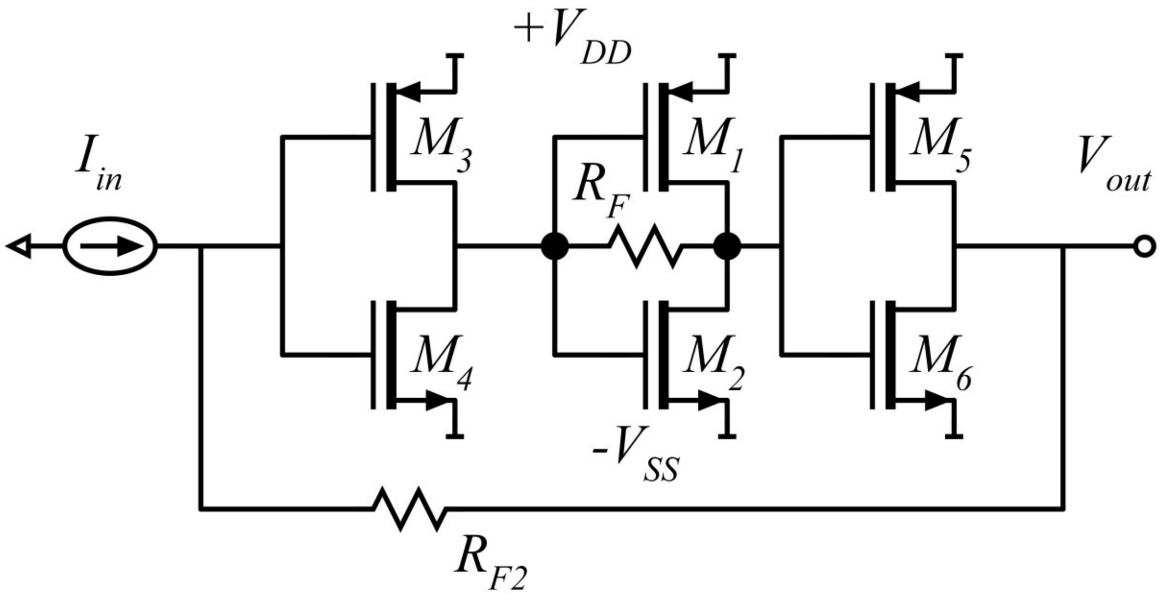
3 [but $I_x = 2 g_m V_{in}$.



$\Rightarrow$ 4 [$\frac{V_{out}}{V_{in}} = 2 \cdot g_m \cdot 10k\Omega = 1mS \cdot 10k\Omega = 10.$

b

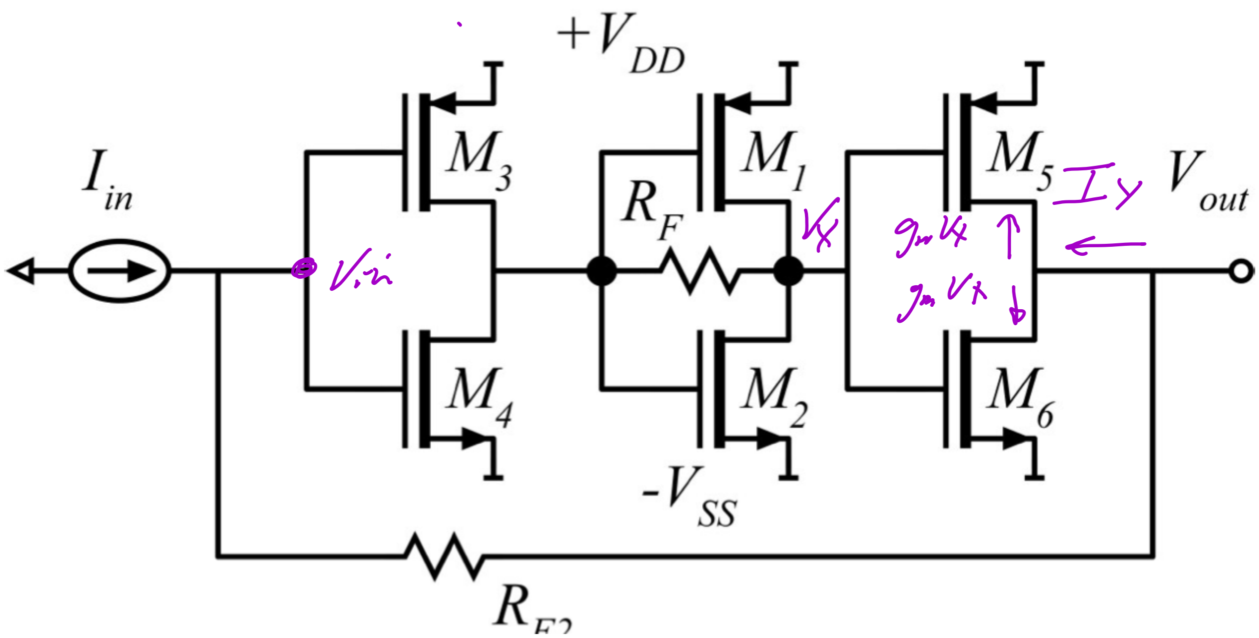
Part c, 10 points



The NFETs and the PFETs all have transconductance gm. The NFETs and the PFETs have have output resistance Rds of infinity.
gm=0.5 mS, Rf=11 kOhm, Rf2=1 kOhm.

Compute, from nodal analysis, the small-signal gain Vout/Iin. This is a transimpedance gain. Hint: you can save a great deal of work by using the results from parts A and B.

Vout/Iin = −960 Ω



b

2 from part B, $V_x = 10V_{in}$.

so $I_Y = 2g_m \cdot V_x = 2 \cdot 0.5mS \cdot 10V_{in}$

2 $= 10mS \cdot V_{in}$.

2 but $I_Y = I_{in} \rightarrow V_{in} = \frac{I_{in}}{10mS}$.

2 and $V_{in} - V_{out} = I_{in} R_{f2}$

$$\frac{I_{in}}{10mS} - V_{out} = I_{in} R_{f2}$$

2 $$\frac{V_{out}}{I_{in}} = \frac{1}{10mS} - R_{f2}$$

$$= 100\Omega - 1k\Omega$$

$$= \underline{-900\,\Omega}$$

b