ECE 137 A Mid-Term Exam

Wednesday February 9, 2022

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 2 problems on this exam, and you have 75 minutes.

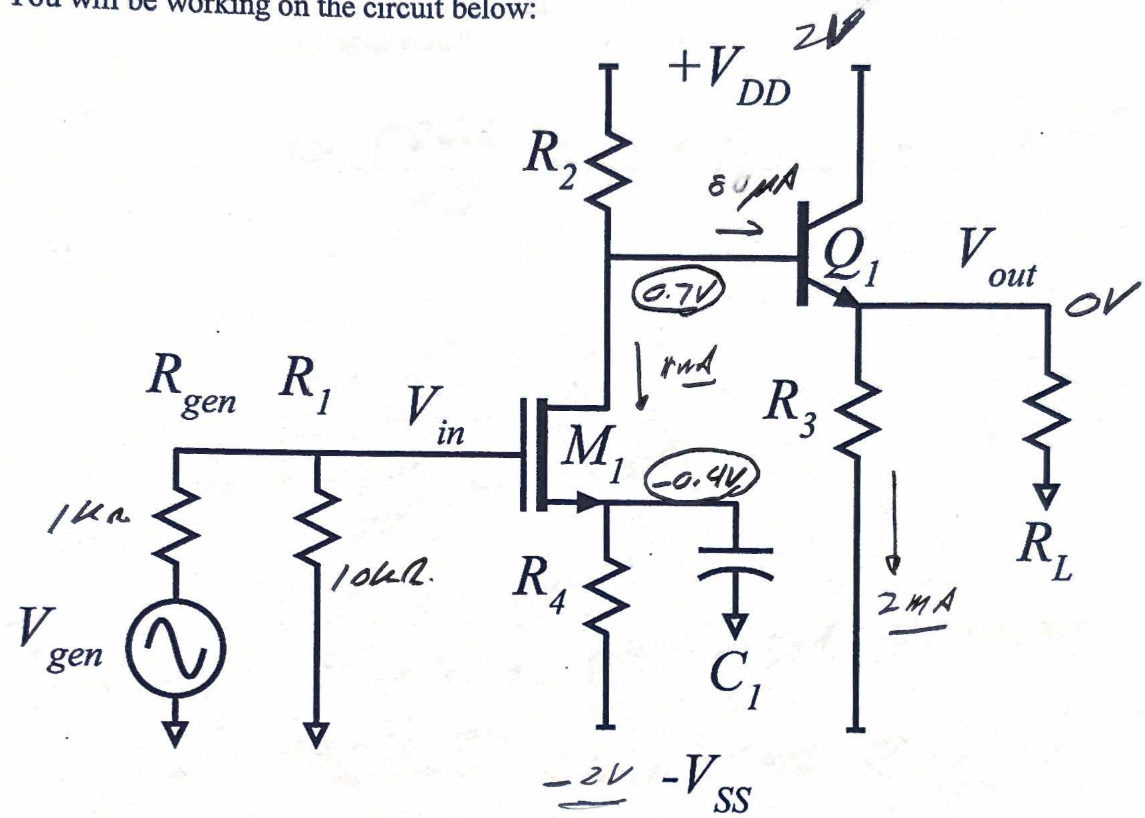
Use any and all reasonable approximations (5% accuracy is fine.), ***AFTER STATING and approximately Justifying them.***

Name: solution.

Part	Points Received	Points Possible
1a		7
1b		7
1c		6
1d		15
1e		15
1f		6
1g		14
2a		12
2b		13
2c		5
TOTAL		100

Problem 1, 70 points

You will be working on the circuit below:



M1: $K_\mu = \mu c_{gs} W_g / 2L_g = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$

$\Delta V = v_{inj} L_g / \mu = 0.1\text{V}$, $V_{th} = 0.3\text{V}$, $1/\lambda = 4\text{V}$

Q2: $\beta = 250$, $V_A = 100$ V

The supplies are +2V and -2V
Rgen=1000 Ohms, RL=1,000 Ohms. R1=10kOhms,
C1 is very large (AC short-circuit)

Part a, 7 points

DC bias.

M1 is to be biased at 1 mA drain current and Vgs=0.4 Volts.

Q1 is to be biased at 2 mA collector current.

The DC value of Vout is *zero volts*.

Find the following:

Wg1= 7.84 μm   R2= 1.3kΩ   R3= 1kΩ   R4= 1.6kΩ

1 $I_{BQ1} = \frac{2\,mA}{250} = 8\,\mu A$.

1 Current in R_2 = $1\,mA + 8\,\mu A = 1.008\,mA$

1 $R_2 = \frac{2V - 0.7V}{1.008\,mA} = 1.29\,k\Omega \cong 1.3\,k\Omega$

1 $R_3 = \frac{2V}{2\,mA} = 1\,k\Omega$

1 $R_4 = \frac{2V - 0.4V}{1\,mA} = \frac{1.6V}{1\,mA} = 1.6\,k\Omega$

Note → M1 is at boundary of μ & v limits → either expression ok.

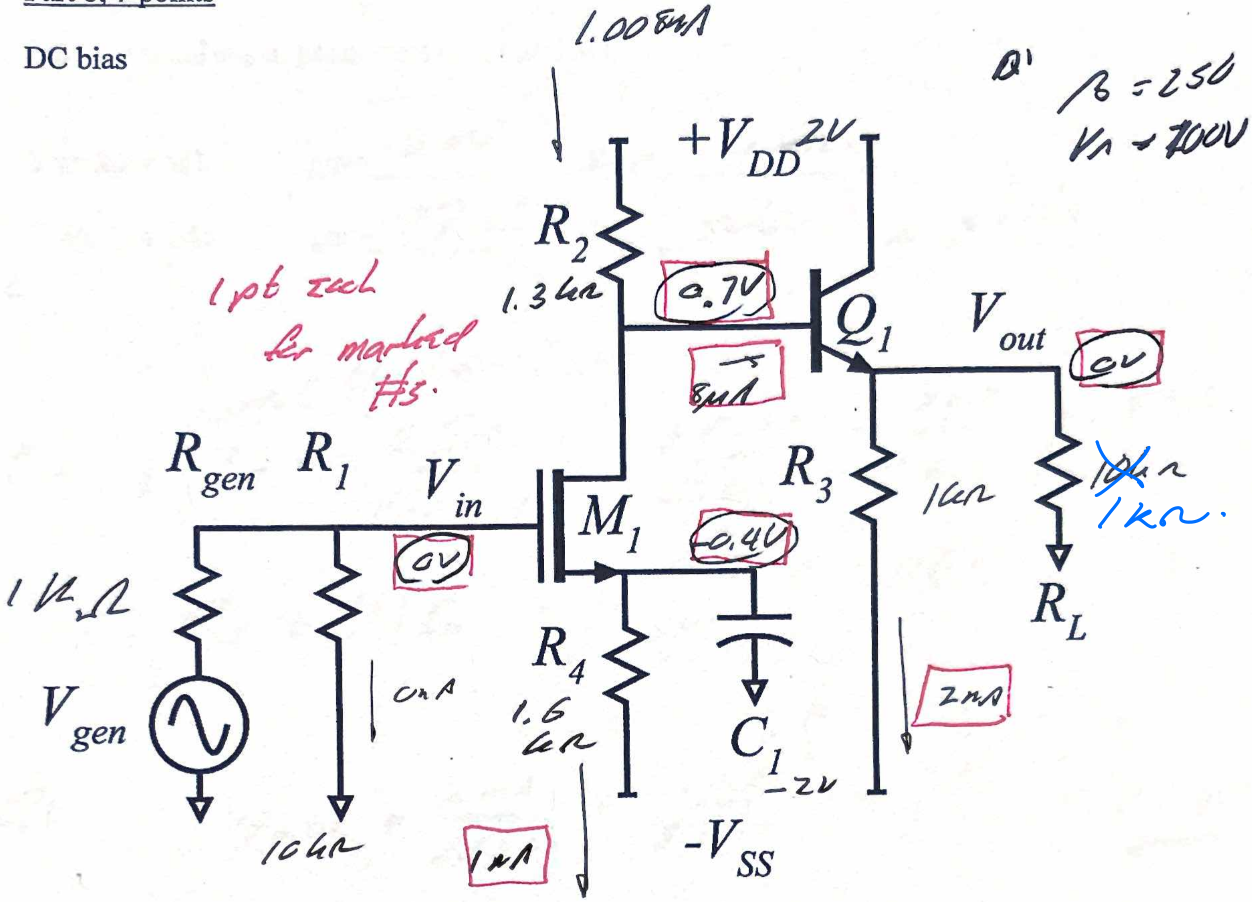
1 $I_{D1} = 1\,mA = 10\,\frac{mA}{V^2}\frac{W_g}{1\mu m}(V_{gs} - V_{th})^2(1 + \lambda V_{DS})$ ($V_{gs} - V_{th}$ = 0.1V; V_{DS} = 1.1V)

1 $1\,mA = 10\,mA \cdot \frac{W_g}{1\,\mu m}(0.1V)^2\left(1 + \frac{1.1V}{4V}\right) \rightarrow W_g = 7.84\,\mu m$

M1: $\frac{1}{1}$ = 4V.

Part b, 7 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

$g_{m,M1}$ = 5mS

$R_{DS,M1}$ = 4kΩ

$g_{m,Q1}$ = 77mS ≈ 1/13Ω

$R_{ce,Q1}$ = 50kΩ

$R_{be,Q1}$ = 3.25kΩ.

Part c, 6 points

Find the small signal parameters of Q1 and M1.

Transistor M1: gm= 5 mS Rds= 4 kΩ.

Transistor Q1: gm= 77 mS Rce= 50kΩ Rbe= 3.25kΩ

1 [mobility limited, so:

M1

1 [$g_m = \frac{2 I_d}{V_{gs} - V_{tl}} = \frac{2(1mA)}{0.4V} = \frac{2mA}{0.4V} = 5mS$

1 [$R_{DS} = 1/\lambda I_D = \frac{4V}{1mA} = 4k\Omega$.

Q1

1 [$g_{m\,Q1} = \frac{2\,mA}{26\,mV} = \frac{1\,mA}{13\,mV} = \frac{1}{13\Omega} = 77\,mS$

1 [$R_{CE} = \frac{V_{CE} + V_A}{I_C} \cong \frac{V_A}{I_C} = \frac{100V}{2mA} = 50k\Omega$

1 [$R_{be} = \frac{\beta}{g_m} = 250\,(13\Omega) = 3.25\,k\Omega$

Part d, 15 points.

Find the small signal voltage gain (Ve1/Vb1) of Q1 and Q1's small-signal input resistance.

Ve1/Vb1= 0.974

Rin,q1= $127 k\Omega$

Q1:

3 [$R_{Leq} = R_3 \parallel R_L \parallel r_o$

2 [$= 1k\Omega \parallel 1k\Omega \parallel 50k\Omega$
$= 495\Omega$

3 [$A_{v1} = \frac{v_{e1}}{v_i} = \frac{R_{Leq1}}{R_{Leq1} + 1/g_m}$

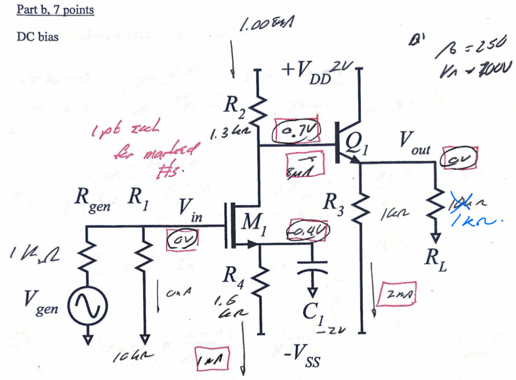
2 [$= \frac{495\Omega}{495\Omega + 13\Omega} = 0.974.$

3 [$R_{in1} = \beta (R_{Leq1} + 1/g_m)$

2 [$= 250(495\Omega + 13\Omega) = 127k\Omega$

Part b, 7 points

DC bias



On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors

Part e, 15 points

Find the small signal voltage gain (Vd1/Vg1) of M1 and the *** resistance.

Vd1/Vg1= -4.86

Rin,amplifier = $10\,k\Omega$

$R_{Leq\,M1} = R_{in2} \parallel R_2 \parallel R_{os1}$

$= 127\,k\Omega \parallel 1.3\,k\Omega \parallel 4\,k\Omega$

$= 974\,\Omega$

$A_{v\,M1} = \dfrac{V_{d1}}{V_{g1}} = -g_{m\,M1} R_{Leq}$

$= -5\,mS \cdot 974\,\Omega$

$= -4.86$

$R_{in\,M1} = \infty$

$R_{in} = R_{in\,M1} \parallel R_1 = 10\,k\Omega$

Part f, 6 points

Find (Vout/Vin), (Vin/Vgen) and (Vout/Vgen)

(Vout/Vin) = – 4.74

(Vin/Vgen) = 0.909

(Vout/Vgen) = –4.32

1 $v_o / v_i = A_{vm1} A_{vo1} = -4.86 \cdot 0.974$

1 $= -4.74$

$v_i / v_{gen} = \frac{R_{in}}{R_{in} + R_{gen}} = \frac{10k\Omega}{10k\Omega + 1k\Omega} = 0.909$

1 1

$\frac{v_o}{v_{gen}} = \frac{v_o}{v_i} \cdot \frac{v_i}{v_{gen}} = -4.32$

1 1

Part g, 14 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q1, and saturation and the knee voltage of M1. ***Give the sign (+ or -) in your answers below.***

Cutoff of Q1; Maximum ΔVout resulting = −0.99V ↓

Saturation of Q1; Maximum ΔVout resulting = 1.5V ↑

Cutoff of M1; Maximum ΔVout resulting = 0.949V ↑

Knee voltage of M1; Maximum ΔVout resulting = 0.986V ↓

Q1 cutoff

/ [$R_{Leq} = R_3 \| R_2 \| R_{Ce} = 495\Omega$.

/ [$I_{CQ1} = 2\,mA$

/ [$\Delta V_{out} = 2mA \cdot 495\Omega = -0.99V$ ↓

Q1 sat

/ [$V_{CE,Q} = 2V - 0V = 2V$

/ [$V_{CE,\,sat\,Q1} = 0.5V$

/ [$\Delta V_{out} = 2V - 0.5V = 1.5V$ ↑

974

m1 cutoff:

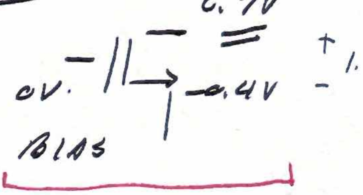
/ [$R_{Leq\,m1} = 974\,\Omega$

/ [$I_{DM1} = 1\,mA$

/ [$\Delta V_{DRAIN1} = 1mA \cdot 974\Omega = 974mV$ ↑

/ [multiply By A_{vQ1} to get ΔV_{out}: $974mV \cdot 0.974 = 0.949V$ ↑

m1 knee voltage



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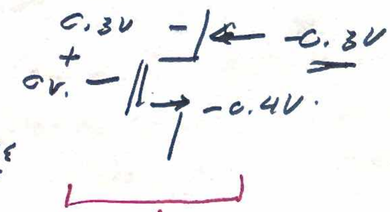

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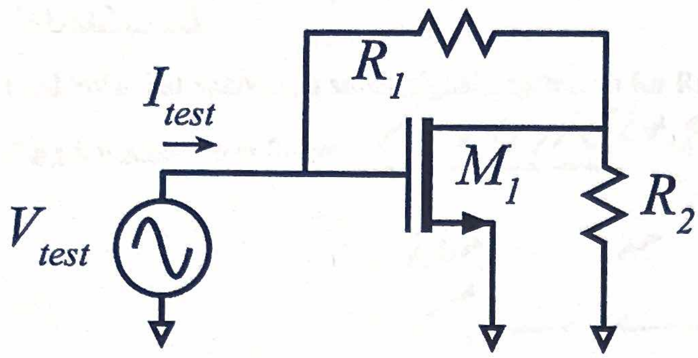
$\Delta V_{DRAIN} = 1V$ ↓] /

$\Delta V_{out} = 1V \cdot A_{vQ1}$
$= 1V \cdot 0.986$
$= 0.986V$ ↓] /

Problem 2, 30 points
nodal analysis



You will be working on the circuit to the left.

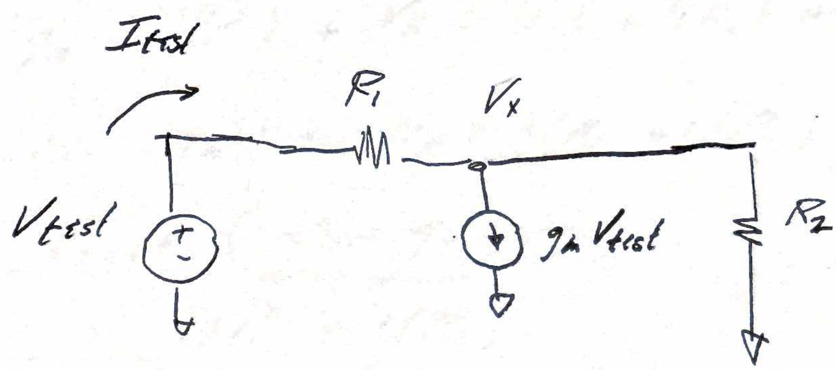
Ignore DC bias analysis. You don't need it.

Transistor M1 has transconductance gm1.

The drain-source resistance Rds of M1 is infinity (so you don't need to draw it!)

Part a, 12 points

Draw the small-signal equivalent circuit

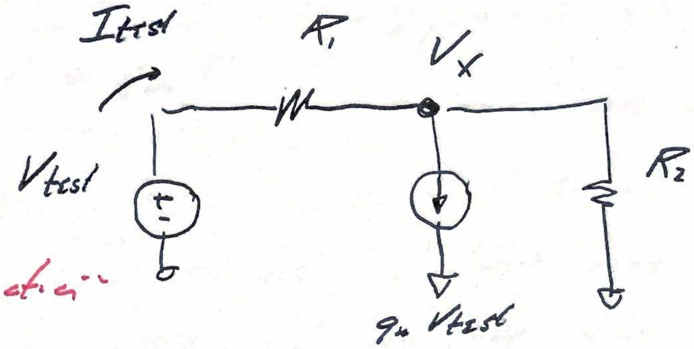


Hard to give partial credit
deduct points if
- topology is wrong
- elements are missing
- elements are not labelled
- location of voltage controlling gm source is not labelled.

Part b,13 points

Find, by nodal analysis, a small-signal expression for Rin,transistor=Vtest/Itest

Rin,transistor=Vtest/Itest= $(R_1+R_2)/(1+g_m R_2)$



6 points for correct equation

$\Sigma I = 0$ @ V_x

$$g_m V_{test} + V_x/R_2 + (V_x - V_{test})/R_1 = 0.$$

$$V_{test}(g_m - 1/R_1) + V_x(1/R_1 + 1/R_2) = 0.$$

$$V_{test}(1/R_1 - g_m) = V_x(1/R_1 + 1/R_2)$$

$$V_x = V_{test}\frac{(1/R_1 - g_m)}{(1/R_1 + 1/R_2)}$$

. . . .

$$I_{test} = \frac{V_{test} - V_x}{R_1} = \frac{1}{R_1}\left[V_{test}\right]\left[1 - \frac{V_x}{V_{test}}\right]$$

$$R_1\frac{I_{test}}{V_{test}} = 1 - \frac{1/R_1 - g_m}{1/R_1 + 1/R_2} = \frac{1/R_1 + 1/R_2 - 1/R_1 + g_m}{1/R_1 + 1/R_2}$$

$$R_1\frac{I_{test}}{V_{test}} = \frac{1/R_2 + g_m}{1/R_1 + 1/R_2}$$

this is correct. — this is correct

$$\frac{V_{test}}{I_{test}} = R_1 \cdot \frac{1/R_1 + 1/R_2}{1/R_2 + g_m} = \frac{1 + R_1/R_2}{1/R_2 + g_m}$$

$$\boxed{\frac{V_{test}}{I_{test}} = \frac{R_1 + R_2}{1 + g_m R_2}}$$

← 7 points for correct answer

grader - be sure that answer is correct ... there are several ways of writing the correct answer.

Part c, 5 points

gm1= 1 mS, R1=2kOhm, R2=1kOhm
Give a numerical value for Rin,transistor

Rin,transistor=Vtest/Itest= 1.5kΩ.

5.

$$R_{int} = \frac{R_1 + R_2}{1 + g_m R_2} = \frac{1k\Omega + 2k\Omega}{1 + 1mS \cdot 1k\Omega}$$

$$= \frac{1k\Omega + 2k\Omega}{2} = 1.5k\Omega$$