## Final Exam, ECE 137A

## Wednesday March 20, 2019, Noon-3 p.m.

Name: $\qquad$
Closed Book Exam:
Class Crib-Sheet and 4 pages (4 surfaces) of student notes permitted Do not open this exam until instructed to do so. Use any and all reasonable approximations (5\% accuracy), after stating \& justifying them.
Show your work:
Full credit will not be given for correct answers if supporting work is missing. Good luck

| Time function | LaPlace Transform |
| :--- | :--- |
| $\delta(t)$ impulse | 1 |
| $U(t)$ unit step-function | $1 / s$ |
| $e^{-\alpha t} U(t)$ | $\frac{1}{s+\alpha}=\frac{1 / \alpha}{1+s / \alpha}$ |
| $e^{-\alpha t} \cos \left(\omega_{d} t\right) U(t)$ | $\frac{s+\alpha}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |
| $e^{-\alpha t} \sin \left(\omega_{d} t\right) U(t)$ | $\frac{\omega_{d}}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |


| Part | Points <br> Received | Points <br> Possible | Part | Points <br> Received | Points <br> Possible |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1a |  | 6 | 2c |  | 15 |  |
| 1b |  | 5 | 2d |  | 10 |  |
| 1c |  | 4 | 3a |  | 7 |  |
| 1d |  | 10 | $3 b$ | 8 |  |  |
| 1e |  | 10 | 3 c |  | 7 |  |
| 2a |  | 10 | 3d |  | 8 |  |
| 2b |  | 10 |  |  |  |  |
| total |  | 100 |  |  |  |  |

## Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts


All the transistors have the same (matched) $I_{S}$, have $\beta=100$, and $V_{A}=\infty$ Volts .
$V_{C E(s a t)}=0.5 \mathrm{~V}$.
$V_{b e}$ is approximately 0.7 V ,
but use $V_{b e}=(k T / q) \ln \left(I_{E} / I_{S}\right)$ when necessary or appropriate.
The supplies are +3 Volts and -3 Volts.
All transistors (and diodes) have the same $I_{S}$
Q1A,1B,5,11 are biased at 5 mA collector current.
Q6 is biased at 20 mA collector current.
Q7 and Q10 are biased at 5 mA collector current.
The DC voltage drops across RE5 and RE14 are both 300 mV .
$\mathrm{RB} 1 \mathrm{~A}=\mathrm{RB} 1 \mathrm{~B}=2 \mathrm{kOhm}$. RE1A=RE1B=44.8Ohm. RL11=1.1kOhm.
$\mathrm{RL}=1 \mathrm{kOhm}$.

Part a, 6 points
DC bias---to simplify , assume $\beta=\infty$ for the DC analysis only.

Find the value of the following resistors:

| Re5= | Re14= | Re4= |
| :---: | :---: | :---: |
| Re15= | Re12 $=$ | Re3= |
| Re7= | Re10= |  |

Part b, 5 points
DC bias---to simplify , assume $\beta=\infty$ for the DC analysis only.


On the circuit diagram above, label the DC voltages at ALL nodes, and the DC collector currents of all transistors. Label the values of all resistors (except RC5).

## Part c, 4 points

find the following

| device | Q1AB | 11 | 12 | 4 | 6 | 15 | 7 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| gm, <br> mS |  |  |  |  |  |  |  |  |

## Part d, 10 points.

Find the following, using the actual value of $\beta$, i.e. $\beta=100$

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Q1AB |  |  |
| Q11 |  |  |
| Q6 |  |  |
| Q7 |  |  |
| Overall differential <br> Vout/Vin |  |  |

Note: with some insight, you can find the combined gain of $\mathrm{Q} 1 \mathrm{AB} / 11$ in a single step. If would would like to do so, omit the separate answers for Q 1 AB and Q 11 in the table above, and instead fill in the table below,

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Q1AB/ Q11 combination. |  |  |

## Part e, 10 points

Maximum peak-peak output voltage (show all your work)
For this, you must use the full circuit diagram, not the half circuit diagram.

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to saturation |
| :--- | :--- | :--- |
| Transistor Q7 |  |  |
| Transistor Q10 |  |  |
| Transistor Q6 |  |  |
| Transistor Q15 |  |  |
| Transistor Q4 |  |  |
| Transistor Q11 |  |  |
| Transistor Q1A |  |  |
| Transistor Q1B |  |  |

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1 -sentence statement below as to why it is not relevant. Q7/10 form a push pull stage, so be careful about your answer there. .

## Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input $\mathrm{Vi}+$ is zero volts, and that we must determine the DC value of the negative input voltage (Vi- ) necessary to obtain this.


The NMOSFETs have $K_{\mu}=\mu c_{g s} W_{g} / 2 L_{g}=0.55 \mathrm{~mA} / \mathrm{V}^{2} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right)$
$K_{v}=c_{g s} v_{i n j} W_{g}=0.69 \mathrm{~mA} / \mathrm{V} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right), \Delta V=v_{i n j} L_{g} / \mu=0.625 \mathrm{~V}, V_{t h}=0.3 \mathrm{~V}$,
$1 / \lambda=20 \mathrm{~V}$
The PMOS have identical parameters, except, of course, $V_{t h}$ is negative.
$V_{D D}=+1 \mathrm{~V},-V_{S S}=-1 \mathrm{~V}, \quad R_{\mathrm{L}}=50 \mathrm{kOhm}$
All transistors have $|\mathrm{Vgs}|=0.4 \mathrm{~V}$, except for M7 and M15, which have $|\mathrm{Vgs}|=0.5 \mathrm{~V}$, and except for M8,9,10,11, which have $|\mathrm{Vgs}|=0.35 \mathrm{~V}$
M12,13 are biased at $I_{D}=50 \mu \mathrm{~A}$.
M5,7,15 are biased at $I_{D}=35 \mu \mathrm{~A}$.
M8,9,10,11 are biased at $I_{D}=25 \mu \mathrm{~A}$.

## Part a, 10 points

DC bias.
Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input $\mathrm{Vi}+$ is zero volts, and that we must determine the DC value of the negative input voltage (Vi- ) necessary to obtain this.
(Hint, this should give Vi- $=0 \mathrm{~V}$ )
Find the following:
Gate widths of M12 and M13 = $\qquad$
Gate width of M7 =
Gate width of M8 =
Gate width of M9 =

Part b, 10 points
DC bias


On the circuit diagram above, label the DC voltages at ALL nodes, the drain currents of ALL transistors, and the gate widths of ALL transistors

Part c, 15 points.
You will now compute the op-amp differential gain. Find the following

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Transistor combination <br> M1,2,13, 13 |  |  |
| M5,6 combination |  |  |
| Q9 or Q12. |  |  |
| Q8 or Q15 |  |  |
| Overall differential <br> Vout/Vin |  |  |

Notes:

1) You can analye M5 and M6 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.
2) For M8/9and for M10/11, you can assume that M8 and M9 are on for the positive signal swing and M10 and M11 are on for the negative signal swing. More accurately, you can assume, for the signal swing near zero volts, that all are on. If you take the latter approach (and do it correctly), you will receive a couple of extra credit points. One hint (don't ask for any other hints): use symmetry.

## Part d, 10 points

Maximum peak-peak output voltage at the positive output $\mathrm{Vo}+$ (show all your work)

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to: <br> knee voltage (saturation) |
| :--- | :--- | :--- |
| Transistor M9 |  |  |
| Transistor M11 |  |  |
| Transistor M8 |  |  |
| Transistor M10 |  |  |
| Transistor M4 |  |  |
| Transistor M19 |  |  |
| Transistor M6 |  |  |
| Transistor M14 |  |  |

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant".

Problem 3, 30 points


## Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.

## Part b, 8 points

$\mathrm{gm}=10 \mathrm{mS} . \mathrm{L}=1 \mathrm{nH} . \quad \mathrm{R}=1000 \mathrm{Ohms}$
Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with ${ }^{* *}$ correct units** and in ratio-of-polynomials form, i.e.
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$ or (as appropriat e) $\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot(s \tau)^{n} \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$
Note that an expression like
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=\frac{1}{1+\left(3 \cdot 10^{-6}\right) s}$ is dimensiona lly wrong; $\frac{1}{1+\left(3 \cdot 10^{-6} \text { seconds }\right) s}$ is dimensiona lly correct
$\operatorname{Vout}(\mathrm{s}) / \operatorname{Vin}(\mathrm{s})=$ $\qquad$

## Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz :

## Draw a clean Bode Plot of Vout/Vin, <br> LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



Part d, 8 points
$\operatorname{Vin}(\mathrm{t})$ is a 0.1 V amplitude step-function.
Find $\operatorname{Vout}(\mathrm{t})=$ $\qquad$
Plot it below. Label axes, show initial and final values, show time constants.


