Differential Amplifiers

\[ V_{cc} = 10 \text{V} \]

\[ V_{ee} = -10 \text{V} \]

\[ R_{e1} = R_{c2} \]

\[ \beta = 100 \]

\[ V_A = 100 \text{V} \]

\[ R_{e1} = R_{c2} = 3 \text{k}\Omega \]

First consider "single-ended input" \( \Rightarrow u_i^- = 0 \text{V} \)

Emitter follower common base
Stage 2

Common base

\[ V_{c2} = \frac{V_{c2}}{V_{c2}} \approx \frac{R_C}{R_C (1 + R_C/R_0)} \]

\[ \approx \frac{R_C}{R_0} \left( \frac{1}{R_0} \right) \]

\[ r_{in2} = R_0 (1 + R_0/R_0) \approx R_2 \]

Stage 1

\[ V_{c1} = \frac{R_C V_C}{R_C + R_C} \approx \frac{R_C}{R_0} \]

\[ \frac{V_{c1}}{V_{c1}} \approx \frac{R_0}{R_2} \]

\[ r_{in1} = \left( 3R_1 + 1 \right) \left( \frac{R_2}{R_1 + R_2} \right) \]

\[ \approx (3R_1 + 1) \left( \frac{R_2 R_2}{R_1 + R_2} \right) \]

Overall

\[ V_{c2} = \frac{V_{c2}}{V_{c2}} \approx \frac{R_C}{R_0} \]

\[ \frac{V_{c2}}{V_{c2}} \approx \frac{R_C}{R_2} \]

\[ \frac{R_2}{R_2 + R_1} \approx \frac{R_2}{R_1 + R_2} \]
Now consider case when $V_{0+} = 0$

$\begin{align*}
\frac{V_{0+}}{V_{0-}} &= \frac{-R_{eg}}{R_{e2} + R_{e1} || R_{ee}} \\
&\approx \frac{-R_c}{R_{e1} + R_{e2}}
\end{align*}$

We thus get the same gain, except for a reversal of signs.

$\Rightarrow V_{out+} \approx \frac{R_c}{R_{e1} + R_{e2}} (V_{0+} - V_{0-})$
Another - easier - way to analyze the differential amplifier:

**Differential and Common-mode gains:**

**Differential signal:** \( V_d = V_i^+ - V_i^- \)

**Common-mode (average) signal:** \( V_{cm} = (V_i^+ + V_i^-)/2 \)

We want to amplify the differential signal, **not** the common-mode signal.

\[
\begin{align*}
V_{dl/2} & \\
\downarrow & \\
V_i^+ & \quad \downarrow & \quad V_i^- \\
\uparrow & \\
V_{dl/2} & \\
\end{align*}
\]

\[
\begin{align*}
V_d &= V_i^+ - V_i^- \\
V_{cm} &= \frac{V_i^+ + V_i^-}{2}
\end{align*}
\]

\[
\begin{align*}
V_{dl/2} & \\
\downarrow & \\
V_{cm} & \quad \downarrow & \quad V_{cm} - V_d/2 \\
\uparrow & \\
V_{dl/2} & \\
\end{align*}
\]

\[
\begin{align*}
V_i^+ &= V_{cm} + V_{dl/2} \\
V_i^- &= V_{cm} - V_{dl/2}
\end{align*}
\]
Differential Gain

\[
V_o = \frac{-R_{eq}}{(-V_d/2) r_{e2}}
\]

Hence

\[
V_o = \frac{R_{eq}}{V_d} = \frac{R_{eq}}{2 r_{e2} (r_{e1} + r_{e2})}
\]

Because \( r_{e1} = r_{e2} \)

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Common-Mode Gain

\[
A_{CM} = \frac{V_o}{V_{CM}} = \frac{-R_{eq}}{r_{e2} + 2R_{EE}} = \frac{-R_{eq}}{2R_{EE}}
\]

But we have already found that

\[
A_0 = \frac{V_o}{V_d} = \frac{R_{eq}}{2 r_{e2} (r_{e1} + r_{e2})}
\]

So the common-mode rejection ratio:

\[
CMRR = \frac{A_0}{A_{CM}} = \frac{R_{EE}}{r_e}
\]

where \( r_e = r_{e1} = r_{e2} \)
Constant-current source for increased CMRR

The simple voltage divider + diode forces 1.0 Volts across the emitter resistor $\rightarrow$ pick $R_{ee}$ for desired current.

\[
R_{out3} = r_{oc3} \left[ 1 + \frac{R_{ee}}{r_{e3}} \frac{R_{be2}}{R_{be} + R_{e3} + R_{e3}} \right] = 994 \, k\Omega
\]

Constant-current source for increased CMRR

\[
CMRR = \frac{R_{out3}}{r_{s1,2}} = \frac{994 \, k\Omega}{26 \Omega}
\]
Differential pair with degeneration:

\[ V_{o+} - V_{o-} = \frac{R_c}{R_{E} + R_{c}} \]

Common-mode:

\[ \frac{V_o}{V_{cm}} = \frac{R_c}{R_{E} + R_{c}} \]

Differential:

\[ \frac{V_o}{V_{cm}} = \frac{R_c}{R_{E} + R_{c}} \]

Norton equivalent of active or passive current source

\[ \text{virtual ground} \]

\[ \Rightarrow \text{CMRR is reduced} \]
WE can also construct differential amplifiers from MOSFETs, thus

... and the analysis is identical to that of the bipolar implementation.
Differential stage input impedances

\[ V_i = 0u \]

\[ \Gamma_2 = \Gamma_{e2} + \text{Reac} \]

\[ \Gamma_3 = \text{Reac} + \frac{\Gamma_{e2}}{\text{Res}} \approx 2 \times \text{Reac} + \Gamma_{e2} \]

...if Res is large

\[ \Gamma_4 = \beta (\Gamma_{e1} + \Gamma_3) \approx 2/\beta (\Gamma_{e2} + \text{Reac}) \]

\[ \Gamma_5 = \text{Res} \parallel \Gamma_4. \]

This is the single-ended input impedance.
Differential input impedance

\[ V_{1/2} \]

Virtual ground

Again use symmetry:

\[ \frac{V_{1/2}}{I_n} = \beta \left( R_{ac} + r_e \right) \]

\[ \frac{V_{1/2}}{I_n} = \frac{1}{\beta} \left( R_{ac} + r_e \right) \]

\[ N \]

\[ N_{1/2} = \frac{2}{\beta} \left( R_{ac} + r_e \right) \]

\[ = R_{in}, \text{ differential} \]

\[ 2/3 \left( r_e + R_{ac} \right) \]
Common-mode input impedance:

\[ \frac{U_{cm}}{I_{cm}} = \beta \left( \frac{1}{R_e} + \frac{1}{R_{cc}} + 2R_s \right) \approx \frac{2}{\beta R_s} \]

\[ = \text{Pin, cm} \]

Total input model:

\[ V^+ \quad 2/\beta(1/R_e + 1/R_{cc}) \quad V^- \]

\[ \beta \cdot 2R_s \quad \beta \cdot 2R_s \]

The input impedance must be modelled by 3 resistors because there are three