MOSFET CURRENT MIRRORS:

Suppose we want to provide some fixed bias current to several circuits: $V_{dd}$

If we assume square law model (assummg $V_{ds} > V_{th}$):

$I_{o1} = \frac{g_m W_3}{2 L} (V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ for all $I_{ds}$.

\[ I_{o2} = \frac{W_2}{W_1} \frac{1 + \lambda V_{ds}}{1 + \lambda V_{ds}} \cdot \frac{W_2}{W_1} \]

(we hope)

\[ \Rightarrow \frac{I_{o2}}{I_{o1}} = \frac{W_2}{W_1} \frac{1 + \lambda V_{ds}}{1 + \lambda V_{ds}} \]

Similarly: $I_{o2}/I_{o1} \approx \frac{W_2}{W_1}$
Example:

250 nm NMOS: \( \sigma \text{d} \omega / \Delta \theta = (1 \text{mA} / \text{V}^2) \left( \lambda \right) \left( \text{W} / \text{L} \right) \)

\( I = 1/100 \quad V_{dd} = 0.5 \text{V} \)

Choose \( W_{g1} = 1 \text{mm} \)

\[ I_0 = \left( 1 \text{mA} / \text{V}^2 \right) \times (V_{gS} - V_{th})^2 \left( 1 + \lambda \right) \left( V_{gS} - V_{th} \right) \]

\[ 1.2 \text{mA} = \left( 1 \text{mA} / \text{V}^2 \right) \left( V_{gS} - V_{th} \right)^2 \]

\[ \Rightarrow V_{gS} - V_{th} = 0.71 \text{V}, \quad V_{gS} = 1.21 \text{V} \]

\[ \Rightarrow I = (4 \text{V} - 1.21 \text{V}) / 0.5 \text{mA} = 5.6 \text{G} \]

Now choose \( W_{g2} = 2 \text{mm}, \quad W_{g3} = 4 \text{mm} \)

\[ I_{d3} = \frac{W_{g3}}{W_{g1}} \left( 1 + \lambda \right) V_{gS}^2 \]

\[ I_{d2} = \frac{W_{g2}}{W_{g1}} \left( 1 + \lambda \right) V_{gS}^2 \]

\[ I_{d1} = \frac{W_{g1}}{W_{g1}} \left( 1 + \lambda \right) V_{gS}^2 \]

\[ I_{d2} \text{ and } I_{d3} \text{ will be approximately } \]

\[ 1 \text{mA} \text{ and } 2 \text{mA}. \]

\[ \Rightarrow \text{but some variation with } V_{gS}. \]
MOS multistage amplifier

This example illustrates the challenges in working with < 100nm LG MOSFETs in analog ICs.

\[ V_{DD} = 1.0V \]

all NMOS: \( W_g = 1\, \mu m \)

all PMOS: \( W_g = 2\, \mu m \)

45mV/L

NMOS: \( V_{th} = 0.2V \)

\[ V_{th} = \frac{1.5\, mA}{1\, \mu m} \]

\[ I_D = \frac{(V_{gs} - V_{th} - \Delta V)}{1\, \mu m} \]

for \( V_{gs} - V_{th} > \Delta V \)

PMOS: \( V_{th} = -0.2V \)

\[ V_{th} = \frac{0.75\, mA}{1\, \mu m} \]

\[ I_D = \frac{(-V_{gs} + V_{th} + \Delta V)}{1\, \mu m} \]

In biasing the fets at \( V_{gs} = 0.5V \), we will operate them at the boundary between mobility- and velocity-limited current. Analysis is therefore somewhat approximate.
DC 6.45 design:

\[ I_{D2} = 2 \times (0.75 \text{mA/V}) \times (0.5 \text{V} - 0.3 \text{V} - 0.2 \text{V}) = 150 \mu\text{A} \]

\[ R = \frac{0.5 \text{V}}{150 \mu\text{A}} = 3.33 \text{ k}\Omega \]

Neglecting \((1 + V_{GS})\) terms, \(I_{D4} = I_{D5} = I_{D3} = 150 \mu\text{A}\)

Q1: \[ I_D = 150 \mu\text{A} = (1.5 \text{mA/V}) \times (V_{GS} - V_{TH} - AV) \]

\[ \Rightarrow V_{GS1} = 0.5 \text{V} \]

\[ V_{GS2} = V_{GS1} = 0.5 \text{V} \]

\[ \Rightarrow I_{D2} = (1.5 \text{ mA/V}) \times (V_{GS} - V_{TH} - AV) = 150 \mu\text{A} \]
Small signal analysis

Q2: Common Source

\[ R_{eq} = \frac{R_{os2}}{1 + \frac{R_{os1}}{R_{os2}}} \]

\[ \frac{1}{\lambda} \frac{dI_d}{dV} = \frac{3V}{150mA} = 20k \Omega \]

\[ \frac{1}{\lambda} \frac{dI_d}{dV} = 20k \Omega \]

\[ R_{eq} = 10k \Omega \]

\[ m = -g_m \cdot R_{eq} = -(1.5mAV) \cdot 10k \Omega \]

\[ m = -15 \text{ not very big (!)} \]

\[ R_{m2} = 0 \]

Q1: Source Follower

\[ R_{eq} = \frac{R_{os1}}{1 + \frac{R_{os4}}{R_{os2}}} = 20k \Omega / 20k \Omega = 10k \Omega \]

\[ m = \frac{R_{eq}}{R_{eq} + 1/g_m} = \frac{10k \Omega}{10k \Omega + 667m} = 0.94 \]
Maximum signal swings

Use constant velocity formulas: \[ V_{peak} = \frac{V_{dc}}{\ln 2} \]
\[ = 0.2 \text{ V} \]

Q2: \[ V_{dc}, \text{ bias} = 1.2 \text{ V} \]
\[ V_{dc}, \text{ knee} = 0.2 \text{ V} \]
\[ \Delta V_{out} = -0.3 \text{ V} \]
\[ \text{Maximum} \]
\[ 0.3 \text{ V} \text{ output} \]
\[ (600 \text{ mV/pp}) \]

Q1: \[ V_{dc}, \text{ bias} = 1.2 \text{ V} \]
\[ V_{dc}, \text{ knee} = 0.2 \text{ V} \]
\[ \Delta V_{out} = +0.3 \text{ V} \]
Active loads - constant current source:

Note that the bias network is not shown. Stages with c.c. loads usually have high gain and often can only be properly biased as part of a feedback amplifier.
Output impedance of Q2: $V_A = 100V$, $\beta = 100$

\[
\begin{align*}
300 \Omega & \quad \text{Resistor} = 26 \Omega \\
4 \Omega & \quad \text{4k}\Omega \quad 326 \Omega \\
& \quad = 301 \Omega = R_{e2}
\end{align*}
\]

\[
R_{out2} = R_{e2} \left[1 + \frac{R_{e2}}{\frac{\beta R_{e2}}{R_{e2} + R_{e2} + R_{e2}}} \right]
\]

\[
\begin{align*}
& = 1.04 \text{M}\Omega \\
& \quad \text{very large, much larger than } R_{e2}
\end{align*}
\]

$\Rightarrow$ Norton model of constant current source:

1 mA

\[
1.04 \text{M}\Omega
\]
The gain of the amplifier:

\[ \frac{V_o}{V_i} = -\frac{R_{E}'}{R_{E}} \frac{R_{S}'}{R_{S}} \frac{R_{E}}{R_{E}'} \]

Because \( R_{E} \) is very large, 10:1 larger than \( R_{S} \) in this example, it has little effect upon the gain.

In contrast, had a biasing resistor been used, its value would have been a few k\( \Omega \) in order to supply 1mA – less gain.

If \( R_{E} \) and \( R_{E}' \) are both \( \gg \) \( R_{i} \), then:

\[ \frac{V_o}{V_i} = -\frac{R_{E}'}{R_{E}} = -\frac{V_{i}}{I_{E}} = -\frac{V_{i}}{I_{E}'} \]

In this case, constant current loads allow large stage gains.
Darlington Pairs:

2 Cascaded emitter followers.

In both cases Q1 is used to further increase the input impedance.
alternate - and most common - form of Darlington:

look frightening? \( \Rightarrow \) relax and let's work it out:

Simplify analysis: set \( R_{C} = \infty \) \( \beta = \infty \)
apply $S V e_1$

then $S V e_1 = S V_{i,1} \cdot \frac{R E E_1}{R E E_1 \cdot \Gamma R_1}$

\[ S I c_1 = \frac{S V e_1}{R E E_1} = \frac{S V_{i,1}}{\Gamma R_1 + R E E_1} \]

\[ S I c_2 = \frac{S V e_1}{R E E_2 + \Gamma R_2} = \frac{S V_{i,1} \cdot A v_1}{R E E_2 + \Gamma R_2} \]

\[ S I o_{ct} = S I c_1 + S I c_2 \]

\[ S V o_{ct} = -S I o_{ct} \cdot R L \]

\[ \Rightarrow \frac{S V o_{ct}}{S V_{i,1}} = -\frac{A v_1 \cdot R L}{\Gamma R_2 + R E E_2} - \frac{R L}{R E E_1 + \Gamma R_1} \]

answer a bit more complex for \( \Gamma \text{iter } R L, R E E \ldots \)
Cascode stage

Starting point: Common Source stage

Break Point

$V_{DD} = 2V$

$R_{g1}$

$R_2$

$R_{ss} = 300 \mu A$

$V_{g1} = 0.8V$

$R_{g1} = 600k \Omega, R_2 = 400k \Omega \rightarrow V_{g1} = 0.8V$

$R_L = R_{ss} / (R_{DD} || R_2) = 2.33k \Omega$

$A_v = -9m R_L = -1m5 \cdot 2.33k \Omega = -2.33$

Low - a weak design.
Now break the circuit as indicated, and insert a common-gate stage.

![Circuit Diagram]

check bias conditions - note that both FETs are in linear active region.
Small-signal analysis:

\[ V_2 \Rightarrow \text{Req} = \frac{\text{Rout}}{1+\frac{\text{R} \cdot \text{Req}}{\text{R} \cdot \text{R}} = 2.5 \Omega } \]

\[ R_{\text{in}2} = \frac{1}{g_m2} \left( 1 + \frac{\text{Req}}{R_{\text{in}2}} \right) \]

\[ = 1.2 \left( 1 + \frac{2.5 \Omega}{37 \Omega} \right) = 1.08 \text{ } \Omega \]

\[ A_{\text{in}2} = \frac{\text{Req}}{R_{\text{in}2}} \]

\[ = 2.5 \Omega / 1.08 \Omega = 2.31 \]

\[ Q1: \text{Req1} = \frac{R_{\text{in}2}}{R_{\text{os}1}} = 1.08 \Omega / 37 \Omega = 0.03 \text{ } \]

\[ A_{\text{in}1} = -g_m1 \cdot \text{Req1} = -0.03 \cdot 1.05 \Omega = -0.05 \]

Overall:

\[ A_v = -\frac{\text{Req} \cdot (R_{\text{in}2} / R_{\text{os}1}) \cdot g_m1}{R_{\text{in}2}} \]

\[ = -g_m1 \cdot \text{Req} \cdot \frac{R_{\text{os}1}}{R_{\text{os}1} + R_{\text{in}2}} \]

Gain expression is almost similar to that of CS stage!

Q2 passes output current of Q1 to R_{\text{in}2}
Folded cascode stage

Q3 and Q4 are a constant-current source. Pick \( W_g = 2 \mu m \) so that

\[ V_{gs} = 0.6 V \]

As before, all FETs: \( g_m / W_g = 1 \mu S / \mu m \)

\[ |V_{dd}/2| = 0.3 V \]

\[ |V_A| = 10 V \]

Q1 and Q2 have \( W_g = 1 \mu m \)
Folded cascode - dc analysis

\[ R_{in2} \]

\[ C_S \quad | \quad C_G \]

\[ R_{o2} \parallel R_C = R_{eq2} \]

\[ R_{g1} \parallel R_{g2} \]

Q2
\[ R_{eq2} = R_{o2} \parallel R_C \]

\[ R_{in2} = (1/g_m) \left( 1 + \frac{R_{eq2}}{R_C} \right) \]

\[ A_V = \frac{R_{eq2}}{R_{in2}} \]

Q1
\[ R_{eq1} = (R_C \parallel R_{o3}) \parallel R_{in} \]

\[ A_V = -g_m \cdot R_{eq1} \]

Overall
\[ A_V = -g_m \left( R_C \parallel R_{o3} \right) \parallel R_{in2} \cdot \frac{R_{eq2}}{R_{in2}} \]

\[ = -g_m \cdot \frac{R_{eq2} \cdot (R_C \parallel R_{o3})}{(R_C \parallel R_{o3}) + R_{in2}} \]
Simpler Method: Track the currents!

Current divider:

\[ I = \frac{I_1}{R_1} \cdot \frac{R}{R_1 + R} \]

\[ I_2 = I \cdot \frac{R_2}{R_1 + R_2} \]

\[ I_{d1} \]

\[ I_{d2} \]

\[ \text{Volt} \]

\[ V_{ot} = -I_{d2} \cdot R_{20g2} = -I_{d2} \cdot R_{20g2} \]

done!
B. polar cascade examples:

Folded:

DC coupled versions will evolve naturally as we become more practiced with active biasing.