Mid-Term Exam, ECE-137B
Tuesday, April 28, 2015

Closed-Book Exam

There are 2 problems on this exam, and you have 75 minutes. 
1) show all work. Full credit will not be given for correct answers if supporting work is not shown.
2) please write answers in provided blanks
3) Don’t Panic!
4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.
Use any, all reasonable approximations. 5% accuracy is fine if the method is correct.

Do not turn over the cover page until requested to do so.

Name: 

<table>
<thead>
<tr>
<th>Time function</th>
<th>LaPlace Transform</th>
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<tbody>
<tr>
<td>δ(t)</td>
<td>1</td>
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<tr>
<td>U(t)</td>
<td>1/s</td>
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<tr>
<td>e^{-αt}U(t)</td>
<td>s + α</td>
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<tr>
<td>e^{-αt}cos(ωt)U(t)</td>
<td>(s + α)^2 + ω^2</td>
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<tr>
<td>e^{-αt}sin(ωt)U(t)</td>
<td>(s + α)^2 + ω^2</td>
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<tr>
<th>Problem</th>
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<th>Points Possible</th>
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<tr>
<td>1a</td>
<td>6</td>
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<tr>
<td>1b</td>
<td>8</td>
<td>8</td>
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<tr>
<td>1c</td>
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<td>8</td>
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<tr>
<td>1d</td>
<td>14</td>
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<td>1e</td>
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<td>1f</td>
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<tr>
<td>total</td>
<td>100</td>
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Problem 1, 60 points

Q1 has 1.0 nm oxide thickness, $\varepsilon_s = 3.8$, 22 nm gate length, and a 0.2 V threshold. Mobility is 400 cm²/V-s, $\lambda = 0$ Volts⁻¹, $C_D = \varepsilon_s \varepsilon_o L_g W_g / T_{ox} = 0.5(\mu F/\mu m) W_g$ and $C_{ox} = (0.5(\mu F/\mu m)) W_g$.

Hints:
$\varepsilon_s \varepsilon_o / T_{ox} = 3.36 \times 10^{-2} F/m^2$, $(\mu_s \varepsilon_s W_g / 2 L_g) = (3.06 \times 10^{-3} A/V²) (W_g/1 \mu m)$

The power supplies are +2V and -2V. The drain currents of Q1 is 2 mA. $V_{ds}$ of Q1 is 0.30 V. The drain of Q1 is at +1.0V.

$R_{gen} = 500 \Omega$, $R_c = 3 \cdot R_D$
$C_{in} = 1 \mu F$, $C_{ox} = 2 \mu F$

\[ +V_{DD} \]
\[ R_D \]
\[ C_{out} \]
\[ V_{out} \]
\[ R_{gen} \]
\[ C_{in} \]
\[ V_{in} \]
\[ Q_1 \]
\[ R_L \]
\[ V_{gen} \]
\[ R_{SS} \]
\[ \text{Q1 has 1.0 nm oxide thickness, } \varepsilon_s = 3.8 , 22 \text{ nm gate length, and a 0.2 V threshold. Mobility is } 400 \text{ cm}^2/(\text{V} \cdot \text{s}), \lambda = 0 \text{ Volts}^{-1}, C_D = \varepsilon_s \varepsilon_o L_g W_g / T_{ox} = 0.5(\mu F/\mu m) W_g \text{ and } C_{ox} = (0.5(\mu F/\mu m)) W_g. \]

\[ \text{Hints:} \]
\[ \varepsilon_s \varepsilon_o / T_{ox} = 3.36 \times 10^{-2} F/m^2, (\mu_s \varepsilon_s W_g / 2 L_g) = (3.06 \times 10^{-3} A/V²) (W_g/1 \mu m) \]

\[ \text{The power supplies are +2V and -2V. The drain currents of Q1 is 2 mA. } V_{ds} \text{ of Q1 is 0.30 V. The drain of Q1 is at +1.0V. } \]

\[ R_{gen} = 500 \Omega, R_c = 3 \cdot R_D \]
\[ C_{in} = 1 \mu F, C_{ox} = 2 \mu F \]
Part a. 6 points

Find the following:

\[ V_{DD} = 3.3 \text{V} \]

\[ V_{SS} = -2 \text{V} \]

\[ R_n = 8.5 \Omega \]

\[ R_o = 800 \Omega \]

\[ W_x = 0.34 \text{mA} \]

\[ R_x = 1800 \Omega \]

Draw all DC node voltages on the circuit diagram below.

\[ V_{DD} = 3.3 \text{V} \]

\[ V_{SS} = -2 \text{V} \]

\[ R_n = 8.5 \Omega \]

\[ R_o = 800 \Omega \]

\[ W_x = 0.34 \text{mA} \]

\[ R_x = 1800 \Omega \]

\[ V_{in} = 0.2 \text{V} \]

\[ V_{gs} = 0.5 \text{V} \]

\[ V_{out} = V_{gs} + V \]

\[ I_0 = 3.3 \text{mA} \]

\[ w_x \]}

\[ w_x = 0.23 \text{mA} \]

\[ I_0 = 27.5 \text{mA} \]

\[ R_0 = \frac{1 \text{V}}{1 \text{mA}} = 1 \text{k}\Omega \]

\[ R_{SS} = 1.7 \text{V} \]

\[ \frac{1 \text{mA}}{8.5 \Omega} = 0.12 \text{mA} \]

\[ R_L = 1.8 \text{V} \]
Part b. 8 points
small-signal parameters
Find the following

\[ C_D = 6 \times 10^{-6} \quad C_{PA} = 4,156 \quad C_{PB} = 305 \text{ GHz} \]
\[ g_n = 27.8 \text{ mS} \quad f_s = 30 \text{ kHz} \]

\[ C_P = 3.78 \times 10^{-9} \quad g_m = 47.9 \text{ mW} + 0.3 \text{ mW} \quad g_y = 2.3 \text{ mW} \quad g_y = 2.3 \text{ mW} \]
\[ C_{GD} = 0.5 \text{ pF} \quad g_m = 41.5 \text{ mW} \]
\[ T_0 = 27.8 \text{ mA} \quad T_0 = 27.8 \text{ mV} \quad T_0 = 27.8 \text{ mV} \]

\[ f_0 = \frac{1}{2 \pi R C} = 0.59 \quad f_0 = \frac{1}{2 \pi R C} = 0.59 \text{ GHz} \]
\[ f_0 = \frac{1}{2 \pi R C} = 0.59 \quad f_0 = \frac{1}{2 \pi R C} = 0.59 \text{ GHz} \]
Part c: 8 points

Mid Band Analysis:

Find the following:

\[ R_{\text{in,loopback}} = 3.4 \, \Omega \quad R_{\text{eq}} = \frac{375 \, \Omega}{3} \]

\[ V_{\text{in}}/V_{\text{in}} = 10.4 \quad V_{\text{out}}/V_{\text{in}} = 0.408 \]

\[ \frac{R_{\text{load}}}{R_{\text{eq}}} = 4.25 \]

\[
\begin{align*}
R_1 &= 850 \, \Omega \\
V_{\text{in}} &= 50 \, \text{mV} \\
R_2 &= R_3 \quad R_4 \\
V_{\text{out}} &= 150 \, \text{mV}
\end{align*}
\]

\[ R_{\text{eq}} = 35.97 \, \Omega \]  
\[ = 39.5 \, \Omega \]

\[ R_{\text{eq}} = R_1/(R_1 + R_2) = 50 \, \Omega / (150 \, \Omega + 50 \, \Omega) = 28.5 \Omega 
\]

\[ R_2 = R_1 \times R_2/R_1 = 20 \, \Omega 
\]

\[ R_1 = R_1 \times R_1/R_1 = 34.5 \, \Omega \]

\[ = 34.5 + 5 \, \Omega 
\]

\[ \frac{R_{\text{load}}}{R_{\text{eq}}} = \frac{375 \, \Omega}{34.5 + 5 \, \Omega} = 10.4 \]
Part d: 14 points
High-Frequency Analysis:
Find the frequencies, in Hz, of the two poles limiting the high-frequency response of the amplifier. Show your analysis (do not simply state that the input pole of a common-gate amplifier is approximately at $f_c$).

$$f_{H,1} = \frac{1}{2\pi C_{gs}}$$

$$f_{H,2} = \frac{1}{2\pi C_{gd}}$$

**Circuit Diagram**

- $V_{in}$
- $R_{in}$
- $C_{gs}$
- $C_{gd}$
- $R_{out}$
- $V_{out}$

**Calculations**

$$C_{gs} = 10 \mu F$$

$$C_{gd} = 10 \mu F$$

$$f_{H,1} = \frac{1}{2\pi (10 \mu F)} = 1.59 \text{ Hz}$$

$$f_{H,2} = \frac{1}{2\pi (10 \mu F)} = 106.86 \text{ Hz}$$

**Note**

The high-frequency response is limited by the poles at $f_{H,1}$ and $f_{H,2}$.
Part e: 14 points

Low-Frequency Analysis:
Find the frequencies, in Hz, of the two poles limiting the low-frequency response of the amplifier. Show your analysis.

\[ f_{1,2} = \frac{1.8 \text{ kHz}}{2} \quad f_{3,4} = \frac{19.8 \text{ kHz}}{2} \]

\[ V_{in} \quad R_2 \quad C_2 \quad R_{2,3} \quad C_{2,3} \quad 2 \text{nF} \]

\[ M = R_C \]
\[ = \frac{(R_{in} + R_{2,3}) C_2}{54.52 \text{ kHz}} \]
\[ = 84.52 \text{ kHz} \]
\[ \frac{1}{M} \text{ kHz} = 0.92 \text{ MHz} = \frac{0.92}{\pi} \text{ rad/s} \]

\[ f_{M1} \text{ kHz} = \frac{0.92}{\pi} \text{ rad/s} \approx 0.3 \text{ kHz} \]

\[ f_{M2} \text{ kHz} = \frac{0.92}{\pi} \text{ rad/s} \approx 0.3 \text{ kHz} \]

\[ M_1 = (R_{in} + R_2) C_2 \]
\[ = 2000 \text{ ohms} \times 2 \text{nF} \]
\[ = 4 \mu\text{F} \]

\[ f_{M3} \text{ kHz} = \frac{0.92}{\pi} \text{ rad/s} \approx 0.3 \text{ kHz} \]

\[ f_{M4} \text{ kHz} = \frac{0.92}{\pi} \text{ rad/s} \approx 0.3 \text{ kHz} \]
Part f: 10 points
Draw a clean asymptotic Bode Magnitude plot of $V_{oc}/V_{in}$ as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly.

$\frac{V_{oc}}{V_{in}} = 4.25 \text{ at } 0 \text{ dB}$

- NF: 0.55 Hz, 58.0 dB
- HF: 118 kHz, 102.6 dB

Plot has too small a range, try to split scales.
Problem 2, 40 points
Part a 10 points

Small signal analysis. Ignore the DC bias; you don’t need it.

The FET has $\lambda=0$ hence $G_{ds}=0$. Also, $C_{gs}=C_{gd}=0.1\text{F}$

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.
Part b, 10 points

**USING NODAL ANALYSIS**, compute $V_{out}(s)/V_{gen}(s)$ in ratio-of-polynomials form:

\[
\frac{V_{out}(s)}{V_{gen}(s)} = \frac{\frac{V_{in}}{V_{in_{load}}} \times (sT)^m}{1 + \frac{b_2 s^2 + b_3 s^3 + \ldots}{1 + a_2 s + a_3 s^3 + \ldots}}
\]

Here $m$, an integer, can be positive or negative or zero.

\[sI = 0 \Rightarrow V_{in}
\]

\[V_{out} \left[ G_1 + G_2 + \Delta G_i \right] + V_{gen} \left[ -G_2 - Q_m \right] = 0.
\]

\[\frac{V_{out}}{V_{gen}} = \frac{G_2 + Q_m}{G_1 + G_2 + \Delta G_i}
\]

\[= \frac{G_2 + Q_m}{G_1 + G_2} \frac{1}{1 + \Delta G_i (G_1 + G_2)^{-1}}
\]

\[\frac{V_{out}}{V_{gen}} = \left( Q_m + \frac{1}{R_1 R_2} \right) R_1 R_2 = \frac{1}{1 + A \Delta G_i (R_1 R_2)}
\]
Part c, 10 points
\[ g_n = 10 \text{ mS}, \quad R_1 = 1 \text{ kOhm}, \quad R_2 = 3 \text{ kOhm}, \quad C_1 = 1 \text{ pF} \]

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:
\[ f_1 = \ldots, \quad f_2 = \ldots, \quad \ldots \]

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:
\[ f_p = 212 \text{ MHz}, \quad f_m = \ldots \]

If there are 2 poles, and they are complex, give \( f_s = \omega_n / 2\pi \) and the damping factor \( \zeta : \)
\[ f_s = \omega_n / 2\pi = \ldots, \quad \zeta = \ldots \]

There is a single, \( n = 1 \) pole in the transfer function.

\[ \frac{1}{\tau} = \left( \frac{R_1 R_2}{11 R_2} \right) \zeta \]
\[ = 750 \cdot 1 \text{ nF} \]
\[ = 750 \text{ pS} \]

\[ f_{ns} = \frac{0.159}{\zeta} = 212 \text{ MHz} \]

Low frequency gain:
\[ (2m + 1 \text{ Hz})(R_1 R_2) \]
\[ = 10.33 \text{ mS} (750 \text{ kHz}) \]
\[ = 7.73 \]
Part d, 10 points

If \( V_{in}(t) \) is a 100mV step-function, find and plot \( V_{out}(t) \). Be sure to label and dimension the axes clearly, and to clearly label key features of the time waveform.

\[
\begin{align*}
V_{in}(t) &= 0.1 \cdot 7.73 \cdot e^{-\frac{t}{10}} \\
V_{out}(t) &= 0.773 \cdot e^{-\frac{t}{1000}}
\end{align*}
\]