ECE 137B Notes set 2: Common Emitter / Common Source

In 137B we can almost always save effort by using a Norton model of the generator.

\[ i_i = i_{\text{gen}} = \frac{v_{\text{gen}}}{r_{\text{gen}}} \]
Side issue: setting up a relationship for later.

\[ V_{in} = \frac{R_{inA}}{R_{inA} + R_{gen}} V_{gen} \]

- or -

\[ I_{i} = \frac{V_{gen}}{R_{gen}} \]

\[ V_{in} = \frac{R_{i}}{R_{i} + R_{inA} + R_{gen}} \]

\[ V_{in} = R_{i} \cdot I_{i} = \frac{(R_{gen} + R_{inA})}{R_{gen}} V_{gen} \]

\[ = \frac{R_{gen} \cdot R_{inA}}{R_{gen} + R_{inA}} \frac{V_{gen}}{R_{gen}} = \frac{R_{inA}}{R_{gen} + R_{inA}} \cdot V_{gen} \]

Learn to recognize that \[ R_{i} \cdot I_{i} = \frac{V_{gen} \cdot R_{inA}}{R_{inA} + R_{gen}} \]
We are working this problem for 2 reasons:

1) to get the answer

2) to review nodal analysis & transfer functions from ECE221BC. Please review

Easier if we write:

\[ Y_i = \frac{1}{R_i} + A C_{be} = G_i + A C_{be} \]

\[ Y_f = \frac{1}{R_{eg} + A C_{e}} = G_{eg} + A C_{e} \]

\[ Y_f = A C_{eb} \]

Please note: only on this problem will I show all details of Nodal Analysis. Please review ZABC tonight.
Always solve circuits using Kirchhoff's current laws (not KVL)
do \( \Sigma I = 0 \) at each node with an unknown voltage.

\[ \Sigma I = 0 \quad @ \quad V_{in} \]

\[-I_i + V_i \times Y_i + (V_i - V_0) \times Y_f = 0 \]

\[ \Rightarrow V_i \times (Y_i + Y_f) + V_0 \times (-Y_f) = I_i \]

\[ \Sigma I = 0 \quad @ \quad V_{out} \]

\[ g_m \times Y_i + (V_0 - V_i) \times Y_f + V_0 \times Y_L = 0 \]

\[ \Rightarrow V_i \times (g_m - Y_f) + V_0 \times (Y_f + Y_L) = 0 \]

- Combine - or at least organize these -

\[(Y_i + Y_f) \times V_i + (-Y_f) \times V_0 = I_i \]

\[(g_m - Y_f) \times V_i + (Y_f + Y_L) \times V_0 = 0 \]

This can be written in matrix form if we prefer:

\[
\begin{bmatrix}
Y_i + Y_f & -Y_f \\
g_m - Y_f & Y_f + Y_L
\end{bmatrix}
\begin{bmatrix}
V_i \\
V_0
\end{bmatrix}
= \begin{bmatrix}
I_i \\
0
\end{bmatrix}
\]

Use your favorite method to solve
- one method is Cramer's rule:
\[ V_0 = \begin{vmatrix} y_i + y_f & I_i \\ g_m - y_f & 0 \\ y_i + y_f & -y_f \\ g_m - y_f & y_f + y_f \end{vmatrix} = \frac{N}{D} \]

\[ N = -I_i (g_m - y_f) = -I_i (g_m - A(1 - \frac{C6}{g_m})) \]

\[ D = (y_i + y_f)(y_f + y_f) + y_f(g_m - y_f) \]

\[ = y_i y_f + y_i y_f + y_f y_f + y_f y_f + y_f g_m - y_f y_f \]

\[ = y_i y_f + y_i y_f + y_f y_f + y_f g_m \]

\[ = (G_i + A C6) A C6 + (G_i + A C6) (G_{6g} + A C6) \]

\[ + A C6 (G_{6g} + A C6) + A C6 g_m \]

as you multiply out, organize in group \( A^0, A^1, A^2, \) etc.

\[ D = G_6 G_{6g} \]

\[ + A \left[ G_6 C6 + G_6 C6 + G_{6g} C6 + G_{6g} C6 + C6 C6 \right] \]

\[ + A^2 \left[ C6 C6 + C6 C6 + C6 C6 \right] \]

now we must organize \( D \) into the following form:

\[ D = \mu \cdot (1 + a_1 A + a_2 A^2 + \cdots) \]
Do this by dividing each term in D by \( G_i \).

In other words, multiplying by \( R_i \cdot \text{Reig} \):

\[
D = \frac{1}{R_i \cdot \text{Reig}} \left[ 1 + a_1 A + a_2 A^2 \right]
\]

Where

\[
a_1 = R_i \cdot \text{Reig} \cdot C_{i6} + R_i \cdot C_{i6} \cdot C_{iC6} + R_i \cdot C_{iC6} + R_i \cdot C_{i6} \cdot C_{iC6} + R_i \cdot \text{Reig} \cdot C_{i6}
\]

\[
= R_i \cdot C_{i6} + R_i \cdot \text{Reig} \cdot C_{iC6} + R_i \cdot C_{iC6} + R_i \cdot (1 + 9m \cdot \text{Reig}) \cdot C_{iC6}
\]

\[
a_2 = R_i \cdot \text{Reig} \cdot (C_{i6} C_{i6} + C_{i6} C_{iC6} + C_{iC6} C_{iC6})
\]

So we can write it all up:

\[
V_0 = -\frac{I_i \cdot \text{Reig} \cdot (1 - aC_{iC6} / 9m)}{R_i \cdot \text{Reig}} \cdot \frac{1 + a_1 A + a_2 A^2}{1 + a_1 A + a_2 A^2}
\]

but, from before, \[I_i \cdot \text{Reig} \cdot R_i \cdot \text{Reig} = -9m \cdot \text{Reig} \cdot R_i \cdot \text{Reig}\]

\[
= -9m \cdot \text{Reig} \cdot \text{Vgen} \cdot \frac{R_i A}{R_i A + \text{Vgen}}
\]
$$\frac{V_o}{V_{gen}} = \frac{R_i A}{(1 + 9m \text{ Re}_g) (1 + A_1 A + A_2 A^2)}$$

\[A_1 = R_i C_{6e} + R_i (1 + 9m \text{ Re}_g) C_{6e} + \text{ Re}_g (C_{6e} + C_{6c})\]

\[A_2 = R_i \text{ Re}_g (C_{6e} C_{6e} + C_{6e} C_{6c} + C_{6e} C_{6c})\]

Zero = \(-\frac{C_{6e}}{9m} - \text{ negative or RNP zero}\)

This has been slow because all steps of nodal analysis have been shown - just this once

* Note that the answer contains both the DC gain and the frequency response.

* Since we know easier methods (from 13.74) to find DC gain, we will often drop constants during AC analysis.

The answer, though complicated, makes perfect physical sense, and you will become familiar with it.
Separated pole approximation:

\[
\frac{V_0(\lambda)}{V_{\text{gen}}(\lambda)} = \frac{V_0}{V_{\text{gen/mid-band}}} \frac{1 + b_1 A}{1 + a_1 A + a_2 A^2}
\]

- Consider a system with 2 real poles:

\[(1 + A \tau_1)(1 + A \tau_2) = 1 + A(\tau_1 + \tau_2) + A^2 \tau_1 \tau_2
\]

- Now suppose that \(\tau_1 \gg \tau_2\):

\[(1 + A \tau_1)(1 + A \tau_2) \approx 1 + A \tau_1 + A^2 \tau_1 \tau_2
\]

This means we can approximately factor \(1 + a_1 A + a_2 A^2:\)

\[(1 + a_1 A + a_2 A^2) \approx (1 + a_1 A)(1 + \frac{a_2}{a_1} A)
\]

If \(a_1 \gg \frac{a_2}{a_1}\)

\[
\frac{V_0(\lambda)}{V_{\text{gen}}(\lambda)} \approx \frac{V_0}{V_{\text{gen/mid-band}}} \frac{1 + b_1 A}{(1 + a_1 A)(1 + \frac{a_2}{a_1} A)}
\]

If \(a_1 \gg \frac{a_2}{a_1}\)

the dominant pole \(\tau_1 \approx \frac{1}{a_1}\)

secondary pole \(\tau_2 \approx \frac{1}{2} \frac{a_2}{a_1}\)
The Miller Approximation.

The Miller approximation is not a good way to solve transfer functions. It is a tool for comprehension.

\[ \text{Gain} = AV = \frac{V_0}{V_{in}}, \text{not} \frac{V_v}{V_{gen}} \]

- Analyze first the section to the right of +
- note for now we are ignoring the output impedance

that is why it is an approximation

\[ I_{in} = ACf (V_{in} - V_{out}) = ACf (V_{in} + AV_{in}) \]

\[ = ACf (1 + AS) V_{in} \]

\[ I_{in} = \frac{1}{Z_i} = \frac{1}{ACf (1 + AS)} \]

\[ Z_i = Dc_{in} \]
\[ R_{in} \quad C_{miller} = C_f (1 + AR) \]

Note the Miller-multiplied capacitance.

\[ \frac{V_o}{V_{gen}} = \frac{V_o}{V_{gen}} \frac{1}{1 + AR \cdot C_{miller}} \]

\[ R_\text{C} (1 + AR) \]

Refer back to the common emitter analysis.

You will see \( \frac{C_{eb} (1 + g_m R_{c} \text{ Gain})}{R_i} \).

**e.g.** \( \frac{C_{eb} (1 + AR)}{R_i} \).
Now use the miller approximation to understand the common emitter response.

Low frequency gain from \( V_i \) to \( V_o \) is \(-9 \times R_{\text{REG}}\).

Approximate: replace \( C_{\text{C6}} \) with \( C_{\text{MILLER}} \).

2 poles:

\[
\frac{1}{2\pi f_{p1}} = \frac{1}{R_i C_{\text{be}}} + \frac{1}{R_i C_{\text{C6}} (1 + 9 \times R_{\text{REG}})}
\]

\[
\frac{1}{2\pi f_{p2}} = \frac{1}{R_{\text{REG}} C_i}
\]

\( f_{p1} \) is wrong by a little. \( f_{p2} \) is very wrong.
Exact Solution

\[ a_1 = R_i C_{6b} + R_i (1 + g_m R_{eq}) C_{6b} + C_{6b} R_{eq} + C_{6c} R_{eq} \]

\[ a_2 = R_i R_{eq} \left( C_{6a} C_{6b} + C_{6a} C_{4} + C_{6c} C_{6c} \right) \]

and if we can use the S.P.A.:

\[ \frac{1}{\pi f_{pi}} = a_1, \quad \frac{1}{\pi f_{pi}} = a_2/a_1 \]

Miller approx: good for quick estimate of 1st pole

Miller approx: very bad for 2nd pole

misses zero completely.
Further Comments regarding Miller Approx:

- We have used Miller Approx only as a way to help us understand the results of analysis.

- We will not use Miller approx. to solve problems directly.

  → I have found this causes confusion.
Frequency Response of Common-Source Stage

\[ I_{in} = \frac{V_{gen}}{R_{gen}} \]

\[ R_i = R_{g1} \parallel R_{g2} \parallel R_{gen} \]

\[ R_{gs} = R_{g1} \parallel R_{g2} \parallel R_{gen} \]

\[ R_{cg} = R_{g1} \parallel R_{g2} \parallel R_{gen} \]
This is clearly the same problem as the bit problem!

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1 + 6.4}{1 + 9.14 + 9.25 4^2}
\]

\[b_1 = -g_m E_g d\quad \text{zero in R.H.P.}\]

\[a_1 = R_i E_s + R_i E_g d \left(1 + g_m R_m p\right) + g_d R_d p + g_c E_c p\]

\[a_2 = R_i R_m \left(E_s E_g d + E_s E_c + E_g E_c\right)\]

As with the bit case, there is a dominant pole at \(f = 1/2\pi a_1\), a second pole at \(f = a_2/2\pi a_1\), and a right-half-plane zero at \(f \approx 1/2\pi b_1\).