ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. December 6, 2011

Do not open exam until instructed to.
Open notes, open books, etc
You have 3 hrs.

Use all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***
***Hint: Stop and think before doing complicated calculations.* For some problems, there is an easier way.**

Problem	Points Received	Points Possible
1a		5
1b		5
1c		5
1d		5
1e		5
1f		5
2		10
3a		10
3a		5
4		10
5		10
6		5
7a		10
7b		10

Name: Solution

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\Gamma_{in}^{\,2}} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-\Gamma_{out}^{\,2}} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left[K - \sqrt{K^2-1}\right] \text{if } K > 1$$

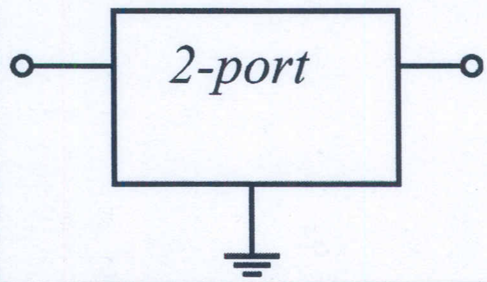
$$G_{MS} = \frac{|S_{21}|}{|S_{12}|} \cdot \text{if } K < 1$$

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta = \det[S]$$

LPh

Problem 1, 30 points
Gain definitions

At a signal frequency of 1 GHz, a two-port has $S_{11} = 0.7071$, $S_{12} = 0.1$, $S_{21} = 10$ and $S_{22} = 0.316$, as defined with a 50 Ohm impedance reference.



part a, 5 points

If the 2-port were directly connected to 50 Ohm load, and a 50 Ohm generator with 1 mW available power, what would be the power dissipated in the load ?

FIRST, is it potentially unstable ?

FOR part B.

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{21} S_{12}|} = \frac{1.003}{2(0.1)(10)} \approx \underline{\underline{0.5}}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = \cancel{(1+j}\,0.7766$$

potentially unstable

Here the gain is the insertion gain S_{21}

$$P_{Load} = |S_{21}|^2 \cdot 1\,mW = 100 \cdot 1\,mW = \underline{\underline{100\,mW}}$$

part b, 5 points

If you were to first stabilize (if necessary) and then impedance match the input and output to 50 Ohms, what would be the gain S_{21} of the resulting amplifier ?

Calculation of k is first needed

- SEE previous page

- k < 1 → potentially unstable

- we must first stabilize then match.

Gain, if we stabilize the minimum amount, will be MSG

Transistor S-parameters

$$MSG = \frac{|S_{21}^T|}{|S_{12}^T|} = \frac{10}{0.1} = 100 \quad \text{in units of } \frac{P_{out}}{P_{in}}$$

After matching & stabilizing, S_{ij}^A ← A = amplifier.

$$|S_{21}^A|^2 = \text{transistor MSG} = 100$$

$$\boxed{|S_{21}^A| = 10}$$

$$\bar{S} = \begin{bmatrix} 0.7071 & 0.1 \\ 10 & 0.316 \end{bmatrix}$$

part c, 5 points

If you were to load the 2-port, without matching or stabilization elements, with a 500 Ohm load, is it possible to select a generator impedance which would cause the 2-port to oscillate ?

0.5 $\left[\Gamma_L = \dfrac{500/50 - 1}{500/50 + 1} = \dfrac{10-1}{10+1} = \dfrac{9}{11} = \ldots \right.$

2 $\left[\Gamma_{in} = S_{11} + \dfrac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \right.$

$$= 0.7071 + \frac{10 \cdot 0.1 \cdot 9/11}{1 - 0.316(9/11)}$$

0.5

$$= 0.7071 + 1.103$$

$$= 1.81$$

2 [$|\Gamma_{in}| > 1$, so some source impedances will result in oscillation

$$\begin{bmatrix} 0.7071 & 0.1 \\ 10 & 0.316 \end{bmatrix}$$

part d, 5 points

If you were to load the 2-port, without stabilization elements, in 50 Ohms, and then impedance-match the input to a 50 Ohm generator with 1 mW available source power, what would be the power in the load ?

Match on input, not on output

→ [operating Gain of transistor] is gain of amplifier.

$\Gamma_L = 0$ (50 Ω load)

$$\Gamma_{in}^T = S_{11}^T + \frac{S_{12}^T S_{21}^T \overbrace{\Gamma_L}^{0}}{1 - S_{22}^T \Gamma_L} = S_{11}^T$$

$$G_p^T = \frac{1}{1 - |\underbrace{\Gamma_{in}}_{S_{11}^T}|^2} \, |\underbrace{S_{21}^T}_{10}|^2 \, \frac{1 - |\overbrace{\Gamma_L}^{0}|^2}{|1 - \underbrace{\Gamma_L}_{0} S_{22}^T|^2}$$

$$= \frac{1}{1 - |S_{11}^T|^2} |S_{21}^T|^2 = \frac{100}{1 - 1/2} = 200$$

$G_p^T = |\hat{S}_{21}|^2 = 200$

power in Load = 1 mW · 200 = $\underline{\underline{200 \text{ mW}}}$

0.7071 0.1

10 0.316

part e, 5 points

If you were to take the 2-port, without stabilization elements, connect the input directly to a 50 Ohm generator with 1 mW available source power, and then impedance-match the output to a 50 Ohm load, what would be the power in the load ?

Match on output, not on input.

Circuit gain $|S_{21}^A|^2$ = transistor <u>available gain</u>

$$G_A^T = \frac{1-|\cancel{\Gamma_s}|^2}{|1-\Gamma_s S_{11}^T|^2} \cdot |S_{21}^T|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

$\Gamma_s = 0$ (50Ω) →

$$\Gamma_{out} = S_{22}^T + \frac{S_{12}^T S_{21}^T \Gamma_s}{1-S_{11}^T \Gamma_s} = S_{22}^T$$

$$G_A^T = 1 \cdot (100) \frac{1}{1-(0.316)} = \frac{100}{0.9} = 111.1$$

$$|S_{21}^A|^2 = G_A^T = 111.1$$

power in load = 1 mW · 111.1 = <u>111.1 mW</u>

$$\begin{bmatrix} 0.7071 & 0.1 \\ 10 & 0.316 \end{bmatrix}$$

part f. 5 points

If you were to take the 2-port, without stabilization elements, connect the input directly to a 500 Ohm generator with 1 mW available source power, connect the output directly to a 50Ω load what would be the power in the load ?

$$\frac{P_L}{P_{AVG}} = G_T = \text{transducer gain}$$

$$\Gamma_L = 0 \qquad \Gamma_S = \frac{10-1}{10+1} = 9/11$$

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_S\Gamma_L|^2}$$

$$= \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-\Gamma_S S_{11}|^2}$$

9/11 0.7071

$$= \cancel{\frac{(100)(0.33)}{(0.421)^2}} \cancel{= 281.5}$$

$$\cancel{P_{load} = 281.5 \text{ mW}}$$

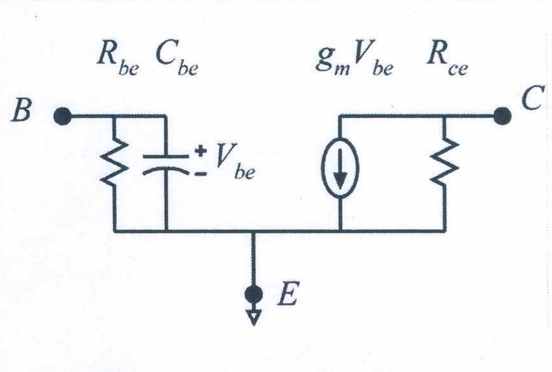
= 186 mW

5

Problem 2, 10 points
Gain definitions

A bipolar transistor is shown at right. R_{be}= 1000 Ohms, g_m=100 mS, R_{ce}=10,000 Ohms, C_{be}=100 fF.

The device is impedance-matched to a 50 Ohm generator and a 50 Ohm load at a 10 GHz signal frequency. Find the **magnitude** of S_{21} of the resulting **amplifier** at that frequency.



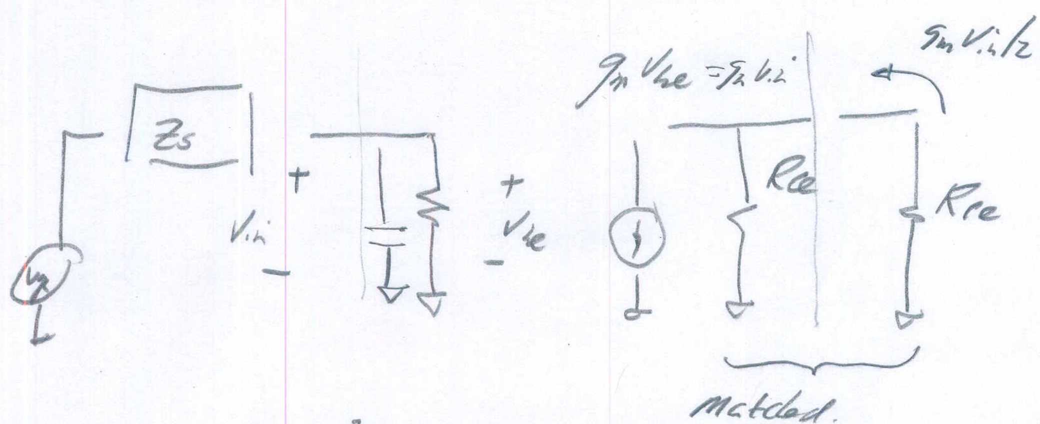
device is unilateral $\rightarrow$ $S_{12} = 0$

$$\rightarrow G_{max} = \frac{1}{1-|S_{11}|^2} \, |S_{21}|^2 \, \frac{1}{1-|S_{22}|^2}$$

but S_{11} is hard to compute - try another approach

If impedance matched on both ports

$$\text{Power Gain} = G_{max} = \frac{P_{recd}}{P_{in}}$$



$$P_{in} = \frac{V_{in}^2}{R_{be}}$$

$$P_{out} = (g_m V_{in}/2)^2 \cdot R_{ce}$$

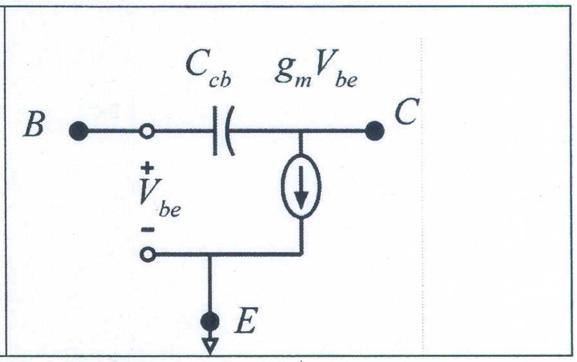
$$\frac{P_{out}}{P_{in}} = \frac{(g_m/2)^2 R_{ce}}{1/R_{be}} = \frac{g_m^2}{4} R_{ce} R_{be} = \frac{(0.1\,S)^2}{4} 1k\Omega \cdot 10k\Omega = \underline{25{,}000}$$

ckt power gain $= |S_{21}^A|^2 = 25000 \longrightarrow |S_{21}^A| = \underline{158}$

Problem 3, 15 points
Gain definitions, computation of Y-parameters

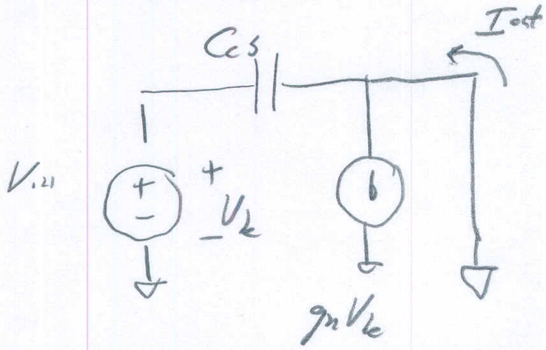
A bipolar transistor is shown at right. $C_{cb}=$ 100 fF, g_m=100 mS.

It can be shown that $S_{21}/S_{12}=Y_{21}/Y_{12}$

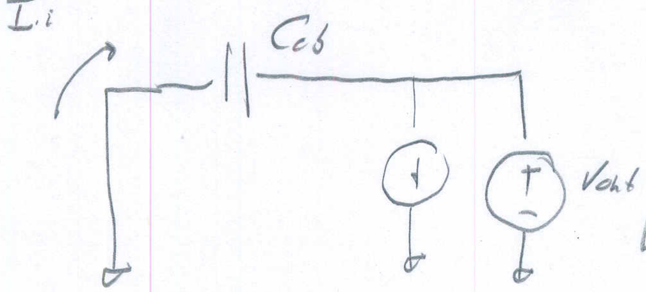


part a, 10 points

compute Y_{21} and Y_{12}



$$I_{out} = g_m V_{in} - j\omega C_{cb} V_{in}$$
$$= g_m - j\omega C_{cb}$$
$$\boxed{Y_{21} = g_m - j\omega C_{cb}}$$



$$I_{in} = -j\omega C_{cb} \cdot V_{out}$$
$$\boxed{Y_{12} = -j\omega C_{cb}}$$

part b, 5 points

The transistor is first stabilized and then impedance-matched to 50 Ohm generator and load. Find the resulting amplifier gain magnitude $\| S_{21} \|$

Amp.fier GAIN = MSG = $\frac{|S_{21}^T|}{|S_{12}^T|} = \frac{|Y_{21}^T|}{|Y_{12}^T|}$

power

$|S_{21}^A|^2 =$

$$= \frac{|\underbrace{g_m}_{0.15} - j\underbrace{\omega C_{cb}}_{6.28 \cdot 10^{-3} S}|}{|-j\omega C_{cb}|} \cong \frac{1}{|6.28 \cdot 10^{-3}|}$$

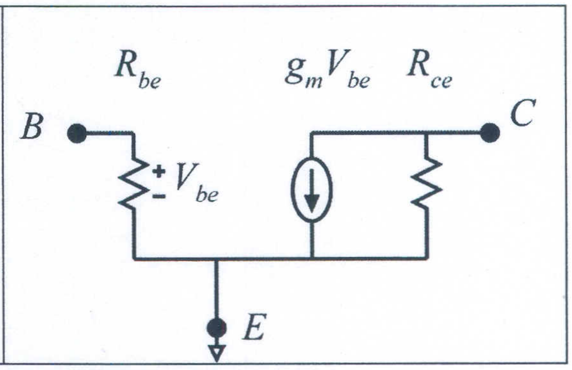
$= 159 = (10^{3}/2\pi)$

$$|S_{21}^A| = \sqrt{159} = 12.6$$

Problem 4, 10 points
Computation of S-parameters

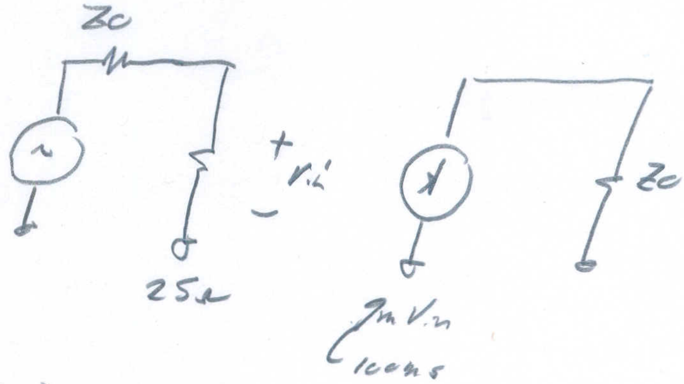
A bipolar transistor is shown at right. R_{be} = 25 Ohms, g_m=100 mS, R_{ce} = infinity Ohms.

Find S_{11} and S_{21}



$$S_{11} = \frac{Z_i/Z_o - 1}{Z_i/Z_o + 1} = \frac{1/2 - 1}{1/2 + 1} = -1/3$$

$$S_{21} = \frac{2V_o}{V_{ga}}\Bigg|_{Z_L = Z_o = Z_{ga}}$$



$$= 2 \cdot \frac{25}{25+50} \cdot (-\underset{100mS}{g_m}\,\underset{50\Omega}{Z_o})$$

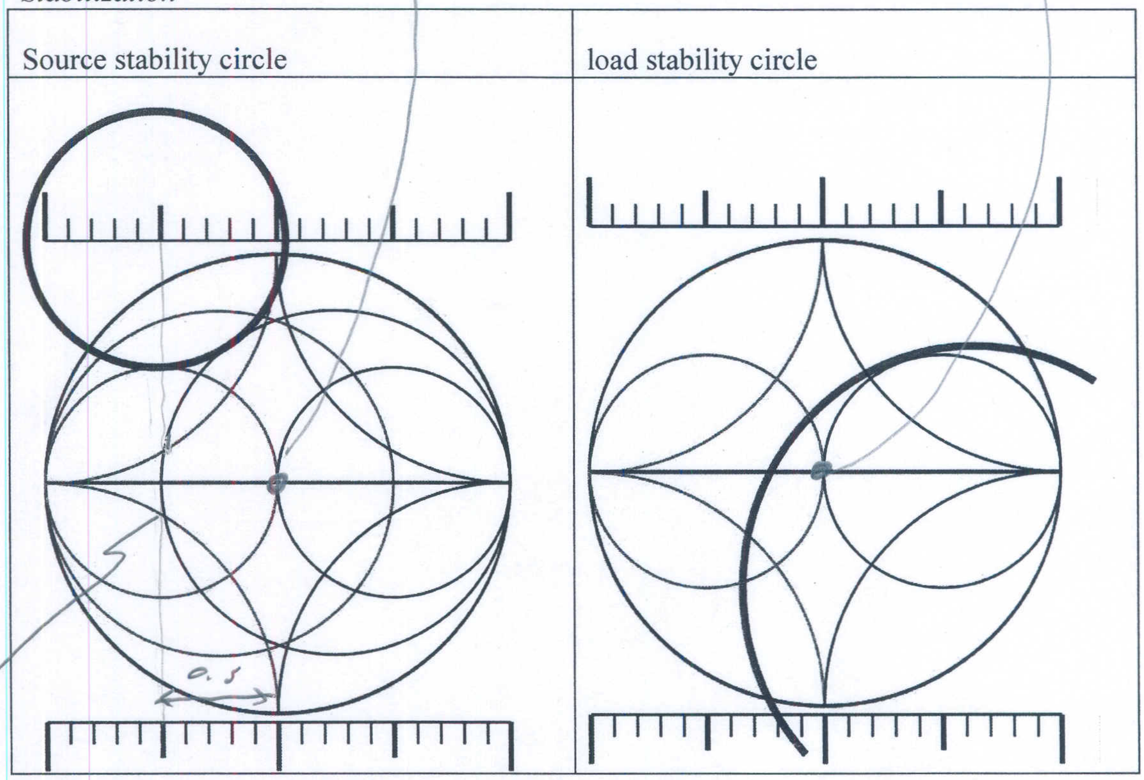
$$= 2 \cdot \frac{1}{3} \; (-5) = \frac{-10}{3}$$

$$\boxed{S_{21} = -\frac{10}{3}.}$$

stable Here because $|S_{22}| < 1$

stable here because $|S_{11}| < 1$

Problem 5, 10 points
Stabilization



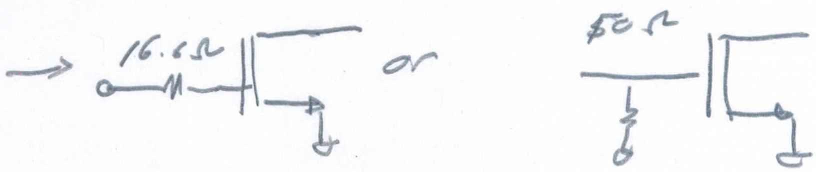
A MOSFET in common-source mode has $S_{11} = 0.5$ and S_{22}=0.3. Source and load stability circles are as shown. Draw **3*** circuit diagrams, giving resistor values, of methods of stabilizing the transistor.

Source: the indicated circle has $\frac{R}{Z_0} = \frac{1-\Gamma}{1+\Gamma} = \frac{1-1/2}{1+1/2} = 1/3$

i.e. $R = \frac{50\Omega}{3} = 16.\bar{6}\Omega$

the stability circle touches the $R = 16.\bar{6}\Omega$ circle

and the $G = (1/(50\Omega))$ circle



Load

$\Gamma_L = 0$ is stable because $|S_{11}| < 1$

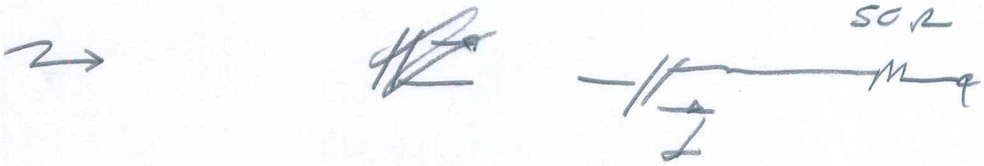
from $\Gamma_{in} = S_{11} + \dfrac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$

so inside of stability circle is stable

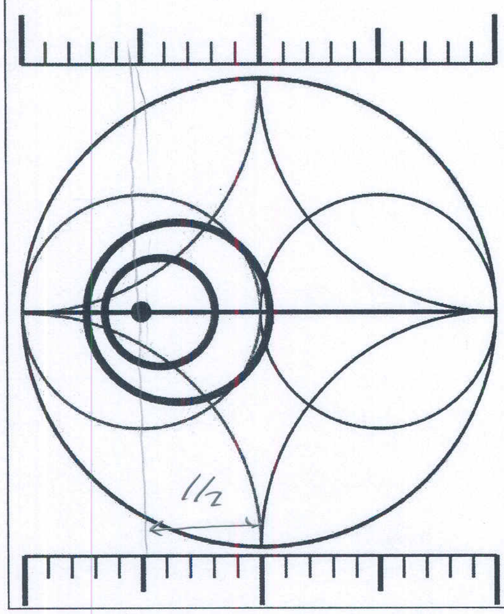
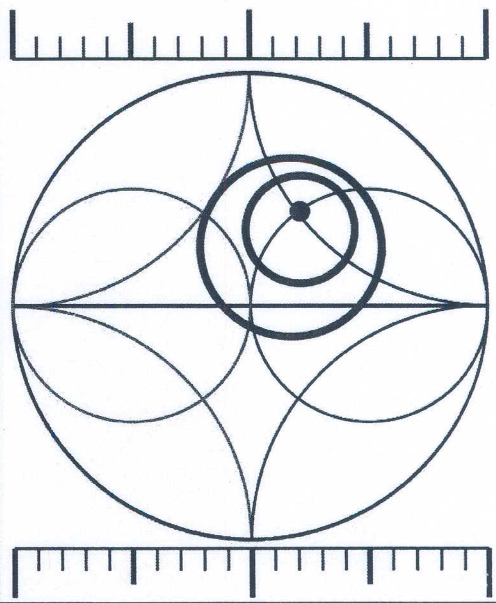
Circle touches the 50Ω series resistance circle

so 50Ω in series will stabilize

A short-ckt on the output is potentially unstable so parrallel stabilization does not work.



Problem 6, 5 points
Gain circles

Available gain circles	Operating gain circles
	

At 10 GHz, a MOSFET in common-source mode operating and available gain circles as shown. Find the optimum generator and load impedances (in complex Ohms).

Available gain - source match

$$\Gamma_s^{opt} = -1/2 \rightarrow Z_{opt} = Z_0 \cdot \frac{1-\Gamma}{1+\Gamma} = Z_0/3 = \underline{16.7\,\Omega} + \underline{j0\,\Omega}$$

<u>operating gain - load match</u>

optimum impedance, as shown, is $\frac{Z_L}{Z_0} = 1 + j \cdot 1$

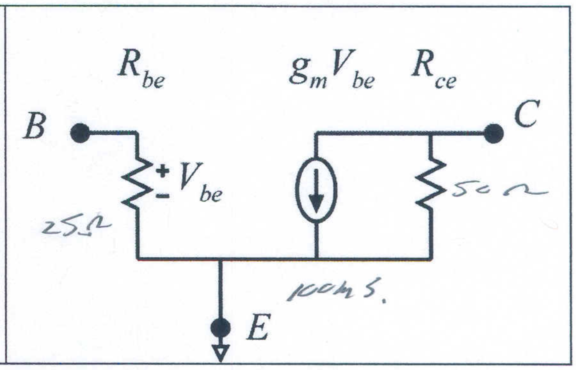
$$Z_{L,opt} = Z_0 + jZ_0 = \underline{50\,\Omega + j\,50\,\Omega}$$

Problem 7, 20 points

Amplifier design

A bipolar transistor is shown at right. R_{be} = 25 Ohms, g_m =100 mS, R_{ce} = 50 Ohms.

The amplifier is to be impedance-matched, using ONLY lumped-element **series** inductors and shunt capacitors.

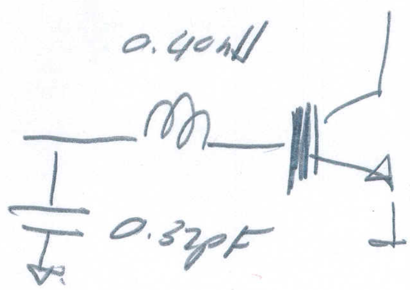


part a, 10 points.

Design on the attached Smith chart the input matching network. Give values of all elements.

output needs no matching network

input → $Z_{in}/Z_0 = 25/50\Omega = 1/2$



point "A" ⇒ $Z/Z_0 = 1/2 + j0$

point "B" = $Z/Z_0 = 0.5 + j0.5$

⟶ must add series element $\frac{Z}{Z_0} = j0.5 \rightarrow j\omega L = j0.5 \cdot 50\Omega$

$$L = \frac{0.5 \cdot 50\Omega}{2\pi(10\text{ GHz})} = 0.40\text{nH}$$

point "B" ⇒ $Y/Y_0 = 1.0 - j1.0$

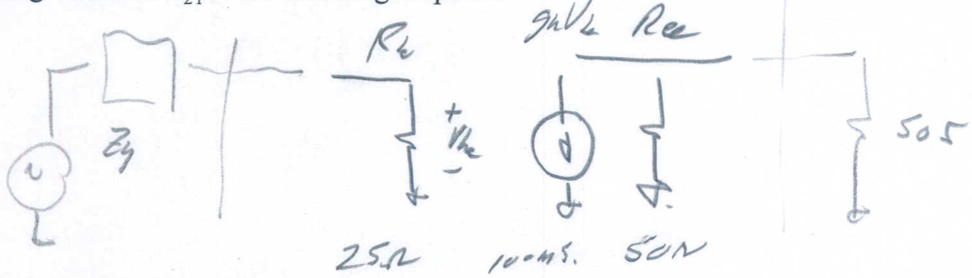
point "C" ⇒ $Y/Y_0 = 1.0 + j0$

→ must add shunt element $Y/Y_0 = j1.0 \rightarrow j\omega C = j1.0/50\Omega$

$$\Rightarrow C = \frac{1}{50\Omega(2\pi)(10\text{ GHz})} = 0.318\text{pF}$$

part b, 10 points.

Determine the magnitude of S_{21} of the resulting amplifier



what is the transistor G_{max}?

Match output & compute P_{out}/P_{in}, because with a matched input, $P_{in} = P_{AVG}$.

$$P_{in} = V_{be}^2 / R_{be}$$

$$P_{out} = (g_m V_{be})^2 (1/2)^2 \cdot \underbrace{50\,\Omega}_{R_{ce} = R_L}$$

$$\frac{P_o}{P_i} = \frac{(g_m/2)^2 R_{ce}}{1/R_{be}} = \frac{g_m^2 R_{ce} R_{be}}{4} = \underline{\underline{3.125}}$$

(1 mS, 50 Ω, 25 Ω)

$$|S_{21}^{Amp}|^2 = G_{max}^{transistor} = 3.125$$

$$|S_{21}^{Amp}| = \sqrt{3.125} = 1.77$$

NAME	TITLE	DWG. NO.
		DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

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