ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 8, 2011

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

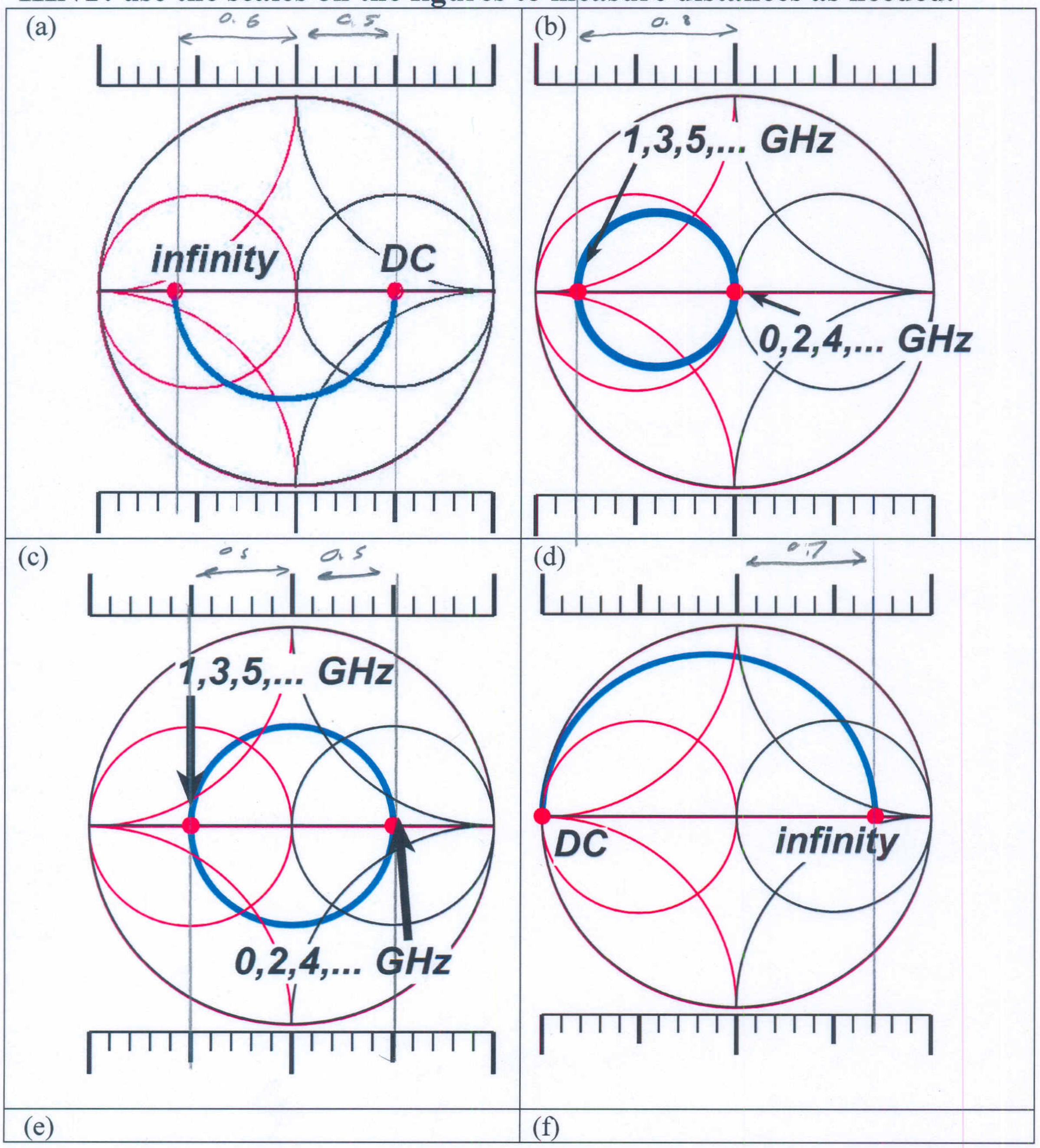
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

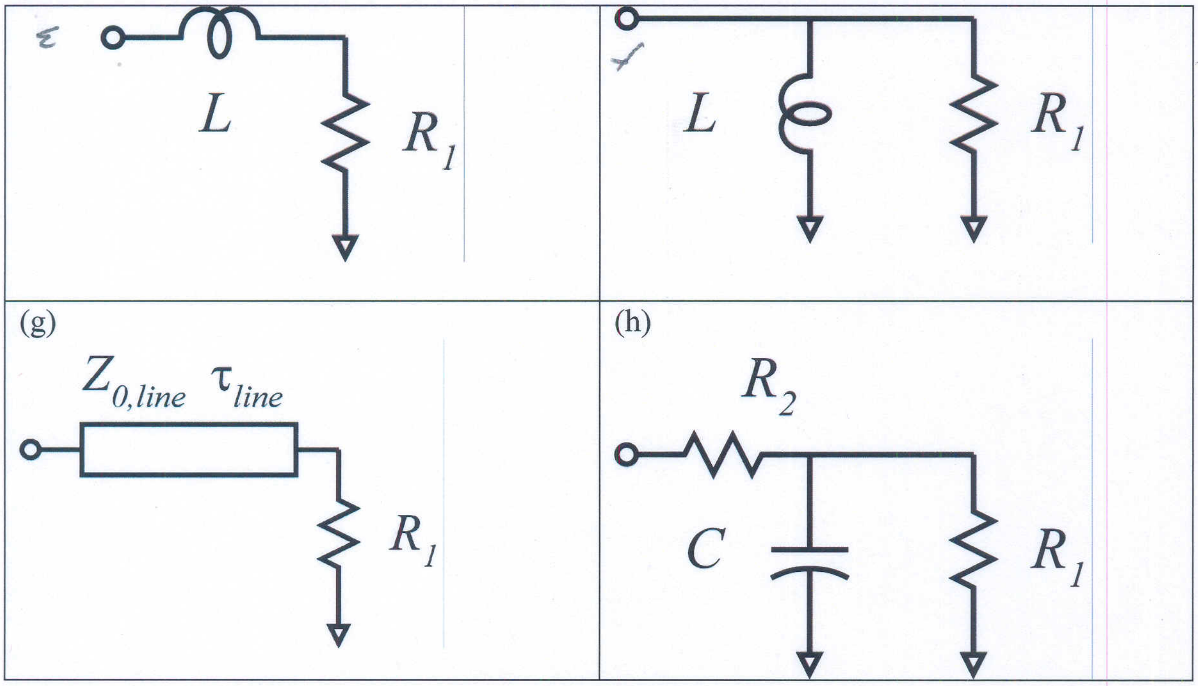
Problem	Points Received	Points Possible
1		20
2a		10
2b		10
2c		10
3a		15
3b		10
4		25
total		100

Name: Solutions.

Problem 1, 20 points
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.





a) @ DC, $\Gamma_L = 0.5 \rightarrow R_L = 50\Omega\,(1+0.5)/(1-0.5) = 150\Omega$ 1pt.

@ $f \rightarrow \infty$, $\Gamma_L = -0.6 \rightarrow R_L = 50\Omega\,(1-0.6)/(1+0.6) = 12.5\Omega$ 1pt

$\rightarrow$ circuit H with $R_2 = 12.5\Omega$, $R_1 = 150 - 12.5\Omega = 137.5\Omega$ 1pt 1pt 1pt

b) @ DC, 2, 4... GHz, $Z_{in} = 50\Omega$

@ 1, 3, ... GHz $Z_{in} = 50\Omega(1-0.8)/(1+0.8) = 5.55\Omega$ 1

$\rightarrow$ Quarter-wave line $Z_{in} Z_{load} = Z_{line}^2 = 50\Omega \cdot (5.55\Omega)$ 2

$\rightarrow Z_{line} = 16.66\Omega$

line is $\lambda/4$ @ 1 GHz $\rightarrow \tau = 1/4$ ns 1

c) @ DC, 2, 4 GHz, $Z_{in} = 50\Omega(1.5)/0.5 = 150\Omega$ 1

@ 1, 3, 5.. GHz $Z_{in} = 50\Omega\,(1-0.5)/(1+0.5\Omega) = 16.6\Omega$ 1

$\rightarrow$ Quarter wave line $Z_{in} Z_{load} = Z_{line}^2 =$

$Z_{line} = (150\Omega \cdot 16.6\Omega)^{1/2} = 50\Omega$ 2

50Ω line, 1/4 ns long, loaded by 150Ω

1

Match each Smith Chart with each circuit, and give all resistor values, and all transmission line delays and characteristic impedances. The charts all use 50 Ohm normalization:

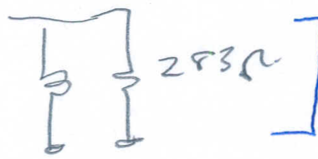
Smith chart (a). Circuit= H . Component values= $R_1 = 137.5\Omega$, $R_2 = 12.5\Omega$

Smith chart (b). Circuit= G . Component values= $R_1 = 50\Omega$, $Z_{o,line} = 16.7\Omega$, $\tau = 1/4\,ns$.

Smith chart (c). Circuit= G. . Component values= $R_1 = 150\Omega$, $Z_{line} = 50\Omega$, $\tau = 1/4\,ns$

Smith chart (d). Circuit= f . Component values= $R_1 = 283\Omega$

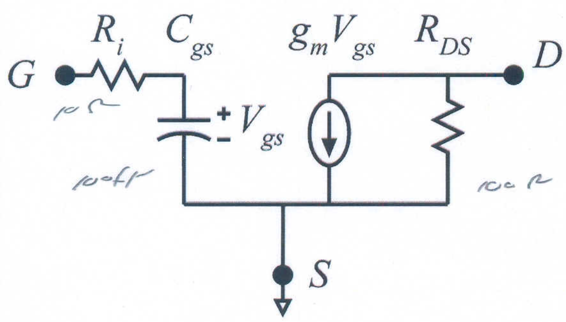
(d) impedance is zero @ dc, is 2 $\dfrac{50\Omega(1+0.7)}{(1-0.7)} = 283\Omega$ @ $f \to \infty$.



3

Problem 2, 40 points

Elementary impedance matching network design.



To the left is the equivalent circuit of a FET. The Transconductance is 100 mS, Ri=10 Ohms, C_{gs}=100 fF, R_{DS}=100 Ohms

Part (a), 15 points.

Using the impedance-admittance charts that have been passed out, design a lumped-element matching network to match the input impedance to 50 Ohms at 10 GHz. Use a series inductor and a shunt capacitor. Give the circuit diagram and the element values.

100fF → $Z = \frac{1}{j\omega C} = 160\Omega \to Z/Z_0 = 3.2$

10Ω → $Z/Z_0 = 10/50 = 0.2$

$Z = 10\Omega - j160\Omega$

$Z/Z_0 = 0.2 - j3.2 = r_A + jx_A = z_A$

2

at point "B", $z_B = 0.2 + j0.4 = r_B + jx_B$ 2

2 $\Delta x = x_B - x_A = 0.4 - (-3.2) = +3.6$

$\Delta X = \Delta x \cdot Z_0 = \Delta x \cdot 50\Omega = 3.6 \cdot 50\Omega = 180\Omega$

2 $\Delta X = \omega L = 180\Omega \to L = 180\Omega / 2\pi f = 2.86nH$

2 @ point "B", $y_B = 1 - j2.0$

@ point "C" $y_C = 1 - j0$

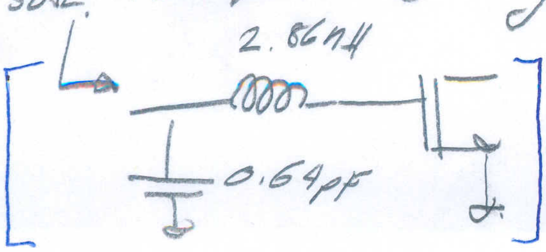
$\Delta y = +2.0 \cdot j$

$\Delta b = 2.0$

$\Delta B = 2.0/50\Omega = 0.040S$

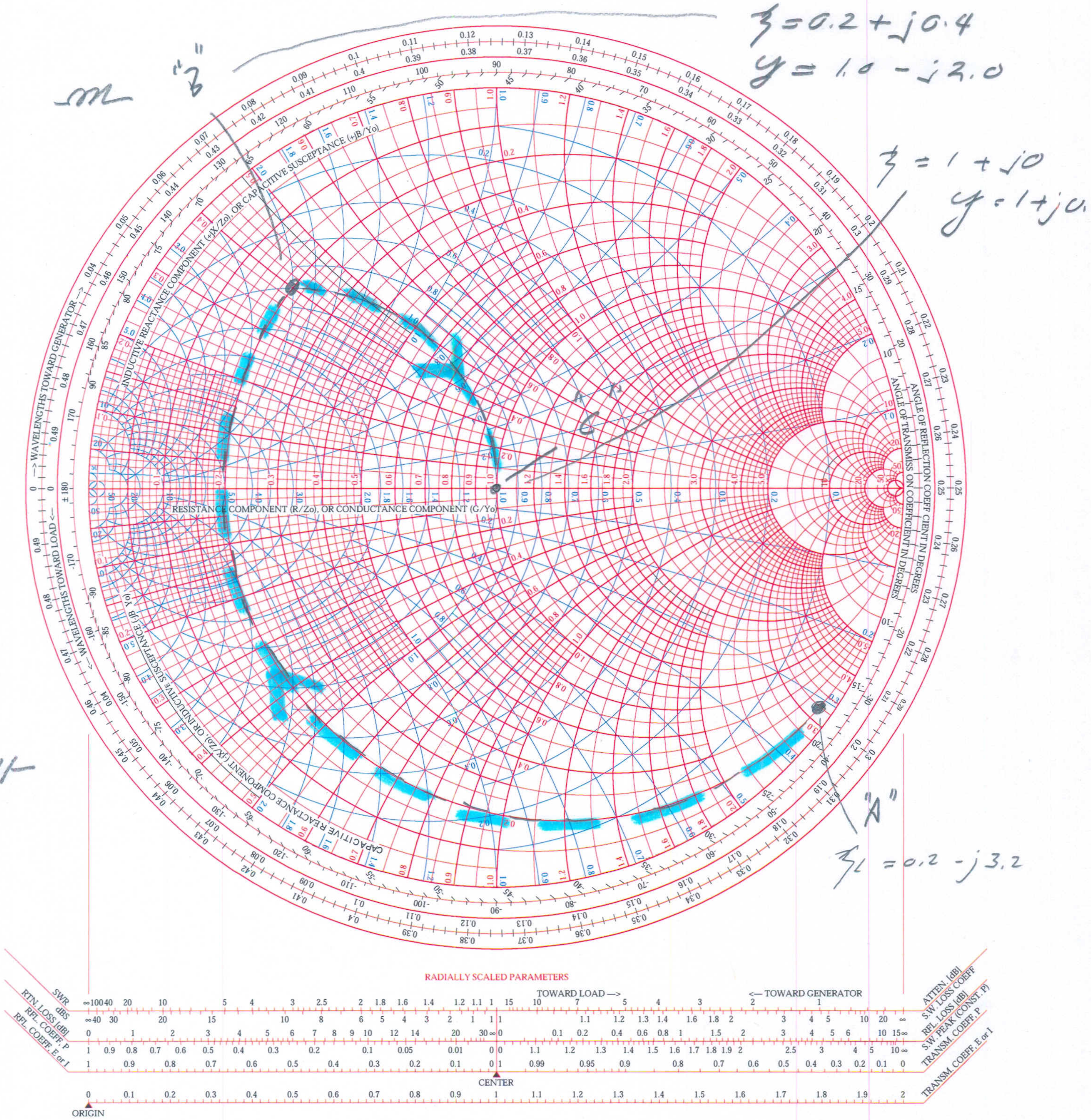
$= \omega C$

2 → $C = \frac{0.040\,S}{2\pi f} = 0.64pF$



NAME	TITLE	DWG. NO.
Solution.	problem 2a	DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Part (b), 15 points.

Using the impedance-admittance charts that have been passed out, design a lumped-element matching network to match the output impedance to 50 Ohms at 10 GHz. Use a shunt inductor and a series capacitor. Give the circuit diagram and the element values.

— point "A" on smith chart - the FET Rout —

100Ω 

$z_L = \frac{100\Omega}{50\Omega} = 2.0$

$y = 0.5 + j0. = g_a + j b_a$ (1)

— point B on smith chart —

(2) $y_B = 0.5 - j\,0.5 = g_b + j b_B$

(2) $\Delta b = b_b - b_a = -0.5$

$\Delta B = \Delta b / Z_0 = -0.5/50\Omega = -0.010\ S = -10\ mS$

(2) $\Delta B = -1/\omega L \rightarrow \quad L = \frac{-1}{\omega \Delta B} = \frac{-1}{2\pi f \Delta B} = \underline{1.59\,nH}$

$z_B = 1 + j1 = r_b + jx_b$

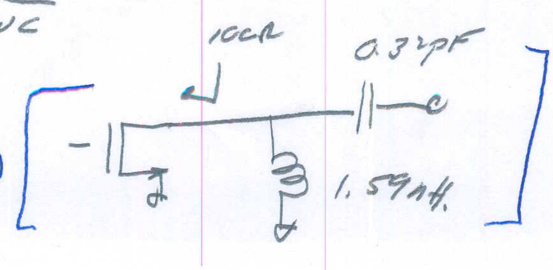
— point C on smith chart

(2) $z_c = 1 + j0 = r_c + j x_c$

(2) $\Delta x = x_c - x_b = -1.0$

$\Delta X = \Delta x \cdot Z_0 = -50\Omega = \frac{-1}{\omega C}$

(2) $\rightarrow C = \frac{1}{2\pi f \cdot 50\Omega} = 0.32\,pF$

(2) 

NAME	TITLE	DWG. NO.
solution	Problem 25	DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

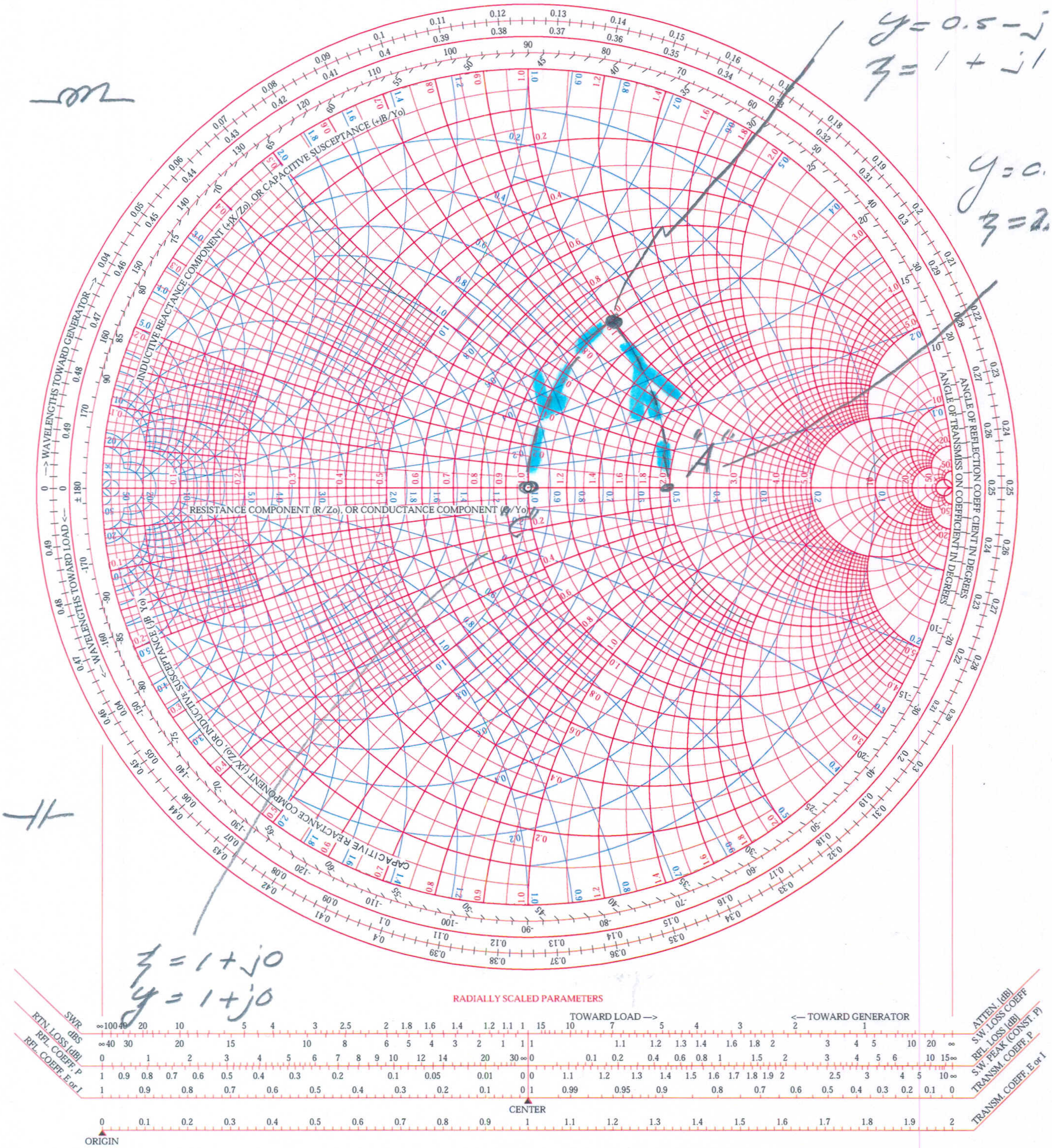
"B"

$y = 0.5 - j0.5$

$z = 1 + j1$

$y = 0.5 + j0$

$z = 2.0 + j0$

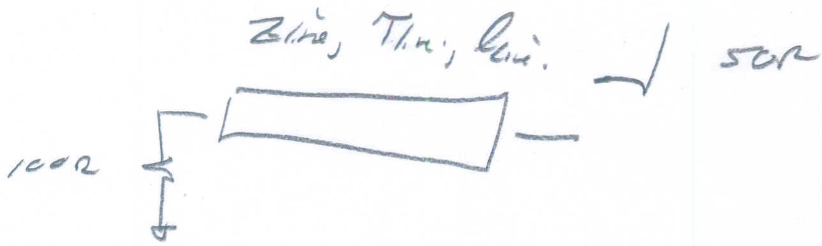


$z = 1 + j0$

$y = 1 + j0$

Part (c), 10 points

Now instead design a quarter-wave line to match the load to 50 Ohms. Find the required line impedance and the physical length.



$$Z_{line} = \sqrt{50\Omega \cdot 100\Omega} = \boxed{70.7\,\Omega}$$

5

$\lambda/4$ @ 10 GHz

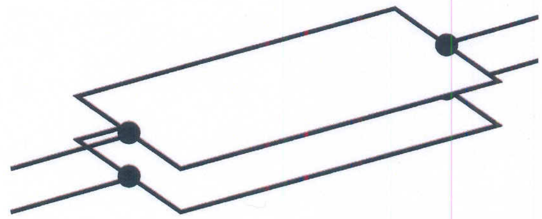
$$\boxed{T_{line} = \frac{1}{4}\,\text{ns}}$$

5

Problem 3, 25 points
Transmission-lines and lumped elements

Part a: 5 points

A transmission line has plates of 300 μm width and 100 μm separation. The line is 1 cm long. The region between the plates has a dielectric constant of 2.0. Neglect the fringing fields at the edges of the plates.



Find the following:
characteristic impedance: 89Ω
propagation velocity: $2.12 \cdot 10^8$ m/s.
propagation delay: 47.1 ps
total inductance: 4.2 nH.
total capacitance: 0.53 pF

$$\sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$Z_o = \frac{377\Omega}{\sqrt{\varepsilon_r}} \cdot \frac{H}{W} = \frac{377\Omega}{\sqrt{2}} \cdot \frac{100\mu m}{300\mu m} = 89\Omega \quad] \; 1$$

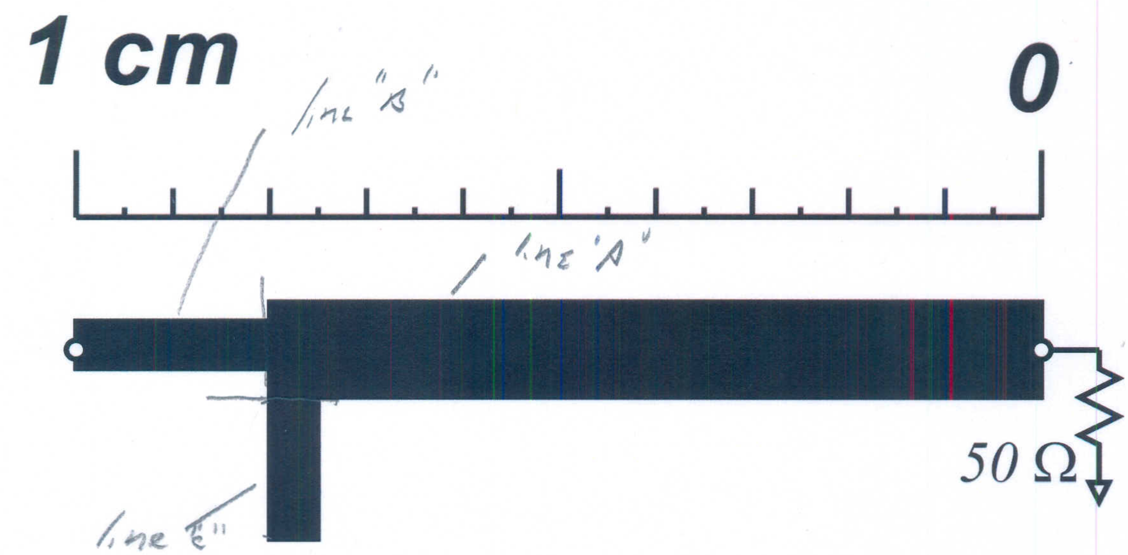
$$v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{2}} = 2.12 \cdot 10^8 \text{ m/s} \quad] \; 1$$

$$T = l/v = 1\text{cm}/v = 47.1\text{ps} \quad] \; 1$$

$$L = Z_o T = 89\Omega \cdot 47.1\text{ps} = 4.2\text{nH} \quad] \; 1$$

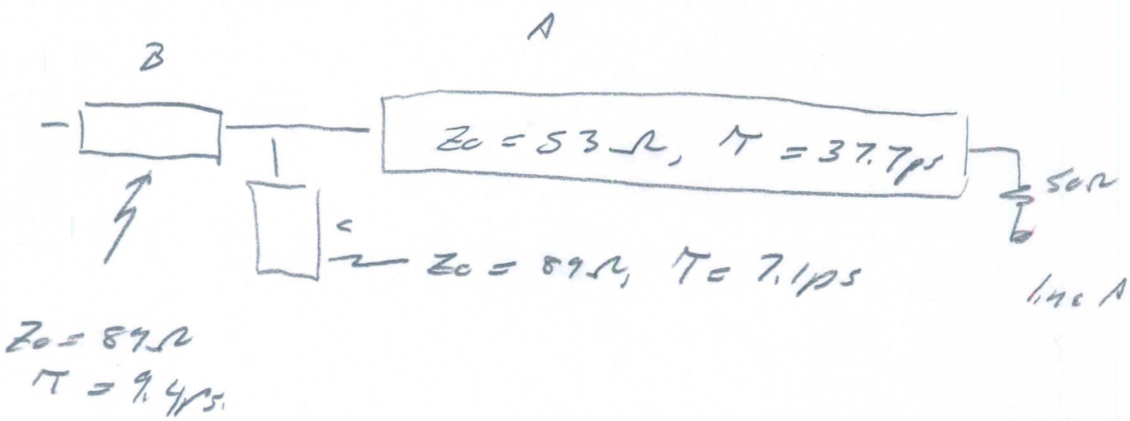
$$C = T/Z_o = 0.53\text{ pF} \quad] \; 1$$

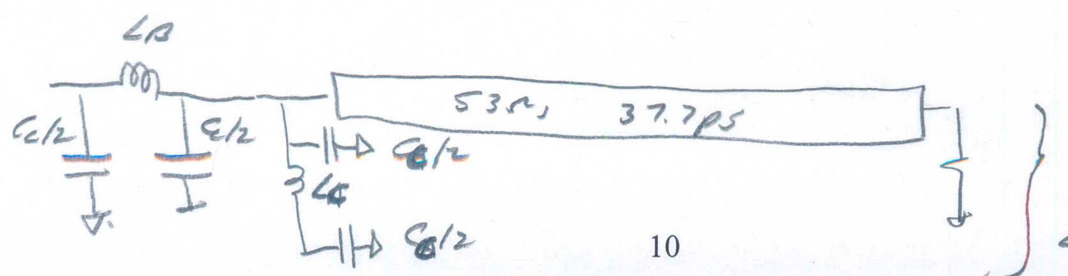
part b, 10 points



Above is an accurate scale drawing of a microstrip circuit. The dots represent the connection points. The circuit board is 0.25 mm thick and has a dielectric constant of 2.0. To approximately model fringing fields, the effective conductor width is taken as the physical conductor width plus the board thickness.

First, draw a transmission-line equivalent circuit for the circuit, giving all characteristic impedances and line propagation delays. ***Second***, assuming a signal frequency of 5 GHz, draw a second circuit diagram in which you replace line sections shorter than an eight-wavelength with T or Pi-section models, giving the computed values of all LC elements of these sections.





$L_B = 0.84\ nH$
$C_B = 0.166\ pF$
$L_C = 0.63\ nH$
$C_C = 79\ fF$

line "A" - width 1 mm, length 8 mm

(1) $\left[Z_c = \frac{377\Omega}{\sqrt{27}} \cdot \frac{0.25mm}{1mm + 0.25mm} = 53.3\Omega \right.$

(1) $\left[v = \frac{c}{\sqrt{\varepsilon_r}} = 2.12 \cdot 10^8 \text{ m/s} \right.$

(1) $\left[\tau = \frac{8mm}{v} = 37.7ps \right.$ # wavelengths $= f \cdot \tau = 0.1885 = \frac{1}{5.3}$

line "B" length 2mm width 1/2 mm

(1) $\left[Z_c = \frac{377\Omega}{\sqrt{27}} \frac{0.25mm}{0.5mm + 0.25mm} = 89\Omega \right.$

$\left[v = 2.12 \cdot 10^8 \text{ m/s} \right.$

(1) $\left[\tau = \frac{2mm}{v} = 9.4ps \right.$ # wavelength $= f\tau = 0.047 = \frac{1}{21}$

line c length 1.5mm width 1/2 mm

1/2 $\left[Z_c = 89\Omega \right.$

1/2 $\left[v = 2.12 \cdot 10^8 \text{ m/s} \right.$

(1) $\left[\tau = \frac{1.5mm}{v} = 7.07ps. \right.$ # wavelengths $= f\tau = 0.035 = 1/28.$

π- sections: lines B & c are $< \lambda/8$ (1)

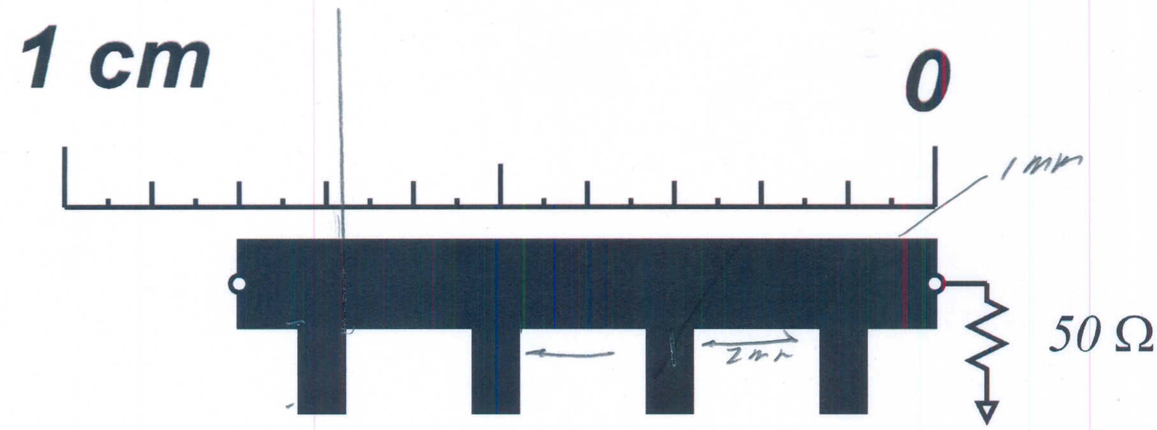
line B $\rightarrow$ $L_B = Z_0 \tau = 0.84nH$ 1/2

$C_B = \tau / Z_c = 0.106pF$ 1/2

line c $\rightarrow$ $L_c = Z_0 \tau = 0.63nH$ 1/2

$C_c = \tau / Z_c = 79fF$ 1/2

part c, 10 points

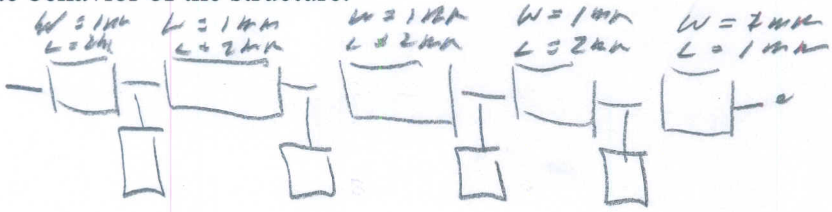


We now repeat the problem for the structure above.

First draw an equivalent circuit using transmission-line sections for all elements, giving all line impedances and delays.

Second draw a transmission line circuit where all elements are replaced by LC ***PI-sections***., giving the values of all L's and C's.

Third, neglecting the ***inductances*** of the ***shunt line elements***, please comment on the approximate behavior of the structure.



Series lines: 1 mm wide → 53 Ω, $2.12 \cdot 10^8$ m/s.] 1/2

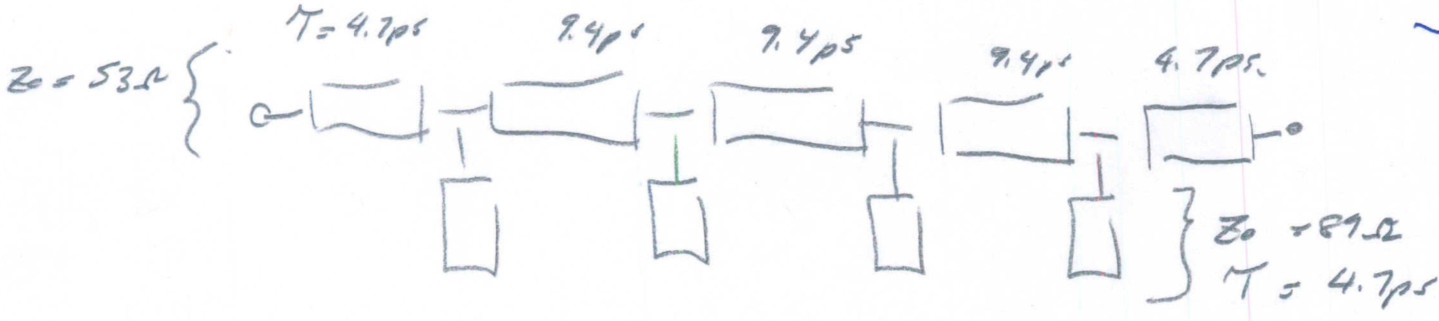
lengths = 1, 2 mm, → τ = 4.7 ps, 9.4 ps] 1/2

Stubs 1/2 mm wide → Z_0 = 84 Ω] 1/2

1 mm long → τ = 1 mm/v = 4.7 ps] 1/2

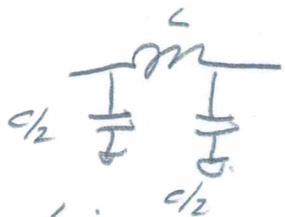
M:

<u>Transmission-line model:</u>



2

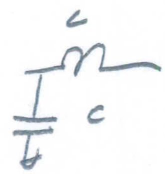
Model longer series lines so:



$L = Z_0 T = 0.5nH$
$C = T/Z_0 = 0.177pF$
$C/2 = 88.5fF$

2

Model shorter lines so:



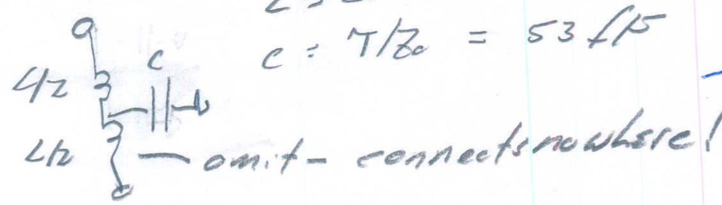
$L = Z_0 T = 0.25nH$
$C = T/Z_0 = 88.5fF$

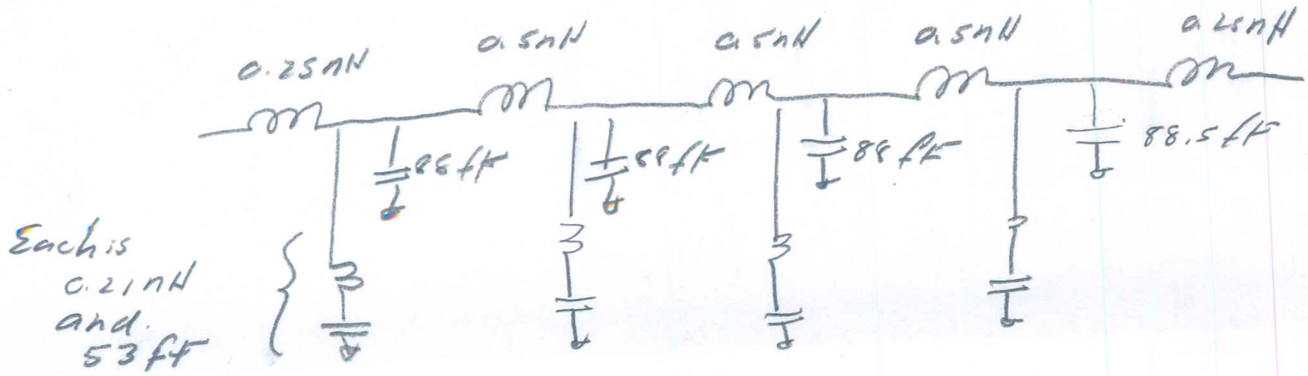
1

<u>Model stubs like so:</u>

$L = Z_0 T = 0.42nH$
$C = T/Z_0 = 53fF$

1

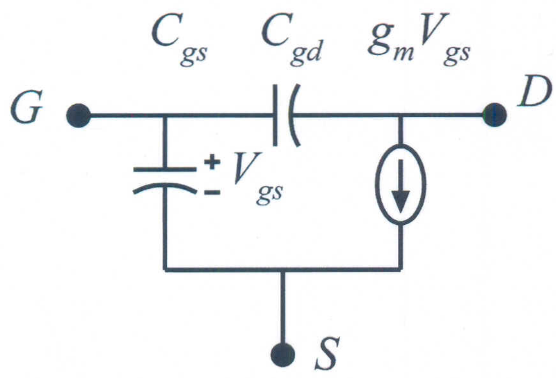




2

If we ignore the series inductance of the shunt section, we have an L-C ladder network — an LC low-pass filter.

Problem 4, 15 points
resistive feedback amplifiers



A FET has a transconductance of 0.3 mS per micron of gate width.
$f_t = g_m /(2\pi(C_{gs} + C_{gd})$ is 100 GHz, and C_{gd} is 20% of C_{gs}.

Design a resistive feedback amplifier with 12 dB gain S21 for a 50 Ohm system using this FET. Draw the circuit diagram with all element values and determine the following:

FET width= 333μm
transconductance= 100mS
C_{gs} = 127 fF
C_{dg} = 32 fF
amplifier 3-dB bandwidth (from nodal analysis or the time constant method). $\boxed{20.2\ \text{GHz}}$

(1) [12dB → $A_v = -4$

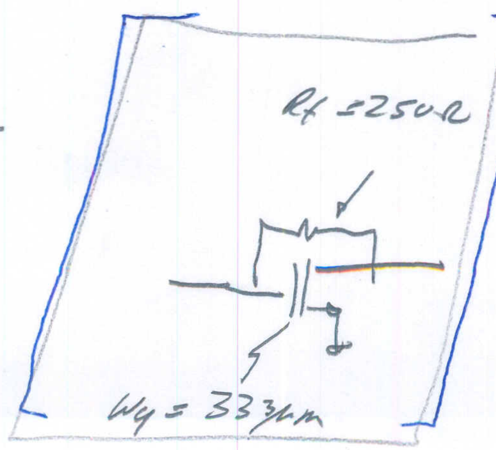
(2) [$g_m = \frac{1-A}{Z_0} = \frac{1-(-4)}{Z_0} = \frac{5}{50\Omega} = 100\text{mS}$

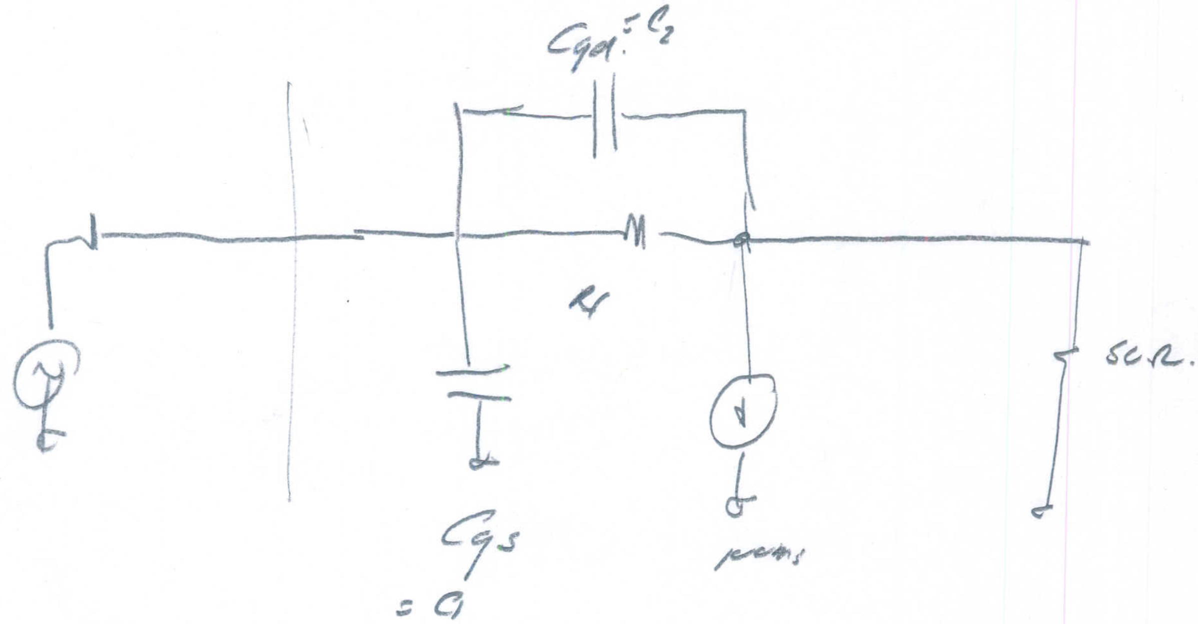
(1) [$W_g = \frac{g_m}{(g_m/W_g)} = \frac{100\text{mS}}{0.3\text{mS}/\mu\text{m}} = \underline{\underline{333\mu\text{m}}}$ → many fingers!

(2) [$C_{gs} = 0.8 \cdot \frac{g_m}{2\pi f_T} = 0.127\text{pF}$

(2) [$C_{gd} = 0.2 \frac{g_m}{2\pi f_T} = 0.032\text{pF} = 32\text{fF}$

(1) [$R_f = Z_0(1-A) = 50\Omega(5) = 250\Omega$





<u>time constant by MOTC:</u>

③ due to Cgs: $\tau = R_{11}^{0} \cdot C_{gs} = 50\Omega \| 50\Omega \cdot C_{gs}$
$= 25\Omega \cdot C_{gs} = 3.2\,ps.$

Cgd time constant is much more tricky, and the answer will be treated as <u>extra</u> <u>credit</u> —

extra 4 pts.

If we <u>remove</u> Rf, then resistance seen by Cgd is:

$R_{22}^{o} = \underbrace{50\Omega}_{R_{gs}} [1 + g_m R_L] + R_L = 50\Omega [1 + 100mS \cdot 50\Omega] + 50\Omega$
$= 350\Omega.$

If we then ~~Replace~~ <u>put</u> <u>back</u> Rf, then

$R_{22}^{o} = 350\Omega \| R_f = 350\Omega \| 250\Omega = 146\Omega$
$\tau = 146\,\Omega \cdot 32\,fF = 4.66\,ps$

② $[a_1 = 4.66ps + 3.2ps = 7.86ps] \rightarrow$ ① $[f_{3db} = 1/2\pi a_1 = 20.2\,GHz.]$