ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. October 30, 2018

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hour and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

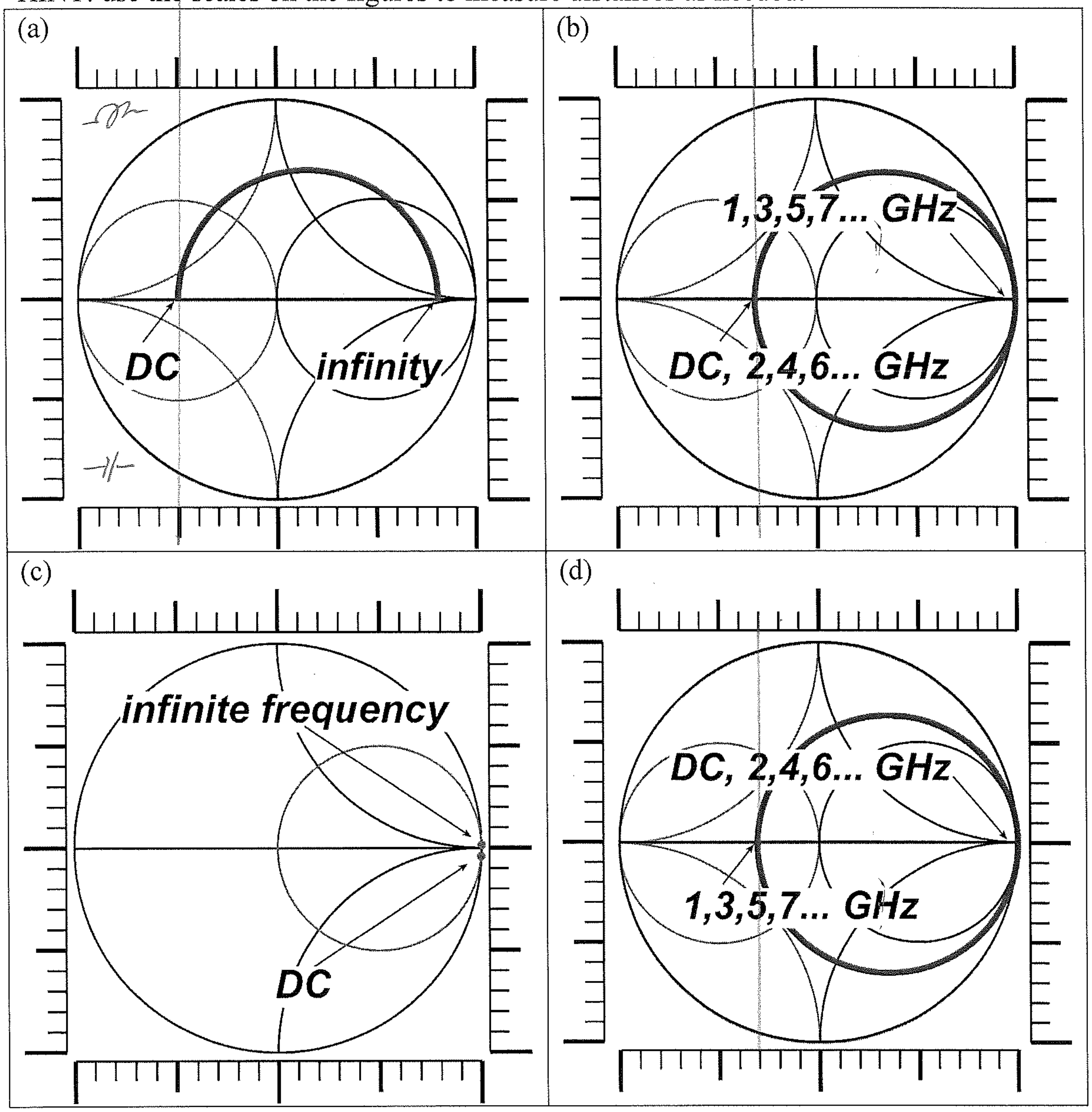
Problem	Points Received	Points Possible
1		15
2a		10
2b		7
2c		8
2d (218 only)		15 (218A only)
3a		7.5
3b		7.5
4		15
5a		5
5b		5
5c		5
5d (218 only)		7.5 (218A only)
total		85 (145), 107.5 (218A)

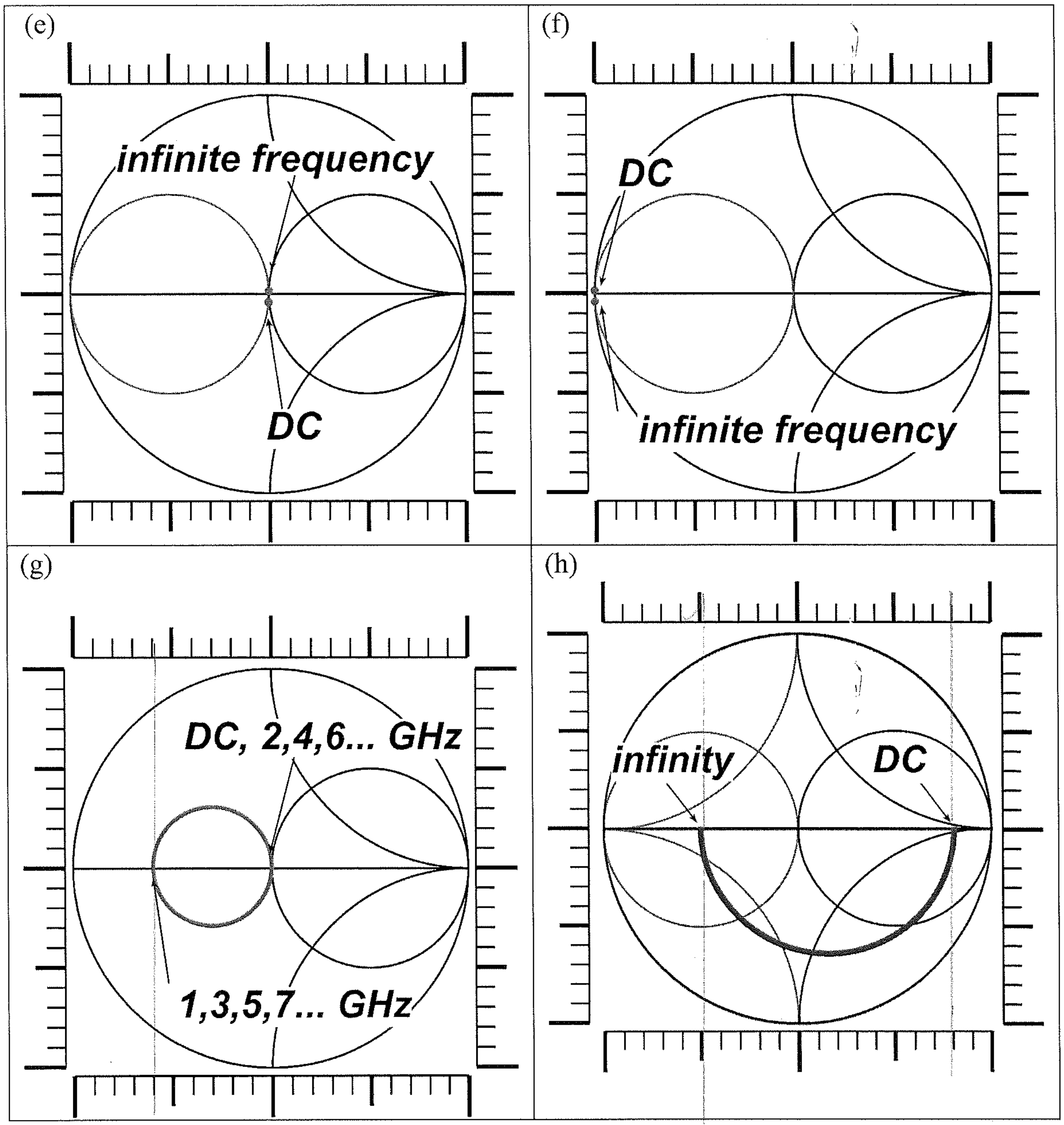
Name: solution

Problem 1, 15 points

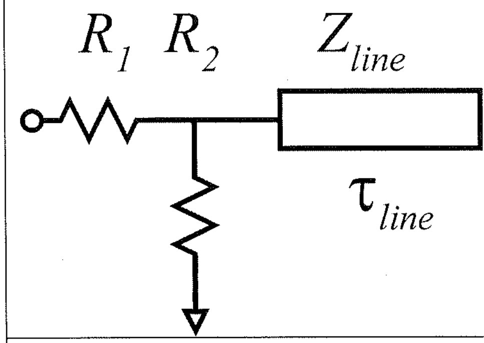
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.

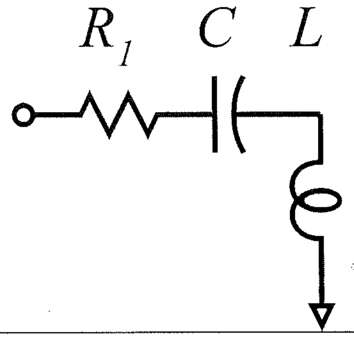



(e)
infinite frequency
DC
(f)
DC
infinite frequency
(g)
DC, 2,4,6... GHz
1,3,5,7... GHz
(h)
infinity
DC

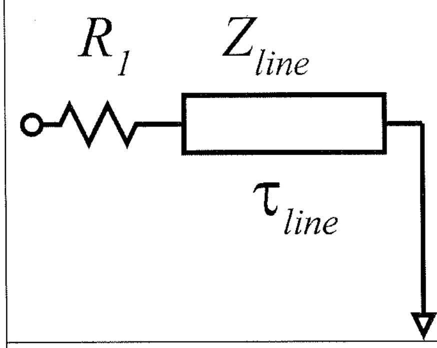
(i)


R_1 R_2 Z_{line}
τ_{line}

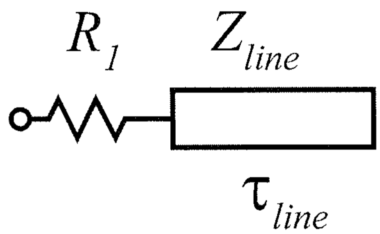
(j)


R_1 C L

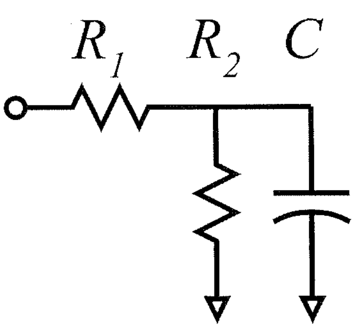
(k)


R_1 Z_{line}
τ_{line}

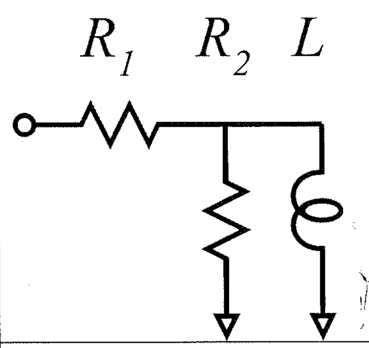
(l)


R_1 Z_{line}
τ_{line}

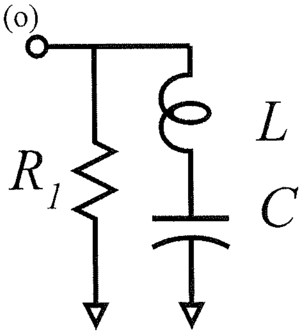
(m)


R_1 R_2 C

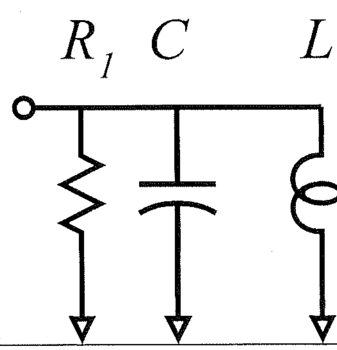
(n)


R_1 R_2 L

(o)


L
R_1
C

(p)


R_1 C L

First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

2 Smith chart (a). Circuit= N .
Component values: $R_1 = 17\Omega$, $R_2 = 433\Omega$, ______,

Smith chart (b). Circuit= k .
Component values: $R_1 = 27\Omega$, ______, ______,

Smith chart (c). Circuit= (i) .
Component values: $R = 50\Omega$, ______, ______,

Smith chart (d). Circuit= l .
Component values: $R = 27\Omega$, ______, ______,

Smith chart (e). Circuit= ERROR! .
Component values: ______, ______, ______,

Smith chart (f). Circuit= (p) .
Component values: $R = 50\Omega$, ______, ______,

Smith chart (g). Circuit= (l) .
Component values: $R_1 = 12.5\Omega$, $R_2 = 37.5\Omega$, ______,

Smith chart (h). Circuit= h .
Component values: $R_1 = 433\Omega$, $R_2 = 17\Omega$, ______,

2

a) goes from $\Gamma = -1/2$ @ DC $\rightarrow 50\Omega \cdot \frac{1-1/2}{1+1/2} = \frac{50\Omega}{3} = 17\Omega$

to $\Gamma = 0.8$ @ $f = \infty \rightarrow 50\Omega \frac{1.8}{0.2} = 50\Omega \cdot 9 = 450\Omega$

trajectory is inductive $\rightarrow$ (n)

$R_1 = 17\Omega$, $R_2 = 450 - 17\Omega = 433\Omega$

2

b) goes between $\Gamma = -0.3 \Rightarrow 50\Omega \frac{1-0.3}{1+0.3} = 27\Omega$
and an open circuit - does so repetitively.
(k)

2

c) goes from open @ DC, to 50Ω, then to open.
$\rightarrow$ (i) $R_1 = 50\Omega$

2 D) goes between open & $50\Omega \cdot \frac{1-0.3}{1+0.3} = 27\Omega$
does so repetitively

$\rightarrow$ (e)

0 (E) goes from 50Ω @ DC to an open
& then 50Ω @ $f=\infty$.
(!) oh no! I've made a mistake!
the right solution is

3 (f) goes from 0Ω @ DC
to 50Ω @ Resonance, to 0Ω @ $f=\infty$
$\rightarrow$ (p) $R_1 = 50\Omega$

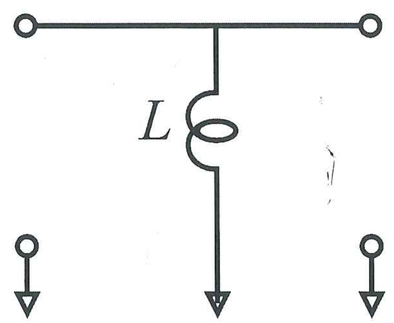
2 (g) goes repetitively between 50Ω starting @ DC
and $50\Omega \left(\frac{1-0.6}{1+0.6}\right) = 12.5\Omega$.
(i) with $R_1 = 12.5\Omega$
& $R_2 = 50\Omega - 12.5\Omega = 37.5\Omega$

2 (h) goes from $50\Omega \frac{1+0.8}{1-0.8} = 450\Omega$ @ DC
to $50\Omega \left(\frac{1-0.5}{1+0.5}\right) = 17\Omega$ @ $f=\infty$
Trajectory is capacitive $\Rightarrow$ (m)
$R_2 = 17\Omega$

$R_1 = 450\Omega - 17\Omega = 433\Omega$

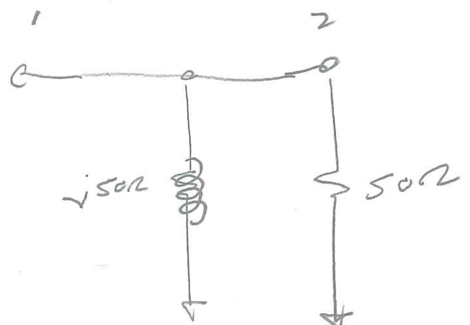
Problem 2, 25 points (ece145A), 40 points (ece218A)
2-port parameters and Transistor models

<u>Part a, 10 points</u>
For the network at the right, give numerical values for the four S-parameters. Assume that the reference Zo is 50 Ohms. The signal frequency is 1GHz and the inductance is 8.0nH.



1 inductive reactance $= j\omega L = j\,2\pi(1\,GHz)\,8\,nH$
$= j\,50\Omega.$

2 S_{11}:



$Z_{in}\big|_{Z_L=50\Omega} = 50\Omega \,//\, j50\Omega$
$= 50\Omega\left(\frac{1\cdot j}{1+j}\right)$
$=$

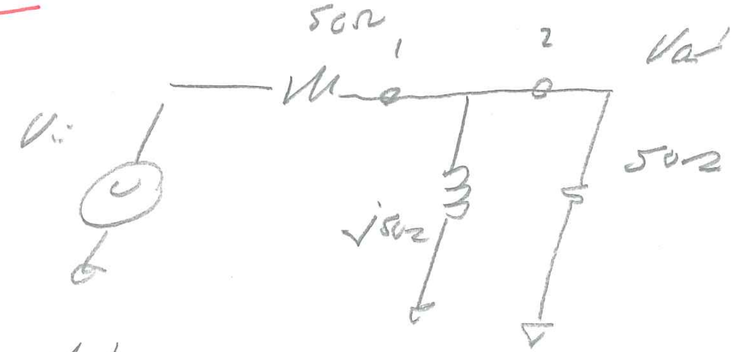
2 $S_{11} = \dfrac{Z_{in}\big|_{Z_L=50\Omega} - 50\Omega}{Z_{in}\big|_{Z_L=50\Omega} + 50\Omega} = \dfrac{\frac{j}{1+j} - 1}{\frac{j}{1+j} + 1} = \dfrac{j - 1 - j}{j + 1 + j}$

$= \dfrac{-1}{1+2j} = \dfrac{1\angle 180^\circ}{\sqrt{5}\angle 63^\circ} = \dfrac{1}{\sqrt{5}}\angle 117^\circ$

all ok.

1/2 by symmetry, $S_{22} = S_{11}$

2



2.

$$\frac{V_{out}}{V_i} = \frac{50\Omega \,||\, j50\Omega}{50\Omega + 50\Omega \,||\, j50\Omega} = \frac{1 \,||\, j}{1 + 1 \,||\, j}$$

$$= \frac{\frac{j}{1+j}}{1 + \frac{j}{1+j}} = \frac{j}{1+2j}$$

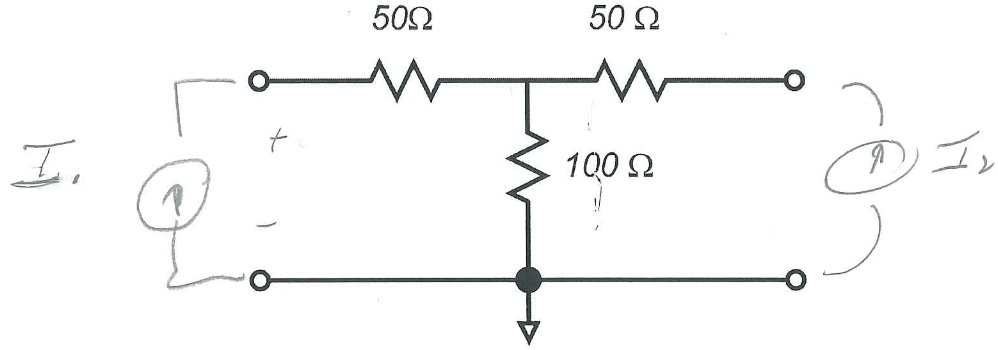
$$= \frac{1\angle 90^\circ}{\sqrt{5}\,\angle 63^\circ} = \frac{1}{\sqrt{5}}\angle 27^\circ$$

$$S_{21} = 2 \cdot \left.\frac{V_o}{V_{gen}}\right|_{Z_L = Z_S = Z_0} = \underbrace{\frac{2}{\sqrt{5}}\angle 27^\circ \text{ or } = \frac{2j}{1+2j}}_{\text{either ok.}}$$

1/2. and, by symmetry, $S_{21} = S_{12}$

Part b, 7 points

Compute the Z parameters for this network



$Z_{11} = Z_{22} = 150\Omega$

$Z_{21} = Z_{12} = 100\Omega$

[by inspection, $V_1 = I_1(150\Omega) + I_2(100\Omega)$] 5

[so this gives Z_{11} and Z_{12}] 1

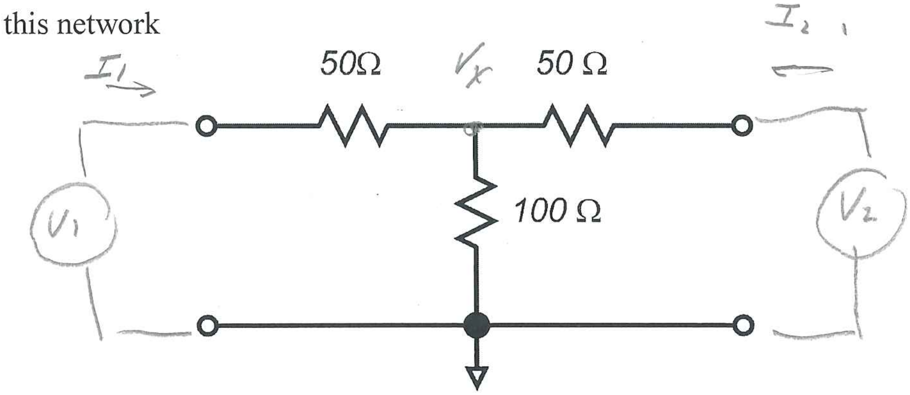
[Z_{22} and Z_{21} by symmetry.] 1

Part c, 7 points

Compute the Y parameters for this network

We could just invert the Z matrix...

...or...



we don't know V_x, so:

2 $$\frac{V_x - V_1}{50\Omega} + \frac{V_x - V_2}{50\Omega} + \frac{V_x}{100\Omega} = 0$$

$$V_x(5\,mS) = V_1 \cdot 2\,mS + V_2(2\,mS)$$

1 $$\boxed{V_x = 0.4 \cdot V_1 + 0.4 \cdot V_2}$$

so $$I_1 = \frac{V_1 - V_x}{50\Omega} = (V_1 - V_x)\,2\,mS = 2\,mS(V_1 - 0.4V_1 - 0.4V_2)$$

1 $$I_1 = 2\,mS(0.6V_1 - 0.4V_2) = 1.2\,mS \cdot V_1 - 0.8\,mS \cdot V_2$$

1 $$I_1 = Y_{11}V_1 + Y_{12}V_2 = 1.2\,mS \cdot V_1 - 0.8\,mS\,V_2$$

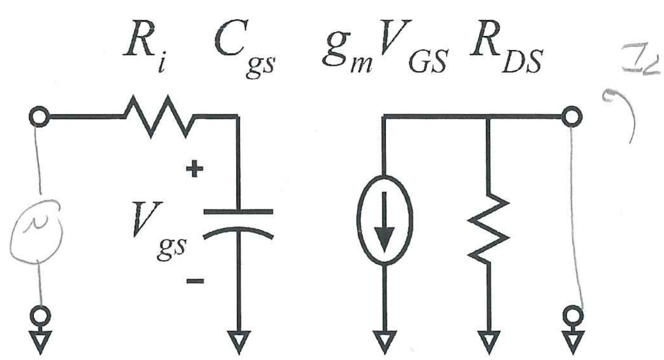
1 so $Y_{11} = 1.2\,mS$, $Y_{12} = -0.8\,mS$

1 by symmetry, $Y_{22} = Y_{11}$ and $Y_{21} = Y_{12}$

Part d, ***ECE218A students only*** 15 points

For the network at the right, give an algebraic expressions for Y_{11} and Y_{21}. Please write as a Taylor series in $j\omega$, omitting terms of power $(j\omega)^3$ and higher.

This is an exercise in device model extraction from measured S/Y/Z parameters.



If $V_1 \neq 0$, $V_2 = 0$ then

2 $\left[\; I_1 = Y_{11} V_1 = \dfrac{V_1}{R_i + \frac{1}{j\omega C_{gs}}} = V_1 \dfrac{j\omega C_{gs}}{1 + j\omega C_{gs} R_i}\right.$

2 $\left[\; \text{so } Y_{11} = \dfrac{j\omega C_{gs}}{1 + j\omega C_{gs} R_i}\right.$ $\cong j\omega C_{gs}\left(1 - j\omega C_{gs} R_i + O((j\omega)^2)\right)$

3 $\left[\; Y_{11} = j\omega C_{gs} + \omega^2 C_{gs}^2 R_i + O(\omega^3)\right.$

2 $\left[\; \dfrac{V_{gs}}{V_1} = \dfrac{1/j\omega C_{gs}}{R_i + 1/j\omega C_{gs}} = \dfrac{1}{1 + j\omega C_{gs} R_i}\right.$

2 $\left[\; I_2 / V_1 = Y_{21} = \dfrac{g_m}{1 + j\omega C_{gs} R_i}\right] = \dfrac{g_m (1 - j\omega C_{gs} R_i)}{1 + \omega^2 C_{gs}^2 R_i^2}$

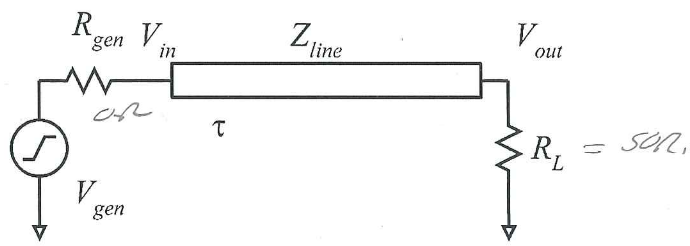
4 $= g_m (1 - j\omega C_{gs} R_i)(1 - \omega^2 C_{gs}^2 R_i^2 + O(\omega^4))$

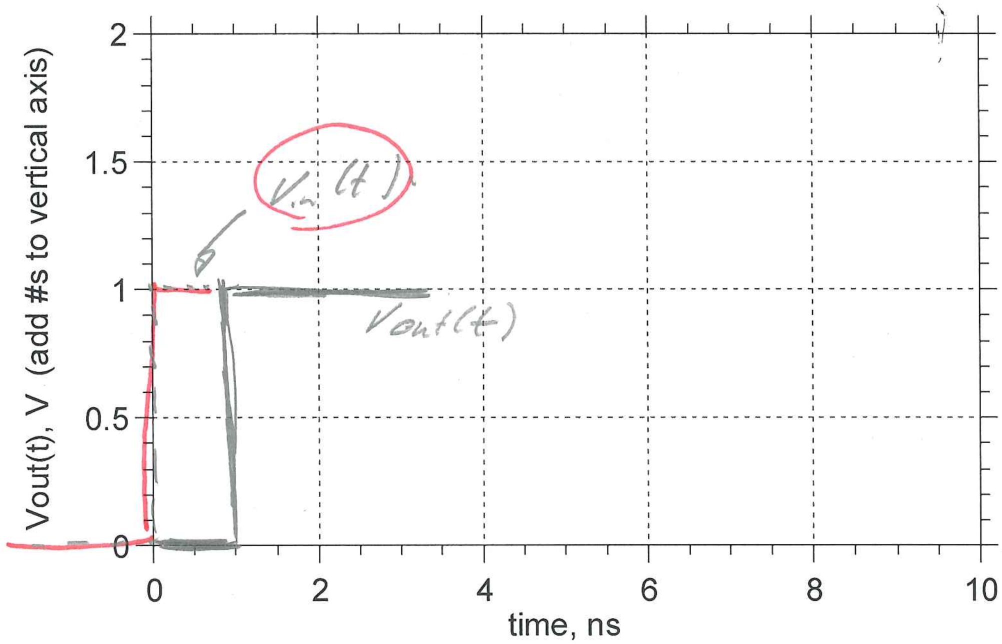
3 $Y_{21} = g_m (1 - j\omega C_{gs} R_i - \omega^2 C_{gs}^2 R_i^2 + O(\omega^3))$

Problem 3, 15 points
Transmission lines in the time domain.

Part a, 7.5 points
Vgen is a 1 V step-function.
Rgen is zero Ohms and RL is 50 Ohms.
tau is 1 ns.
Zline is 50 Ohms.

Plot below Vout(t) and Vin(t)



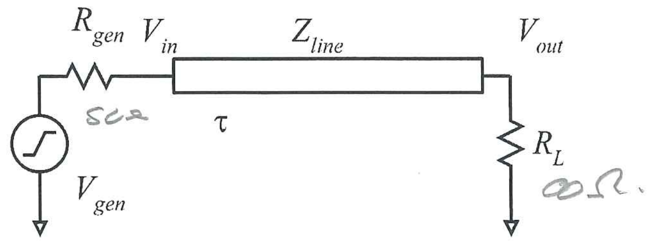


3.75pts each...

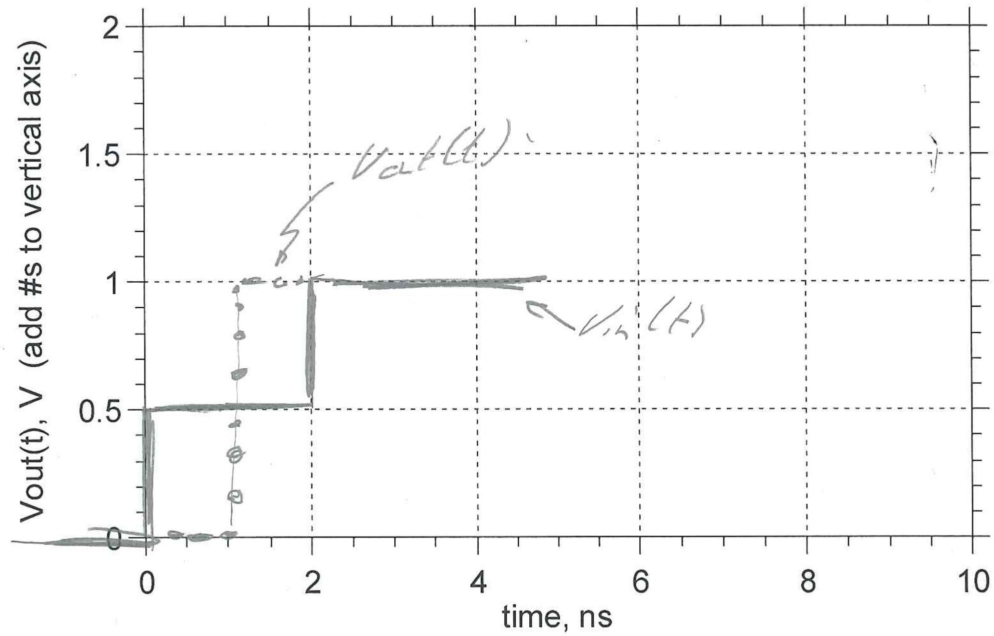
Part b, 7.5 points

Vgen is a 1 V step-function.
Rgen is 50 Ohms and RL is infinite.
tau is 1 ns.
Zline is 50 Ohms.

Plot below Vout(t) and Vin(t)



Please comment on the pulse response of this circuit and that of part a.



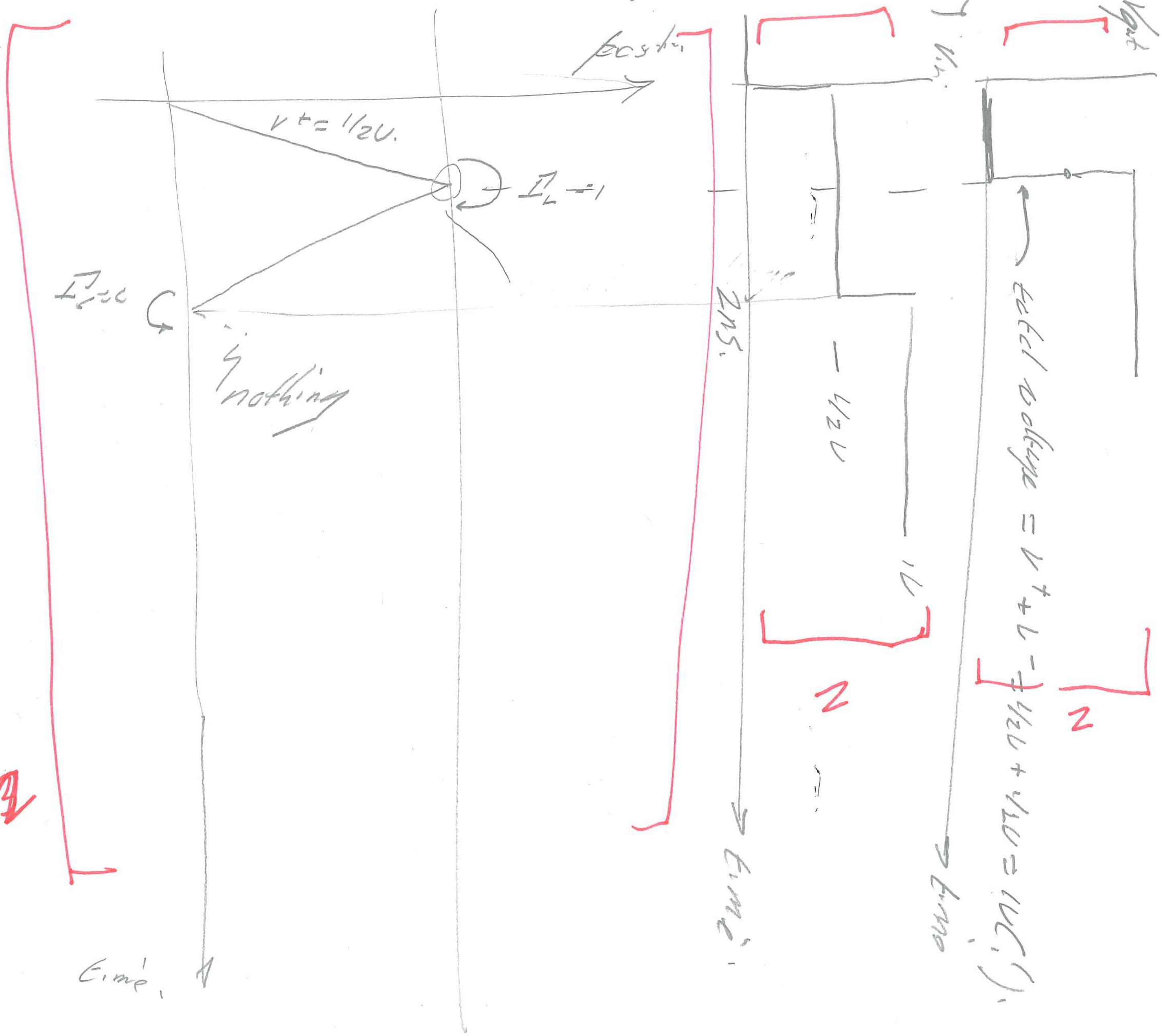
0.5

$$\Gamma_L = \frac{\infty - 1}{\infty + 1} = 1$$

$$\Gamma_S = 0$$

$$\Gamma_S = \frac{Z_0}{Z_0 + R_{gen}} = 1/2.$$

[$\Gamma_L = 1$, $T_S = 1/2$, $\Gamma_S = 0$.]



2

2

2

Comment

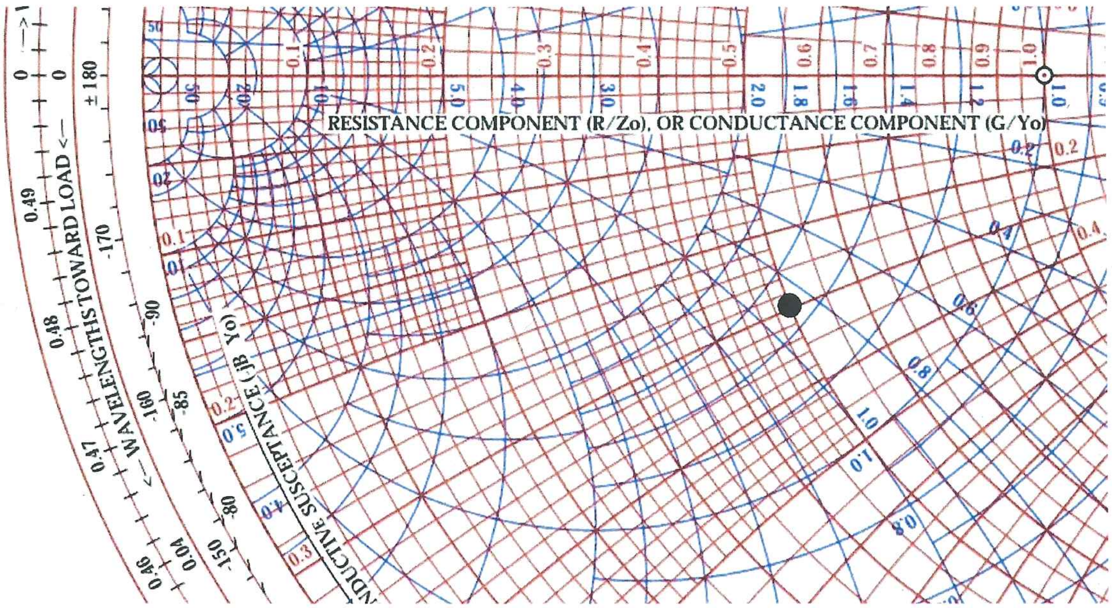
part (a) is called ~~se~~ receiving end termination.
Signal is undistorted at both ends of line.

1

part (b) is called sending end termination.
Signal is undistorted only at receiving end of line.

Problem 4, 15 points
Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 1 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50Ohms*. Give all element values. Use the full impedance-admittance chart which has been provided to you.



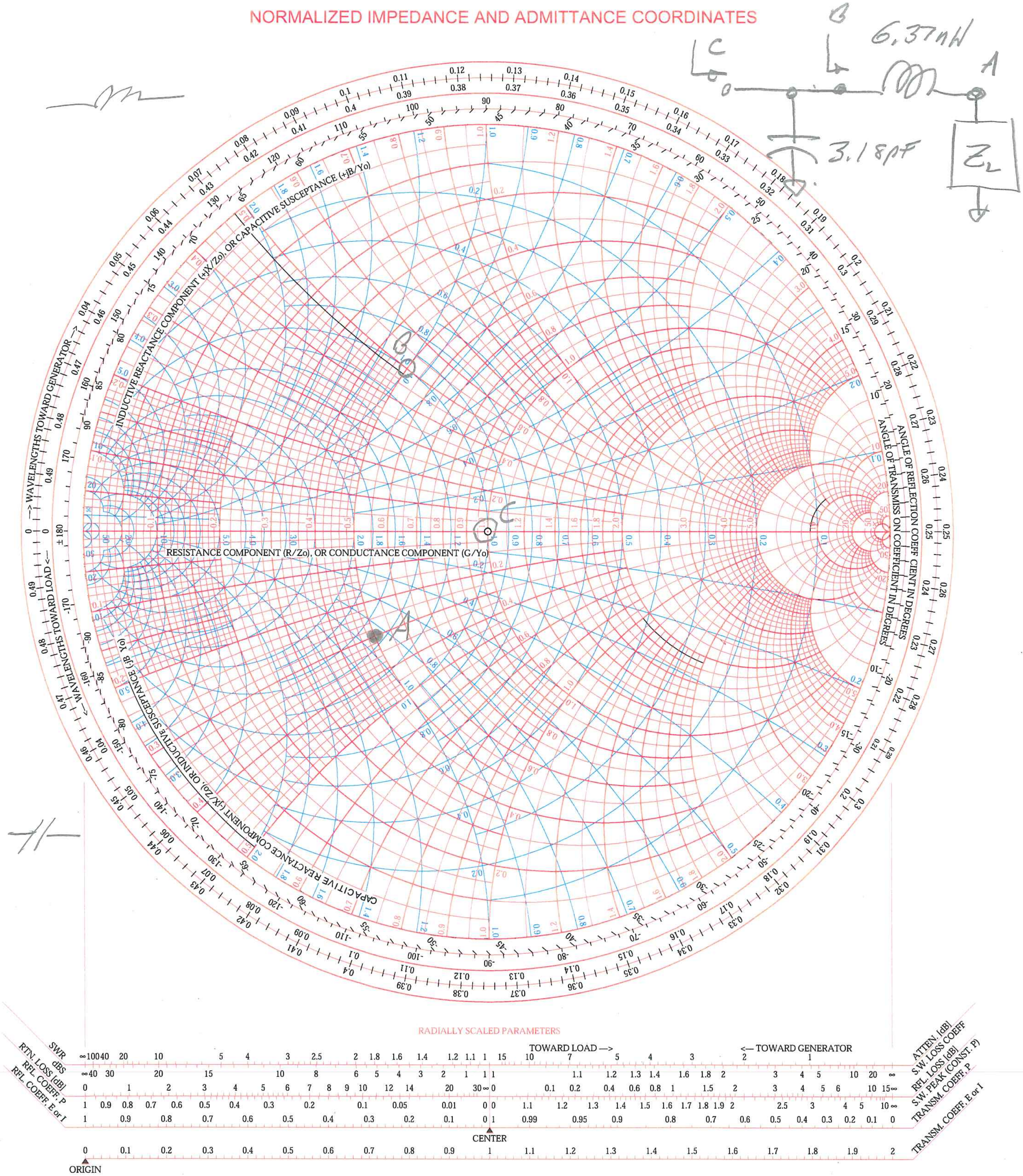
There are two different two-element solutions

Both are shown

Either one is acceptable.

NAME	TITLE *first solution*	DWG. NO.
		DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



First soluti..

2 Point "a" is at an impedance, normalized of $z_A = 0.5 - j0.3$.

2 We move to point "B", which has a normalized impedance at

$$z_B = 0.5 + j0.5$$

2 This movement from "A" to "B" corresponds to the additi.. of $\Delta z = j0.5 - (-0.3j) = 0.8j$

an un-normalized impedance of $\Delta Z = 0.8 \cdot 50\Omega \cdot j$
$= j40\Omega$

This is an inductor $\Delta Z = j\omega L = j40\Omega$

$$\rightarrow L = \frac{40\Omega}{\omega} = \frac{40\Omega}{2\pi(1GHz)} = 6.37\,nH.$$

2 Point "B" has a normalized admittance of

$$y_B = 1.0 - j1.0$$

2 Point "C" has a normalized admittance of

$$y_C = 1.0 + j0.$$

3 This movement from "B" to "C" corresponds to the additi.. of

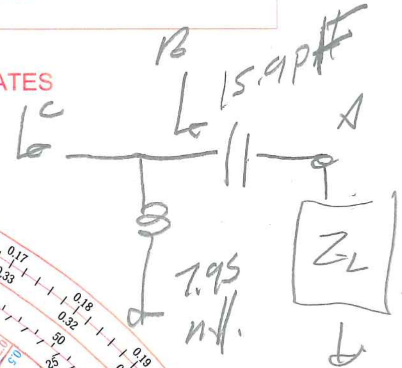
$$\Delta y = j0 - (-j1) = j1$$

an, un-normalized admittance of $\Delta Y = \frac{j1}{50\Omega} = +j(20mS)$.

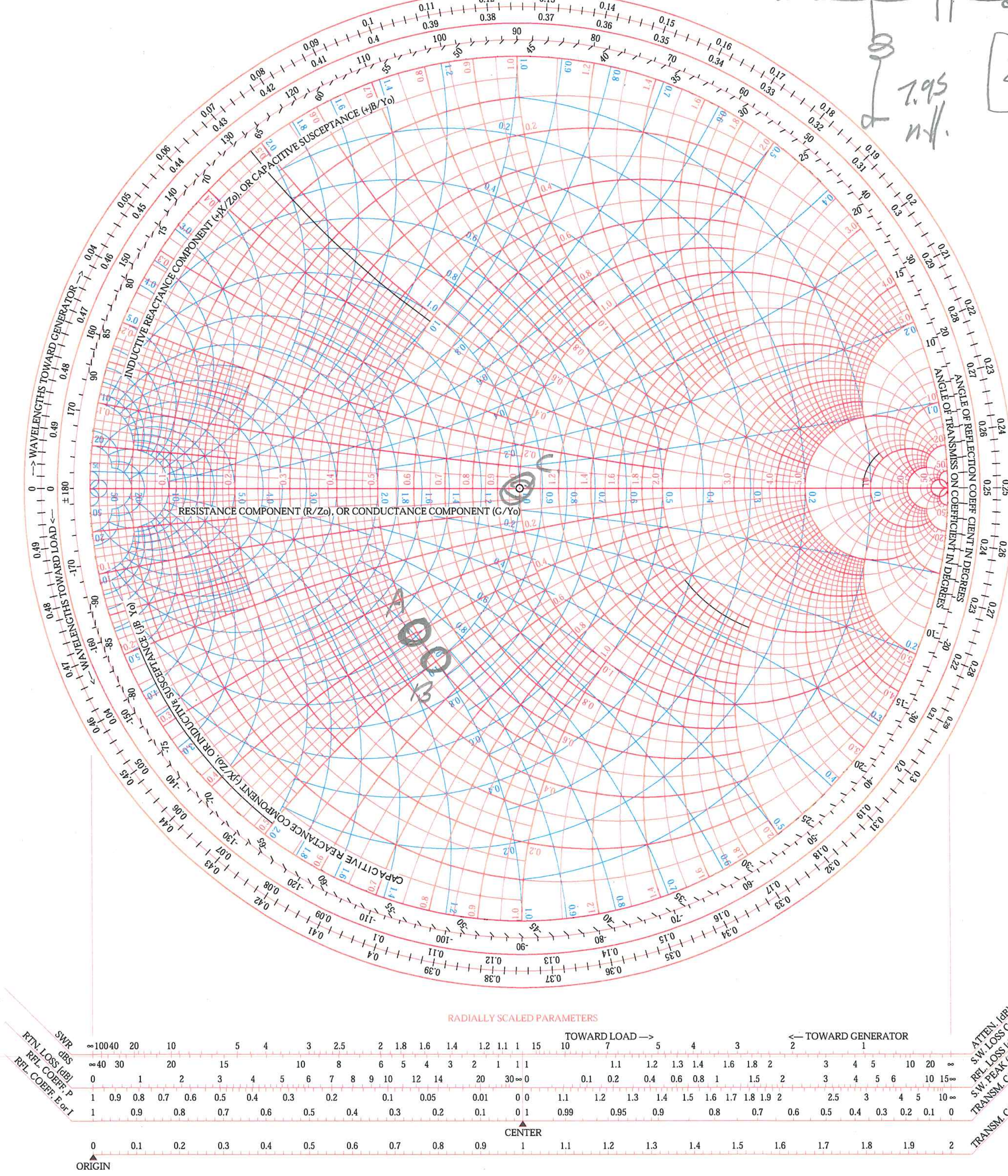
2 This is a capacitor $\Delta Y = j\omega C = j\,20mS$

$$C = \frac{20mS}{2\pi(1GHz)} = 3.18pF$$

NAME	TITLE Second solution	DWG. NO.
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	DATE



NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Second solution

2 point "a" is at the normalized impedance

$z_A = 0.5 - j0.3$

2 we move to point "B", a normalized impedance of

$z_B = 0.5 - j0.5$

2 This movement from "A" to "B" corresponds to the addition of $\Delta z = -j0.5 - (-j0.3) = -j0.2$

an un-normalized impedance of

$\Delta Z = -j0.2 \cdot 50\Omega = -j10\Omega$.

This is a series capacitor $\Delta Z = \frac{1}{j\omega C} = -j10\Omega$

$\rightarrow C = \frac{1}{10\Omega \cdot 2\pi(1\,GHz)} = 15.9\,pF$.

2 Point "B" has a normalized admittance of

$y_B = 1 + j1$

2 point "C" has a normalized admittance of

$y_C = 1 + j0$.

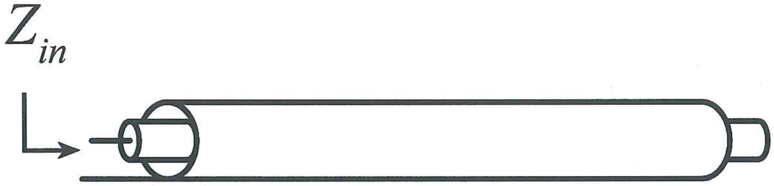
3 The movement from "B" to "C" corresponds to the addition of $\Delta y = j0 - (j1) = -j1$, an un-normalized admittance of $\Delta Y = \frac{-j1}{50\Omega} = -j(20mS)$

2 This is an inductor $\Delta Y = \frac{1}{j\omega L} = -j\,20mS$

$L = \frac{50\Omega}{2\pi(1\,GHz)} = 7.95\,nH$.

Problem 5, 15 points (ece145A), 20 points (218A)
Transmission-line properties.

<u>Part a, 5 points</u>



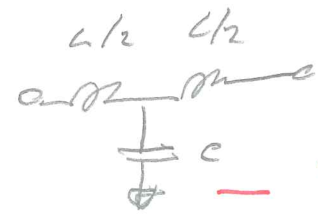
In an undergraduate class, a 1 meter length of 50 Ohm cable connects a circuit under test to an oscilloscope. The input impedance to the oscilloscope is extremely high. The propagation velocity of the transmission line is $2 \cdot 10^8$ m/s. At a signal frequency of 1 MHz, what would the input impedance be, approximately ?

1 speed of light delay $= \tau = \dfrac{\ell}{v} = \dfrac{1\,m}{2 \cdot 10^8\,m/s}$

$\tau = 5ns.$

1 The period of 1 MHz is 1 μs, so $f\tau << 1$, or equivalently, $\ell << \lambda$

use lumped element model: T-section



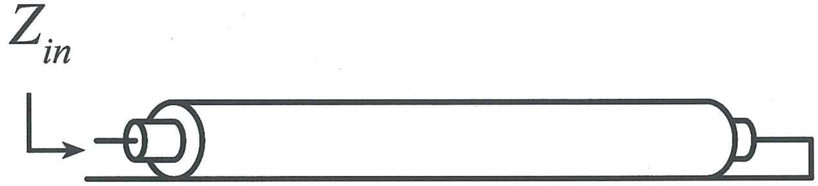
1 $C = \tau / Z_0 = \dfrac{5ns}{50\Omega} = 100pF$

$L = \tau \cdot Z_0 = 5ns \cdot 50\Omega$
$= 0.25\mu H.$

1 $Z = j\omega L/2 + \dfrac{1}{j\omega C} = +j(\dfrac{1.6\Omega}{2}) - j1590\Omega$

$\cong \dfrac{1}{j\omega C} = -j1590\Omega$

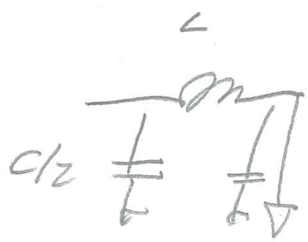
Part b, 5 points



At 1 MHz, if we short-circuit the far end of the cable, what would be the approximate input impedance ?

2.5

now use a π - section



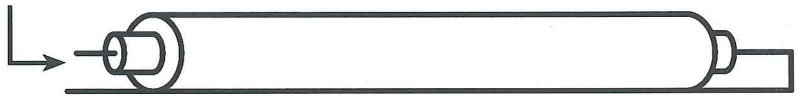
2.5

$Z = j\omega L \,//\, \frac{1}{j\omega C/2} = j(1.6\,\Omega)//(-j\,31.8k\Omega)$

$\cong j\,1.6\,\Omega$

Part c, 5 points

Z_{in}



At what frequencies is the input impedance infinite ?
At what frequencies is the input impedance zero ?

1 line has $\tau = 5ns$

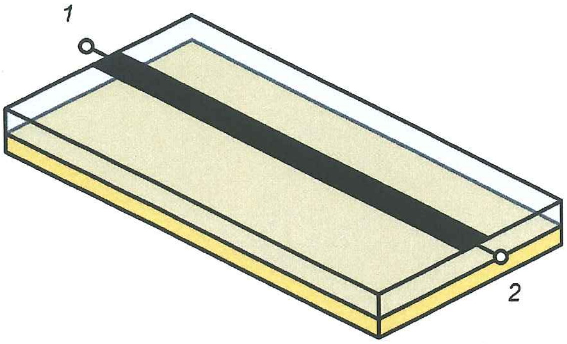
1 so line is one wavelength long @ $f = \frac{1}{5ns} = 200MHz$

1 line is $1/4\,\lambda$ long @ $f = 50MHz$
line is $1/2\,\lambda$ long @ $f = 100MHz$

1 Input impedance will be zero @
$f = 0$, 100MHz, 200MHz, 300MHz, ...

1 Input impedance will be infinite when
$f = 50MHz$, 150MHz, 250MHz, 350MHz, ...

Part d, 7.5 points ECE 218 students only



Danger - confusion of

L = inductance

and

L = length

I've written $\mathcal{L}$ for inductance

-We will make a 50 Ohm microstrip line on a circuit board of 1mm thickness having a dielectric constant of 4. If we ignore fringing fields, how wide must the conductor be ?

-Using transmission-line relationships, and ignoring the fringing fields, derive expressions for the inductance and capacitance of a conductor of length L and width W, on a circuit board of thickness T, where the circuit board has a ground plane on the back surface.

If we ignore fringing fields, then

2.5

$$Z_0 \cong 377\Omega \frac{1}{\sqrt{\epsilon_r}} \frac{H}{W} = 50\Omega \qquad 377\Omega = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\Rightarrow H/W = \frac{50\Omega}{377\Omega} \cdot \sqrt{4} = 0.265$$

$$W = \frac{H}{0.265} = \frac{1mm}{0.265} = 3.77mm.$$

If we ignore fringing fields then the capacitance must be

1

$$\boxed{C = \epsilon_0 \epsilon_r \frac{W \cdot L}{H}}$$

4.

But $Z_0 = \sqrt{\frac{\mathcal{L}}{C}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \frac{H}{W} \rightarrow \mathcal{L} = \left(\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}\right) \frac{H^2}{W^2} \cdot C$

$$\mathcal{L} = \frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r} \frac{H^2}{W^2} \epsilon_0 \epsilon_r \frac{W}{H} \cdot L \rightarrow \boxed{\mathcal{L} = \mu_0 \mu_r \frac{H}{W} \cdot L}$$