## ECE ECE145A (undergrad) and ECE218A (graduate)

## Mid-Term Exam. November 14, 2012

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

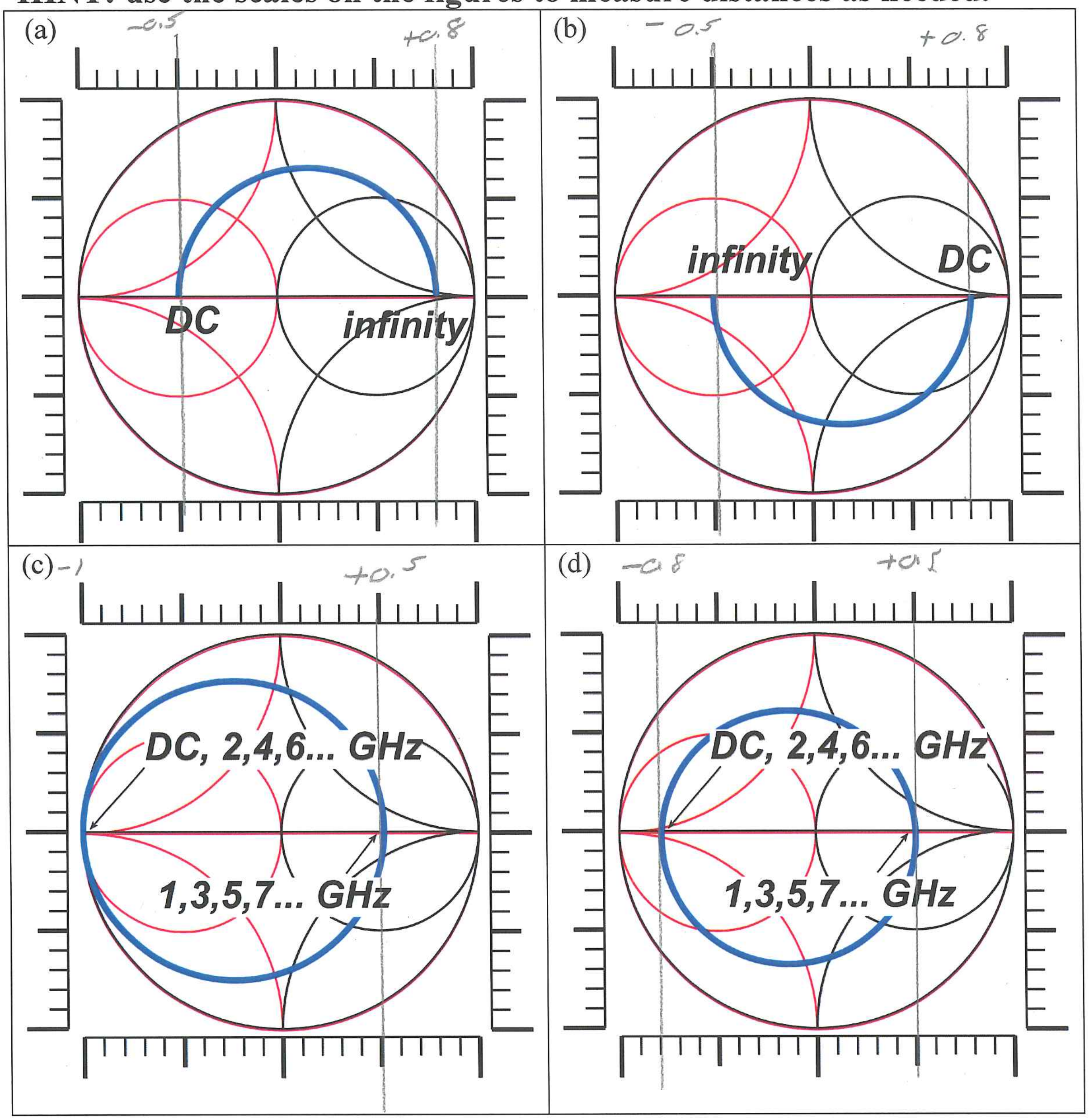
Use any and all reasonable approximations (5% accuracy is fine. ) , ***AFTER STATING THEM.***

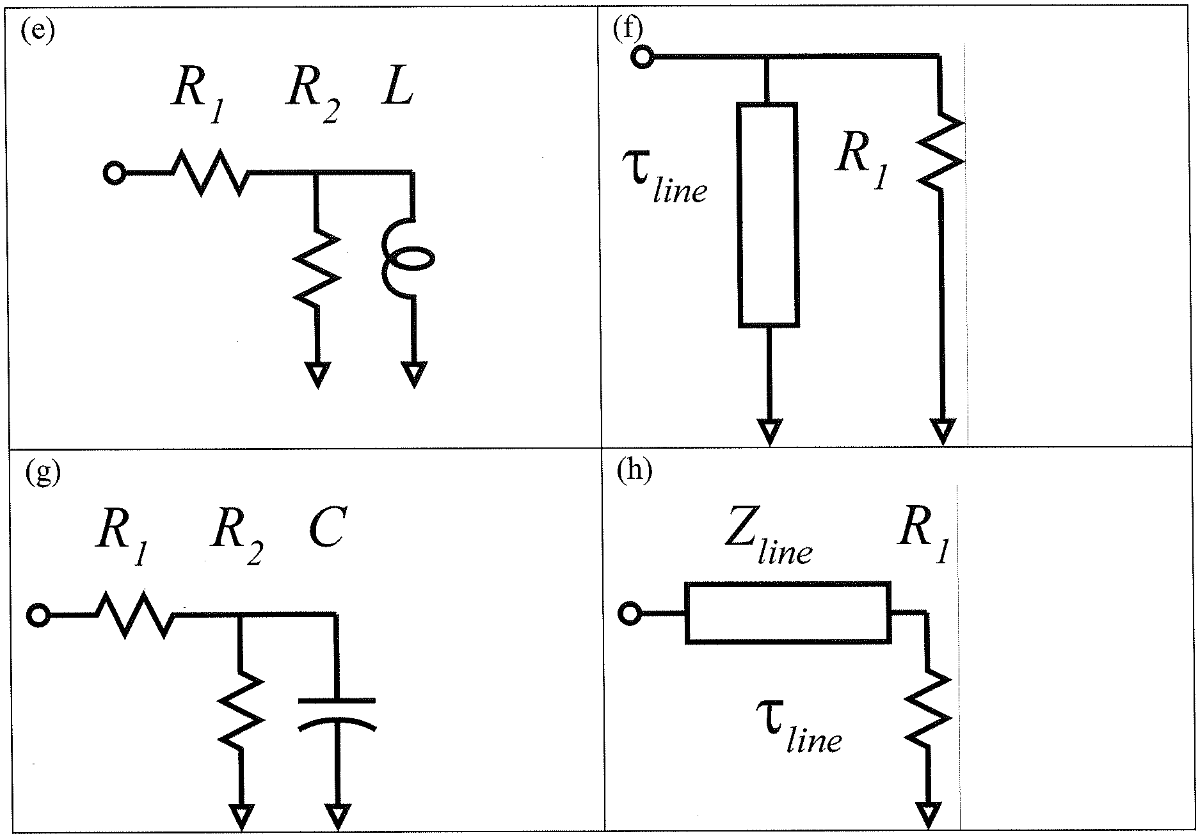
| Problem | Points Received | Points Possible |
|---|---|---|
| 1 | | 20 |
| 2a | | 20 |
| 2b | | 10 |
| 3a | | 10 |
| 3b | | 10 |
| 4a | | 5 |
| 4b | | 5 |
| 4c | | 5 |
| 5 | | 15 |
| total | | 100 |

**Name:** Solution

**Problem 1, 20 points**
*The Smith Chart and Frequency-Dependent Impedances.*

**HINT: use the scales on the figures to measure distances as needed.**






Match each Smith Chart with each circuit, and give all resistor values, and all transmission line delays and characteristic impedances. The charts all use 50 Ohm normalization:

(1)

Smith chart (a). Circuit=__E__. Component values=__$R_1 = 17\Omega$, $R_2 = 433.3\Omega$__

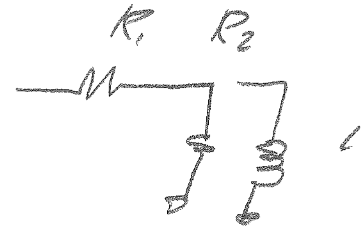
Smith chart (b). Circuit=__~~B~~ G__. Component values=__$R_1 = 433\Omega$, $R_2 = 17\Omega$__

Smith chart (c). Circuit=__F__. Component values=__$R_1 = 150\Omega$, $\tau = 250ps$__

Smith chart (d). Circuit=__H__. Component values=__$R_1 = 5.5\Omega$, $\tau = 250ps$, $Z_0 = 29\Omega$.__

smith chart a

this is network (e)




at DC, $Z = R_1$ & $\frac{R/Z_0 - 1}{R/Z_0 + 1} = \Gamma \rightarrow R_1 = Z_0 \frac{1+\Gamma}{1-\Gamma}$

(2)

$= 50\Omega\left(\frac{1-0.5}{1+0.5}\right)$

$= 16.7\Omega$

at $\omega \rightarrow \infty$, $Z = R_1 + R_2 \rightarrow R_1 + R_2 = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1+0.8}{1-0.8}$

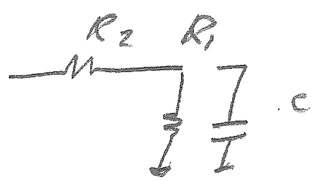
(2)

$R_1 + R_2 = 450\Omega$

$R_2 = 433.3\Omega$

smith chart b

this is network (g)




at DC $Z = R_1 + R_2 \rightarrow R_1 + R_2 = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1.8}{0.2}$

$= 450\Omega$

at $\omega \rightarrow \infty$ $Z = R_2$  $R_2 = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1-0.5}{1+0.5} = 17\Omega$

↳ $R_1 = 433\Omega$

smith chart (c) this is network (f)

$R_1 = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1+1/2}{1-1/2} = 150\Omega$

it is not possible to find the $Z_0$ of the line

line is $\lambda/4$ @ 1 GHz, so line is $\lambda$ @ 4 GHz,

so $T = 1/(4\,GHz) = 250ps$

smith chart (h) this is network (h).

AT DC

$R_1 = Z_0 \frac{1+\Gamma}{1-\Gamma} =$

at

$= 50\Omega \frac{1-0.8}{1+0.8}$

$= 5.55\Omega$

# Smith Chart D

## this is network(h)

at DC, $\Gamma = -0.8$

at DC $R_1 = Z_c \dfrac{1+\Gamma}{1-\Gamma}$

$= 50\Omega \dfrac{1-0.8}{1+0.8}$

$= 5.55\Omega$

At 1GHz, line is $\lambda/4$ long. So line is $\lambda$ long @ 4GHz

$\rightarrow \tau = 250\,ps.$

At 1GHz, the line input impedance is $Z_{in} = Z_c \dfrac{1+0.5}{1-0.5} = 150\Omega$

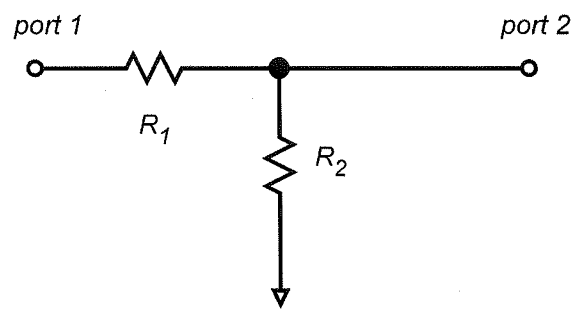
this is the quarter-wave condition,

so $Z_{line} = \sqrt{R_1 Z_{in}} = \sqrt{5.55\Omega \cdot 150\Omega}$

$= 28.9\Omega$

**Problem 2, 30 points**
*2-port parameters and Transistor models*

Part a, 20 points
For the network at the right, give algebraic expressions for the four Z-parameters and for the four S-parameters.



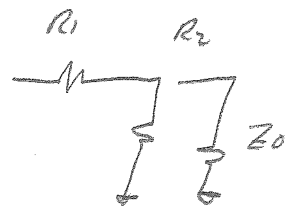

Then using R1=25 Ohms, R2=50 Ohms, , give numerical values.

By inspection:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \quad (3)$$

$$= \begin{bmatrix} 75\Omega & 50\Omega \\ 50\Omega & 50\Omega \end{bmatrix} \quad (1)$$

$S_{11}$: $Z_{in}\big|_{Z_L = 50\Omega}$ →

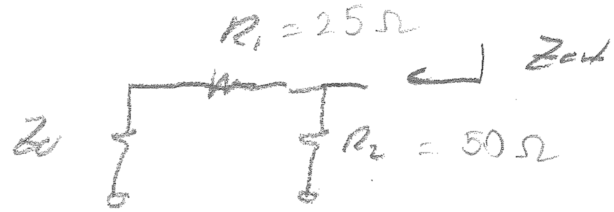



(1)

$$= R_2 \| Z_0 + R_1 = 50\Omega \| 50\Omega + 25\Omega = 50\Omega \quad (1)$$

$$S_{11} = \frac{\dfrac{R_2 \| Z_0 + R_1}{Z_0} - 1}{\dfrac{R_2 \| Z_0 + R_1}{Z_0} + 1} = \frac{1-1}{1+1} = 0 \quad (2) + (1)$$

$S_{22}$: $Z_{out}\Big|_{Z_g = Z_o} \rightarrow$



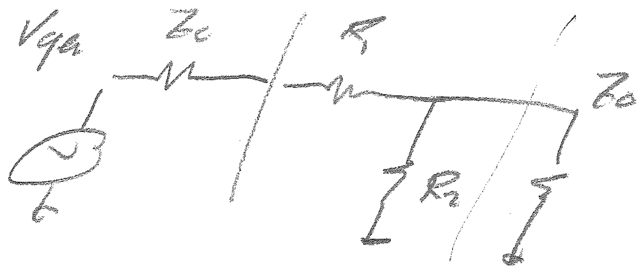

$$= R_2 \,//\, (R_1 + Z_o) = 50\,\Omega \,//\, 75\,\Omega = 30\,\Omega$$

(1) + (1)

$$S_{22} = \frac{\dfrac{R_2 \,//\, (R_1 + Z_o)}{Z_o} - 1}{\dfrac{R_2 \,//\, (R_1 + Z_o)}{Z_o} + 1} = \frac{0.6 - 1}{0.6 + 1} = -0.25$$

(1) + (1)

$S_{21} \Rightarrow$




(1)

(1)

$$S_{21} = 2\frac{V_o}{V_{gen}}\Big|_{Z_L = Z_o = Z_{gen}} = \frac{(Z_o \,//\, R_2)\; 2}{(Z_o \,//\, R_2) + R_1 + Z_o}$$

$$= \frac{(25\,\Omega)\, 2}{25\,\Omega + 25\,\Omega + 50\,\Omega} = 1/2$$

(2)

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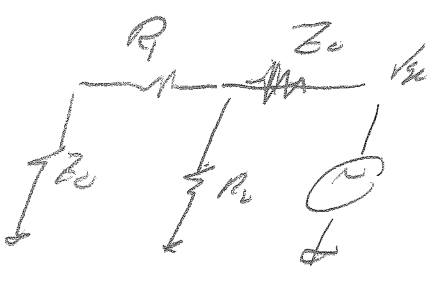
$S_{12} \Rightarrow$




(1)

$$S_{12} = 2\left(\frac{Z_o}{Z_o + R_1}\right) \cdot \frac{[(Z_o + R_1) \,//\, R_2]}{(Z_o + R_1) \,//\, R_2 + Z_o}$$

(1)

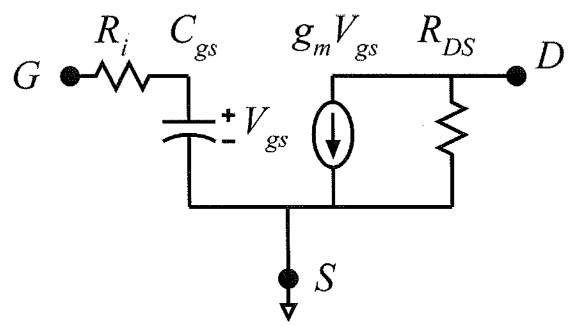
$$= 2\left(\frac{50\,\Omega}{50\,\Omega + 25\,\Omega}\right) \frac{(50\,\Omega + 25\,\Omega) \,//\, 50\,\Omega}{(50\,\Omega + 25\,\Omega) \,//\, 50\,\Omega + 50\,\Omega} = 1/2 \quad (\Rightarrow S_{12} = S_{21}$$

as is required.

(2)

Part b, 10 points

To the right is the equivalent circuit of a FET.




First give algebraic expressions for S11 and S21 as a function of frequency.

Then set the transconductance to 100 mS, Ri=5 Ohms, $C_{gs}$=50 fF, $R_{DS}$ =infinity, and calculate S21 at 10GHz.

$S_{11}$:

$$Z_{in} = R_i + 1/j\omega C_{gs}$$

$$Z_{in}/Z_0 = R_i/Z_0 + 1/j\omega C_{gs} Z_0$$

(2)

$$S_{11} = \frac{(R_i/Z_0 + 1/(j\omega C_{gs} Z_0)) - 1}{(R_i/Z_0 + 1/(j\omega C_{gs} Z_0)) + 1} = \ldots$$

(2)

$$S_{21} = 2\frac{V_o}{V_{gen}}\Bigg|_{Z_L = Z_0 = Z_{gen}} = \frac{2(1/j\omega C_{gs})}{1/j\omega C_{gs} + R_i + Z_0} \cdot -g_m (R_{ds}\|Z_0)$$

(2)

$$= \frac{-2 g_m (R_{ds}\|Z_0)}{1 + j\omega C_{gs}(R_i + Z_0)} = \frac{-2(100\,mS)(50\,\Omega)}{1 + j\,2\pi f(5\,\Omega + 50\,\Omega)\,50\,fF}$$

($f$ ↑ 10 GHz)

$$= \frac{-10}{1 + j\,0.173} = \frac{-10(1 - j\,0.173)}{1 + (0.173)^2}$$

(2)

$$S_{21} = -9.71 + j1.68$$

(2)



0.971

**Problem 3, 20 points**
*Elementary impedance matching network design.*

You are going to design an impedance-matching network to match a 20 Ohm resistor to a 50 Ohm generator at a frequency of 2 GHz.

Part (a), 10 points.

***Using the impedance-admittance charts that have been passed out***, design a lumped-element matching network. Use a series inductor and a shunt capacitor. Give the circuit diagram and the element values.

Point "A" $20\Omega \rightarrow z_L = \frac{20\Omega}{50\Omega} = 0.4 = z_A$ (1)

Point "B"

$z_B = 0.4 + j0.5$

added normalized reactance

$jx = j0.5 \Rightarrow jX = j(0.5)50\Omega$

$= j25\Omega$

$j\omega L = j25\Omega$

$\Rightarrow L = \frac{25\Omega}{2\pi(2GHz)} = 1.99\,nH$ (2) + (1/2)

$y_B = 1.0 - j1.2$

Point "C": $y_C = 1.0 - j0$

(2) + (1/2)

$\Rightarrow jb = j1.2 \Rightarrow jB = j\frac{1.2}{50\Omega}$

$= j0.024\,S$

$jB = j\omega C \longrightarrow C = \frac{1.2}{50\Omega(2\pi f)} = 1.91\,pF$

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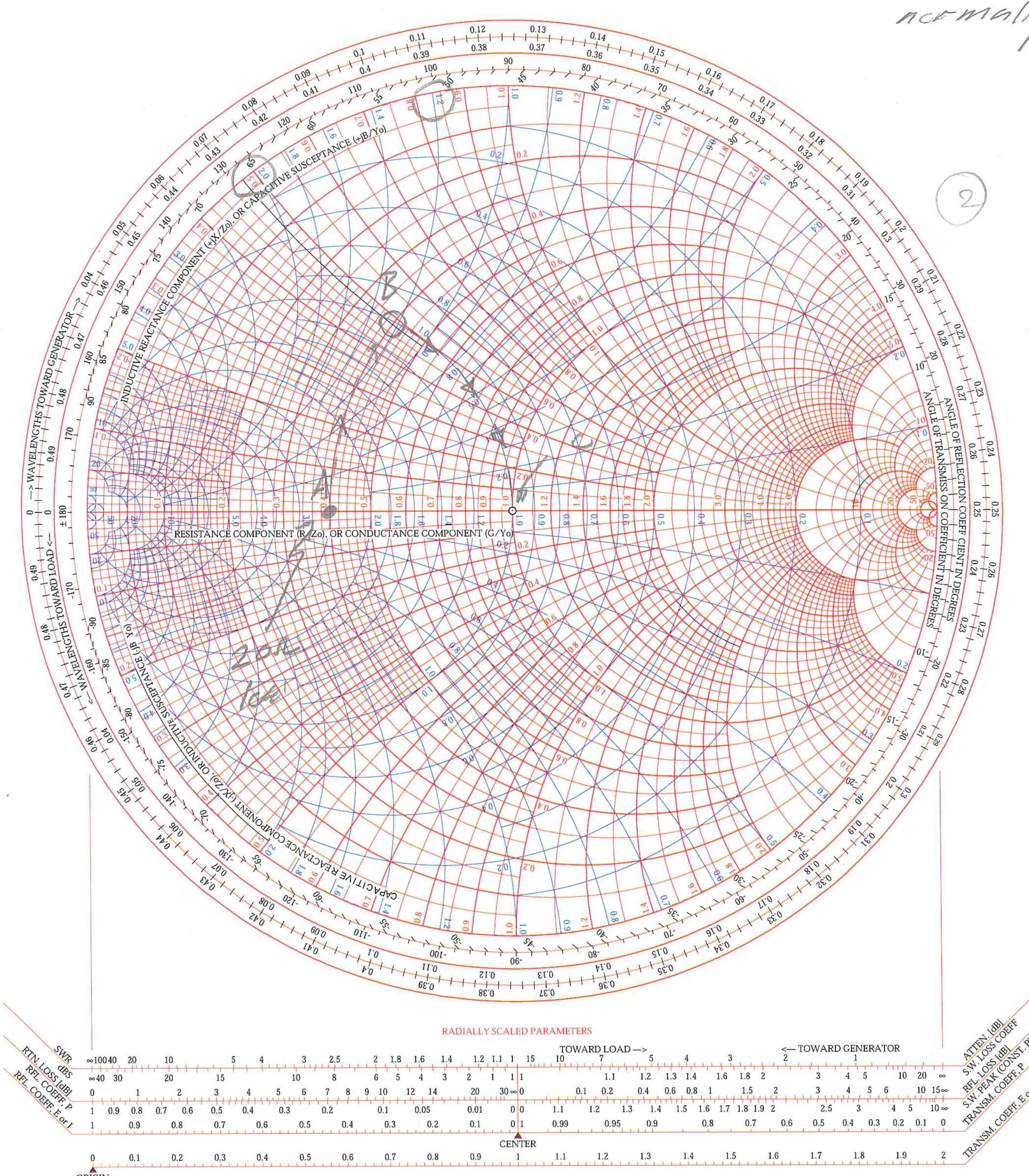
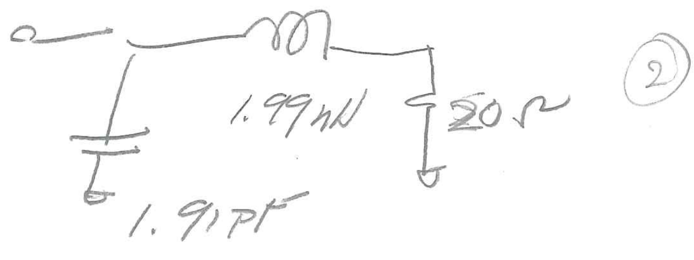
50Ω normalization

(2)




(2)

50

Part (b), ~~10~~ points.

Lumped inductors are not available to you. ***Using the impedance-admittance charts that have been passed out***, design a second matching network using a series transmission-line of ~~100~~ Ohms characteristic impedance and a shunt capacitor. The propagation velocity on the transmission line is 2/3 the speed of light. Give the circuit diagram and the element values (capacitance, length of line). .

from smith chart: A → B

line of 63° length ⇒ (3)

$$\text{Length} = \frac{63}{360°} \cdot \frac{(2/3 \cdot c)/2}{2(10^9)/s} = \underline{18.5\,mm}$$

"point B" $y = 1 - j1$ (1)

"point C" $y = 1 + j0$ (1)

$$j(\Delta b) = -j1 \Rightarrow jB = j\frac{1}{50\Omega} = j\omega C$$

$$\Rightarrow C = \frac{1}{\omega\, 50\Omega} = \frac{1}{2\pi f\, 50\Omega} = 1.6\,pF$$

↑ 2GHz (1)

| NAME | TITLE problem 3(b) | DWG. NO. |
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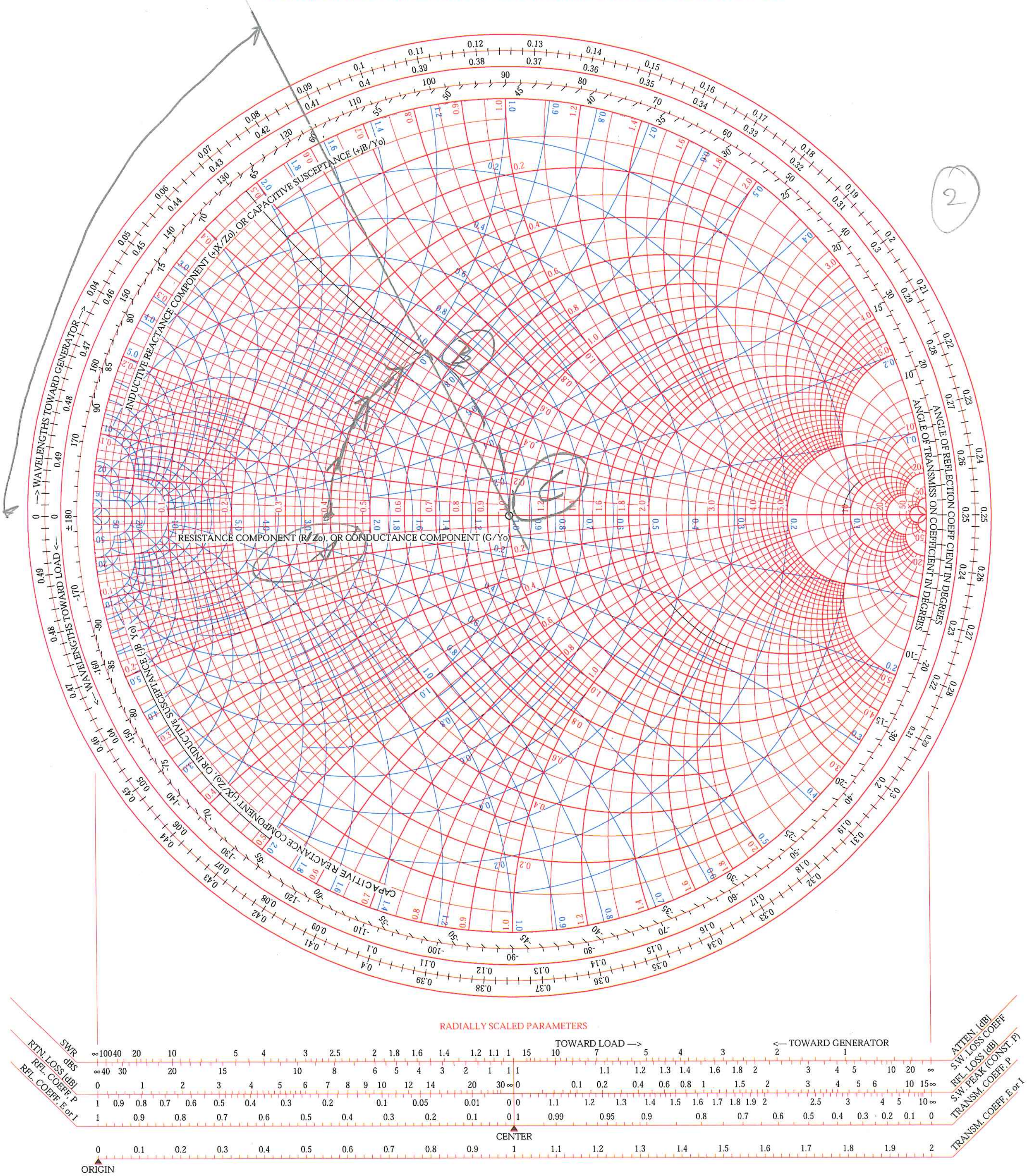
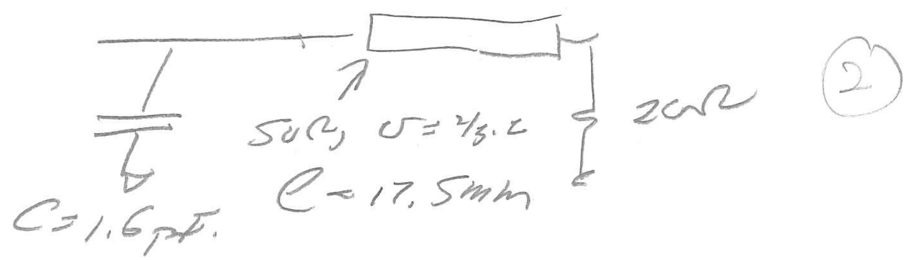
## NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



**Problem 4, 15 points**
*Transmission-lines and lumped elements*

Part a: 5 points

A transmission-line has propagation velocity of 2/3 the speed of light. Its characteristic impedance is 75 Ohms. It is one meter long. Find the total inductance and total capacitance in this cable.

(1)

(2)

$L = Z_0 T = Z_0 \cdot \ell / v$

$= 75\Omega \cdot \frac{1m}{2\cdot 10^8 m/s} = 75\Omega \cdot 5ns = 0.375\mu H$

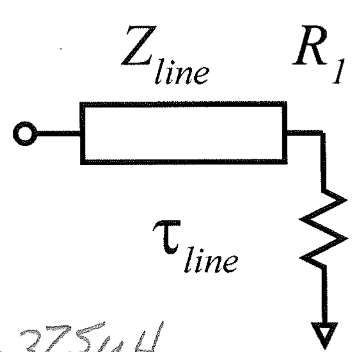
$C = T / Z_0 = 5ns / 75\Omega = 66.6pF$

(2)

Part b: 5 points

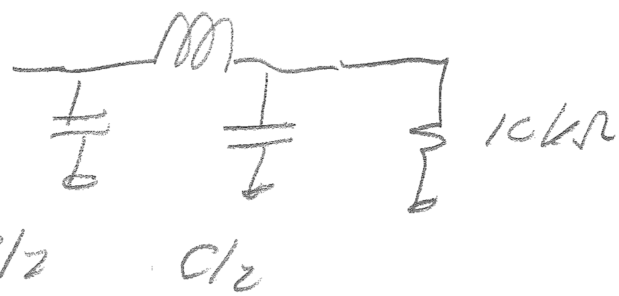
The above cable is loaded in a resistor R1=10 kOhm. Find the input impedance of the cable at a frequency of 1 MHz.

***Critical hint: use any and all reasonable approximations. Do not carry terms in the calculation which are negligible.***




$L = 0.375\mu H$

(2)




$C = 67pF$

$j\omega L = j2.35\,\Omega \ll 10k\Omega. \rightarrow j\omega L$ is negligible (1)

$1/j\omega C = -j\,2.37k\Omega$ (1)

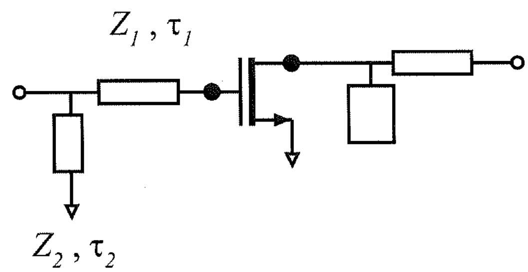
$$Z \cong 10k\Omega \,//\, -j2.37k\Omega = \frac{-j\,23.7\ K\Omega}{(10 - j2.37)}$$

$$= \frac{56.169 - j\,237}{10^2 + (2.37)^2}\ K\Omega$$

$$Z_{in} = 531.81 - j\,224.4\ \Omega$$

Part b: 5 points

You are now going to design a physical matching network for the input to a transistor amplifier. The design frequency is 10 GHz.




The series line #1 must have ~~100~~ 75 Ohm characteristic impedance and is 1/8 (guide) wavelength long at the design frequency.

The shunt line #2 must have 50 Ohm characteristic impedance and is 0.1 (guide) wavelength long at the design frequency.

The lines are microstrip lines. The substrate is 75 micrometer thick and has a dielectric constant of 12. Use the approximations $v = c/\sqrt{\varepsilon_r}$ and $Z_0 = 377\Omega \bullet (\varepsilon_r)^{-1/2} \bullet H/(H+W)$, where H is the substrate thickness and W the conductor width.

Compute the length and width of lines 1 and line 2

line 1 length (m): 1.0825 mm

line 1 width (m): 33.2203 μm

line 2 length (m): 0.866 mm

line 2 width (m): 88.245 μm

Series line (width) (1)

$Z_0 = \frac{377}{\sqrt{\varepsilon_r}} \cdot \frac{H}{H+W} = 75\,\Omega$

$\frac{H}{H+W} = 0.68915$

$0.31085 \cdot H = 0.68915 * W$

$W = 33.8203\ \mu m$

$W = 33.2303\ \mu m$

Shunt line (width) (1)

$Z_0 = 377 \frac{1}{\sqrt{\varepsilon_r}} \cdot \frac{H}{H+W} = 50$

$\frac{H}{H+W} = 0.45943$

$0.540574 * H = 0.45943 * W$

$W = 88.245\ \mu m$

$W = 88.245\ \mu m$

Series line (length) (1)

$v = c/\sqrt{\epsilon_r} = 86.6025 \times 10^5$ m/s

$\lambda_d f = v$

$\lambda_d = 8.66$ mm

length of series line $= 1/8 \cdot \lambda_d$

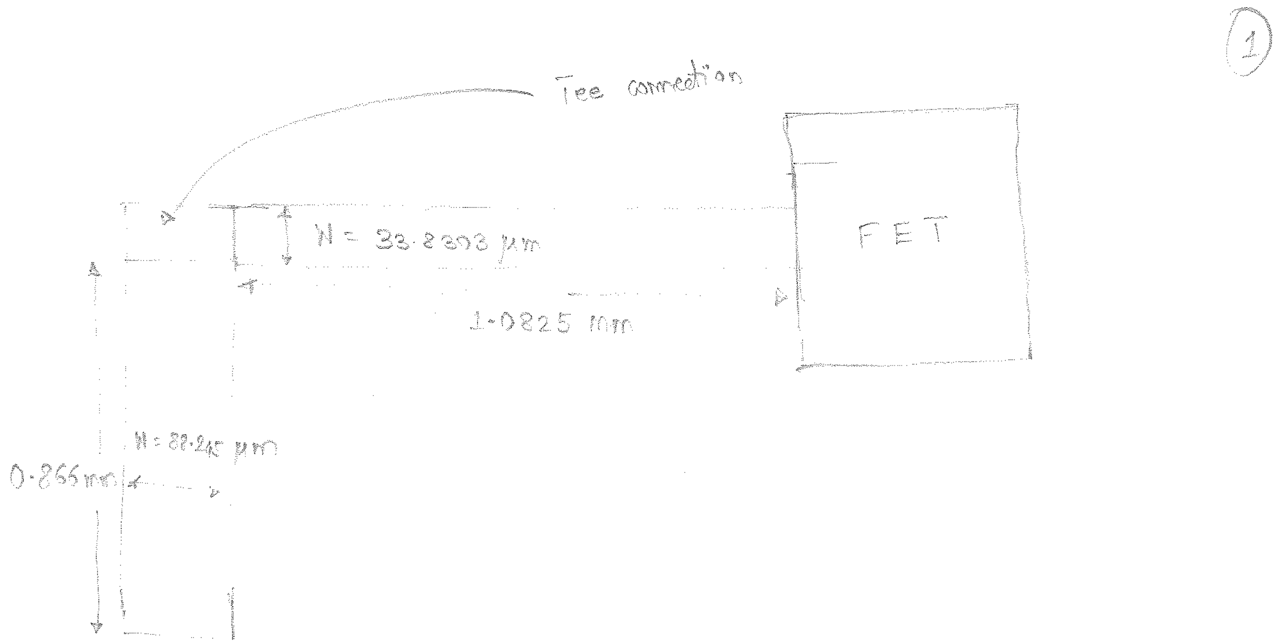
$= 1.0825$ mm

Shunt line (length) (1)

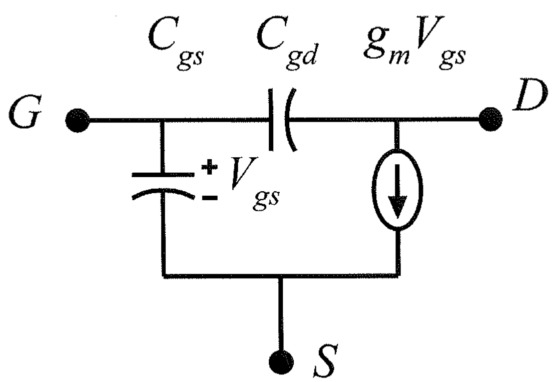
length of shunt line $= 0.1 \lambda_d$

$= 0.866$ mm

Please make a well-scaled sketch (top view) the matching network below with dimensions indicated.



**Problem 5, 15 points**
*resistive feedback amplifiers*




A FET has a transconductance of 1 mS per micron of gate width. $C_{gs}$ is 1 fF per micron of gate width, and $C_{gd}$ is 0.1 fF per micron of gate width

Design a resistive feedback amplifier with 20 dB gain S21 for a *75*Ohm system using this FET. Draw the circuit diagram with all element values and determine the following:

FET width=__146.7 μm__
transconductance=__0.1467 S__
$C_{gs}$ =__146.7 fF__
$C_{dg}$ =__14.67 fF__
amplifier 3-dB bandwidth (from nodal analysis or the time constant method).

$(A_v)_{dB} = 20 \Rightarrow \boxed{A_v = 10}$ $\boxed{Z_0 = 75\ \Omega}$

To avoid reflections, $(S_{11} = 0)$

$R_f = (1 + A_v) Z_0$

$\boxed{R_f = 825\ \Omega}$ (2)

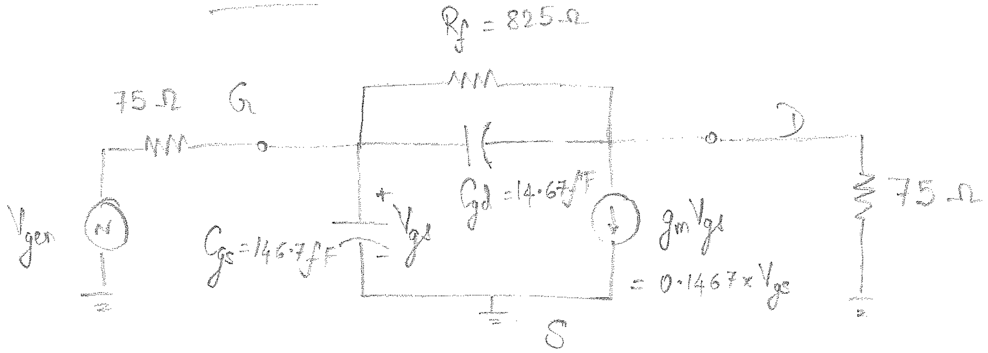
$g_m = (1 + A_v)/Z_0 = 0.1467$ S

$\boxed{g_m = 0.1467\ \text{S}}$ (2)

$\Rightarrow 10^{-3} \cdot W_g = 0.1467 \Rightarrow \boxed{W_g = 146.7\ \mu m}$ (2)

$$C_{gs} = 1\,fF * W_g = 146.7\,fF$$
$$C_{gd} = 0.1\,fF * W_g = 14.67\,fF$$
(2)

Circuit

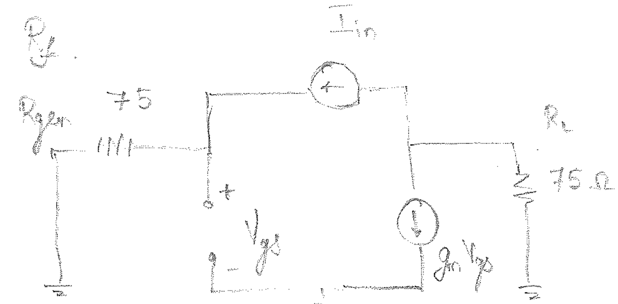


(2)

By MOTC method.

$$R_{11} = (75 \| 75) = 37.5\,\Omega$$ (1)

$R_{22}$ is calculated as follows. First remove $R_f$.




$$V_{gs} = I_{in} R_{gen}$$

$$V_{in} = V_{gs} + (g_m V_{gs} + I_{in}) \times R_L$$

$$= I_{in} \left[ R_{gen} + (g_m R_{gen} + 1) \times R_L \right]$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \left[ Z_o + (1 + g_m Z_o) * Z_o \right]$$

$$R_{22} = R_f \| R_{in}$$ (Now add $R_f$ back again)

$$R_{22} = R_f \| \left[ Z_o + (1 + g_m Z_o) * Z_o \right] = 225 \| \left[ 75 + 900.1875 \right]$$

$$R_{22} = 446.9\,\Omega$$ (3)

$$a_1 = R_{11} C_{gs} + R_{22} C_{gs} = (5.501 + 6.55)\ ps$$

$$a_1 = 12.051\ ps$$

$$\frac{1}{a_1} = \omega_{3dB} \Rightarrow f_{3dB} = \frac{1}{2\pi a_1} = 13.2 \text{ GHz}$$

$$\boxed{f_{3dB} = 13.2 \text{ GHz}}$$

(1)

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