ECE ECE145A (undergrad) and ECE218A (graduate)
TAKE HOME Final Exam. Return before Friday December 15, 2017

HONOR SYSTEM: Open book. You have 3 hrs. WORK WITHOUT HELP.
Use all reasonable approximations (5% accuracy is fine.),
AFTER STATING and justifying THEM.
Think before doing complex calculations. *Sometimes there is an easier way.*

Problem	Points Received	Points Possible
1A		5
1B		5
1C		5
1D		10
1E		10
1F		10 (218A only)
2A		10
2B		5
2C		5
2D		5
3A		10
3B		10
4A		5
4B		10
4C		10 (218A only)
total		95 (145A), 115 (218A)

I certify that

1) I have taken no more that 3 hours to work the exam. (or DSP) limits.

2) I have help from no one

3) I did not read the exam until the time I took the exam.

Signed: ________________________

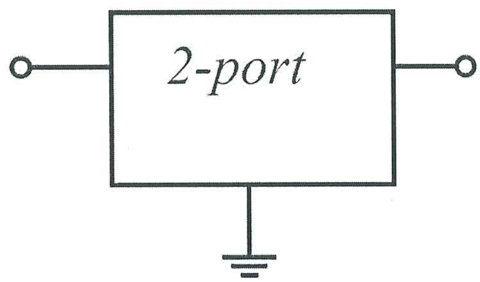
$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\|\Gamma_{out}\|^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]\text{if } K>1$$

$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot\text{if } K<1 \qquad K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta = \det[S]$$

Unconditionally stable if : (1) K>1 **and** (2) $\|\det[S]\|<1$

Problem 1, 35 points (145A), 45 points (218A)
Two-port properties, Power gain definitions

At a signal frequency of 10 GHz, a transistor has $S_{11}=0$, $S_{12}=0.2$, $S_{21}=2$ and $S_{22}=0.8$, as defined with a 50 Ohm impedance reference.	

part a, 5 points

Is the circuit unconditionally stable ? Hint: examine the expressions for Γ_{in} and Γ_{out}

Yes/No: ? NO

2
$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11}\Gamma_s}$$

2
$$= 0.8 + \frac{(0.2)(2)\Gamma_s}{1-(0)\Gamma_s} = 0.8 + 0.4\,\Gamma_s$$

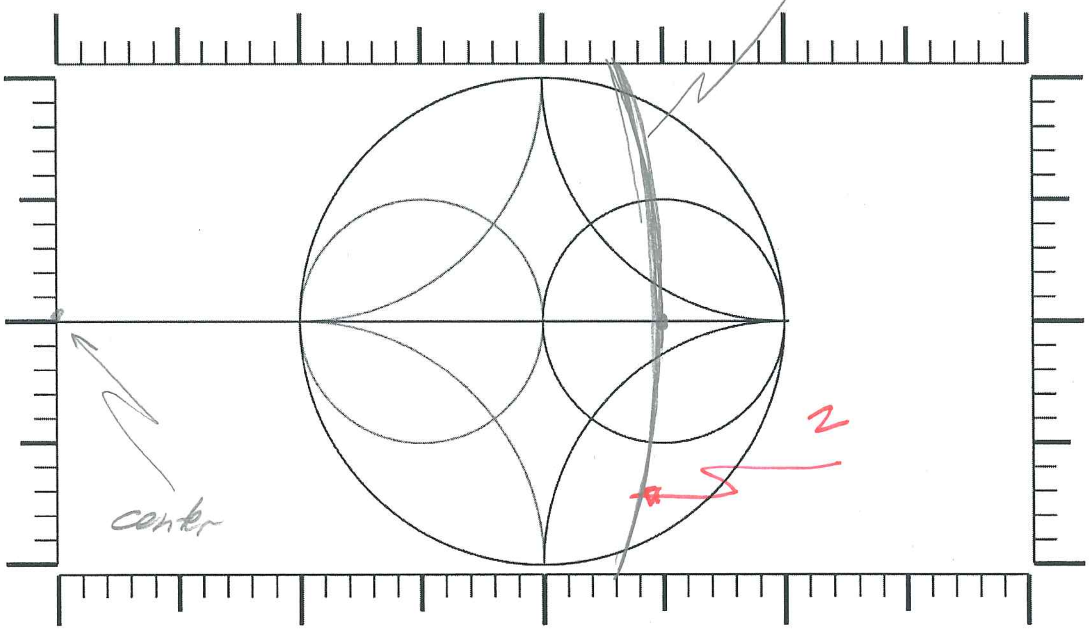
1
clearly, if $\Gamma_s = 1$, $\Gamma_{out} = 1.2$

so the 2-port is potentially unstable.

part b, 5 points

Please draw a SOURCE stability circle

Circle of radius = 2.5 centered at $\Gamma_S = -2$ crosses axis at $\Gamma_S = 0.5$.



1 Source stability circle is values of Γ_S which give $|\Gamma_{out}| = 1$

1 If $|\Gamma_{out}| = 1$ then $\Gamma_{out} = e^{-j\theta}$

1 $\Gamma_{out} = e^{-j\theta} = 0.8 + 0.4\,\Gamma_S$

$0.4\,\Gamma_S = e^{-j\theta} - 0.8$

1 $\boxed{\Gamma_S = 2.5 \cdot e^{-j\theta} - 2}$

Circle of radius = 2.5 centered at $\Gamma_S = -2$

θ	$e^{-j\theta}$	Γ_S
0	1	0.5
-180°	-1	-4.5

part c, 5 points

The transistor is connected to a 100 Ohm generator with 1mW available power. The output of the transistor is impedance matched to the load. How much power will be delivered to the load ?

load power = 1.9mW

$S_{11} = 0$ $S_{12} = 0.2$

$S_{21} = 2$ $S_{22} = 0.8$

1 $$\Gamma_S = \frac{100/50 - 1}{100/50 + 1} = \frac{1}{3}$$

1 this is the available gain

1 $$G_A = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2} \frac{1}{1 - ||\Gamma_{out}||^2} \cdot |S_{21}|^2$$

1 $$\Gamma_{out} = S_{22} + \frac{S_{21} S_{12} \Gamma_S}{1 - S_{11} \Gamma_S}$$

$$= 0.8 + 0.4\,\Gamma_S = 0.8 + 0.4(1/3) = 0.933\overline{3}$$

1 $$G_A = \frac{1 - |1/3|^2}{|1 - 1/3 \cdot 0|^2} \frac{|4|}{1 - ||0.933\overline{3}||^2} = \frac{0.888\overline{8}}{1} \frac{4}{1.871}$$

$$= 1.90.$$

$P_{del} = G_A \cdot P_{avg} = 1.9\,mW$

part d, 10 points

IF necessary, the transistor is stabilized, and is then impedance-matched to a 50 Ohm generator and a 50 Ohm load ? If the generator has 1mW available power, how much power will be delivered to the load ?

load power = 10 mW.

1 - transistor is potentially unstable

2 - so after stabilization & matching, our gain will be the MSG.

1 - $MSG = \dfrac{|S_{21}|}{|S_{12}|} = \dfrac{2}{0.2} = 10.$

1 $P_{Load} = 10 \cdot 1\,mW = 10\,mW$

part e, 10points

In the case of part D, above, if we consider the transistor, the possible stabilization network, and the matching networks to be, in combination, an amplifier, what are ||S21||, ||S11||, ||S22||, ||S12|| of the *amplifier* ?

$||S11^A||$ = 0

$||S12^A||$ = 0.316

$||S21^A||$ = 3.16

$||S22^A||$ = 0

2 - the amplifier has been matched both so $|S_{11}^A| = |S_{22}^A| = 0$

4 - the amplifier power gain is $|S_{21}|^2$ and is equal to the K of MSG = 10

$|S_{21}|^2 = 10 \rightarrow |S_{21}^A| = 3.16$

4 - and (tricky) the matching and stabilizing networks are reciprocal, so

$$\frac{|S_{21}^A|}{|S_{12}^A|} = \frac{|S_{21}^T|}{|S_{12}^T|} = \frac{2}{0.2} = 10$$

$$\Rightarrow |S_{12}^A| = \frac{|S_{21}^A|}{10} = 0.316$$

part f, 10 points (somewhat tricky, ece218c students only)

I now connect the *amplifier* of parts D and E to a 100 Ohm generator of 1 mW available power. The load is 25 Ohms. What is the power delivered to the load ?

Pload=______________

Sorry, I have messed up.

Can't be solved!

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-||\Gamma_L||^2)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_S\Gamma_L|^2}$$

$S_{11} = S_{22} = 0$ $\quad |S_{21}| = 3.16$ $\quad |S_{12}| = 0.316$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-||\Gamma_L||^2)}{|1 - S_{12}S_{21}\Gamma_S\Gamma_L|^2}$$

unfortunately, though we know $|S_{21}|, |\Gamma_S|, |\Gamma_L|$) and though we know $|S_{21}S_{12}|$, we don't know $S_{21}S_{12}$!

10 pts for noting that we can't solve

Cannot answer (!).

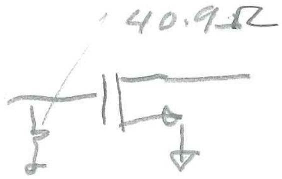
Problem 2, 25 points

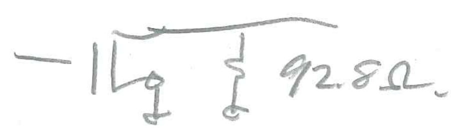
Unconditionally stable and Potentially unstable amplifier design, gain circles

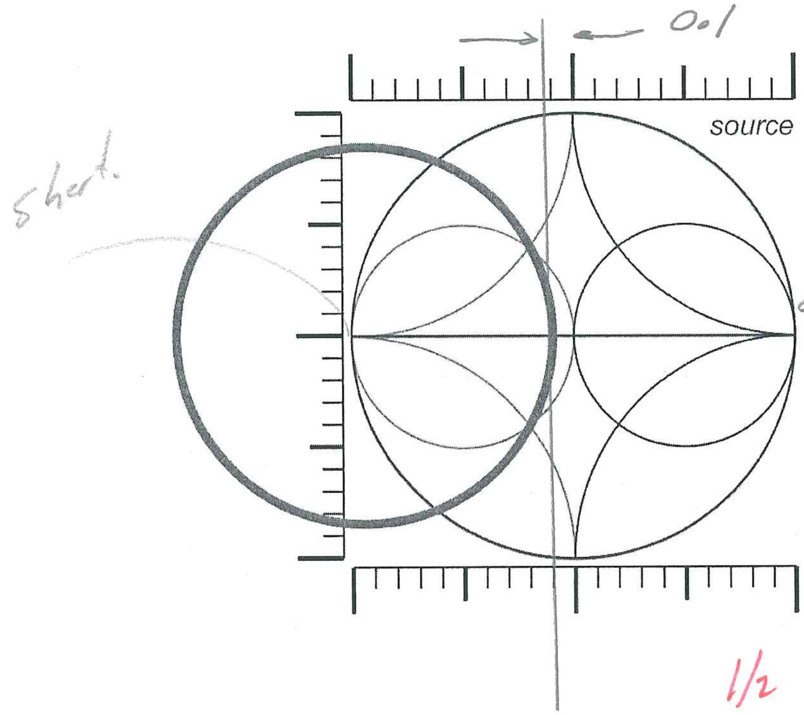
<u>part a, 10 points</u>

A MOSFET in common-source mode has ||S11|| < 1 and |S22|| >1, and has stability circles (50Ohm reference impedance) as below.

In the boxes below, draw circuit diagrams of *two* methods of stabilizing the transistor.







1/2 source stability circle: values of Γ_S giving $|\Gamma_{out}| = 1$

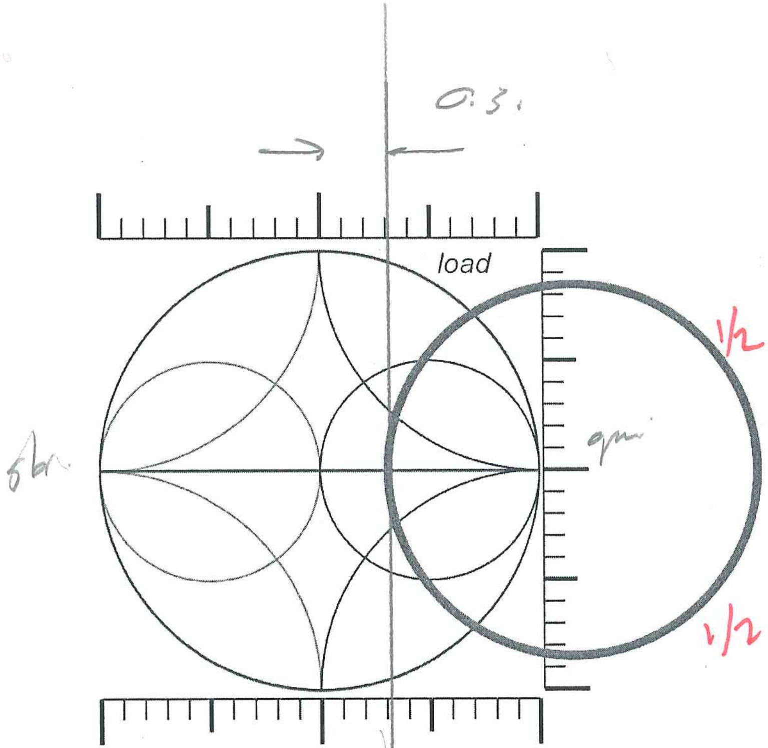
1/2 $\Gamma_{out} = S_{22} - \dfrac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$

1/2 $= S_{22}$ if $\Gamma_S = 0$

1/2 Now $|S_{22}| > 1$ so the point ($\Gamma_S = 0$) is unstable

1 so - inside of circle is stable

For



1/2 Load stability circle. Values of Γ_L giving $|\Gamma_{in}| = 1$

1/2 $$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$= S_{11}$ if $\Gamma_L = 0$

1/2 now $|S_{11}| < 1$ so the point $\Gamma_L = 0$ is stable.

1 so – outside of circle is stable

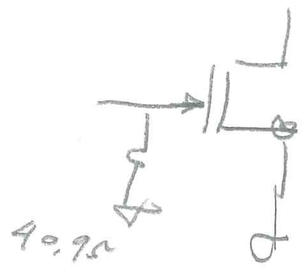
Source

1 stability circle is tangent to $\Gamma_S = -0.1$

$\Rightarrow R = 50\Omega \cdot \frac{1-0.1}{1+0.1} = 50\Omega \cdot \frac{9}{11} = 40.9\Omega.$

~~we need to~~

1 we can stabilize by adding a shunt 40.9Ω

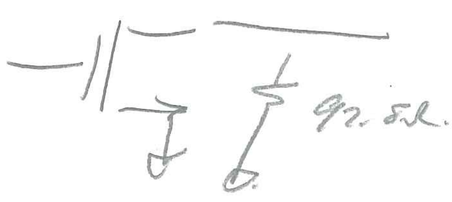


Load

1 stability circle is tangent to $\Gamma = 0.3$

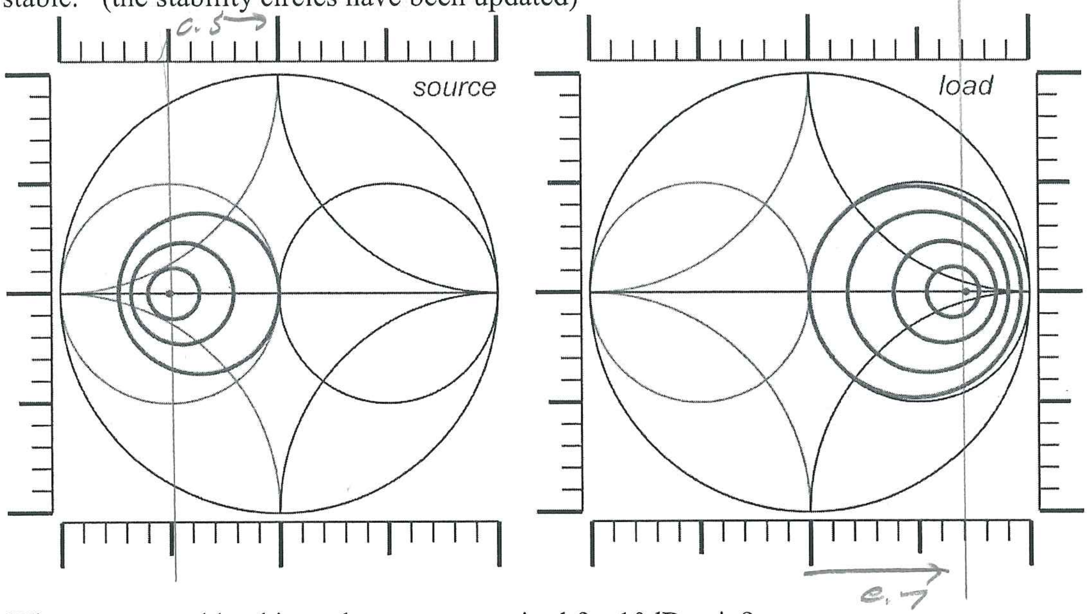
$R = 50\Omega \cdot \frac{1+0.3}{1-0.3} = 50\Omega \frac{13}{7} = 92.8\Omega$

1 we need to keep Z_L less than this, so we need a shunt connection



part b, 5 points

A transistor has available and operating gain circles as below. These are graphed in 1dB gain increments. The transistor MAG is 10dB, and the transistor is unconditionally stable. (the stability circles have been updated)



What source and load impedances are required for 10dB gain?

Zs= 16.7Ω   Zl= 283.3Ω

2.5 $Z_S = 50\Omega \dfrac{1-0.5}{1+0.5} = 50\Omega \cdot \dfrac{1}{3} = 16.7\Omega.$

2.5 $Z_L = 50\Omega \dfrac{1+0.7}{1-0.7} = 50\Omega \dfrac{17}{3} = 283.\bar{3}\Omega.$

part c, 5 points

We continue with the transistor of part B. If the generator impedance is 50Ohms, and the load impedance is matched to the transistor output impedance, then what power gain will the transistor have ?

Power gain =___7dB___

3 [A S/R generator is 3 steps (circles) away from Zopt. → 3dB gain decrease

7dB gain] 2

part d, 5 points

We continue with the transistor of part B. If the load impedance is 50Ohms, and the generator impedance is matched to the transistor input impedance, then what power gain will the transistor have ?

Power gain = 6dB

PKG.

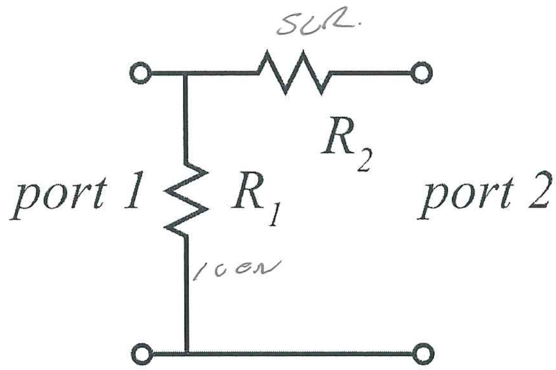
3 [A 50Ω load is 4 circles away

from Zopt → 4dB gain decrease] 2

Problem 3, 20 points
S parameters and Signal flow graphs

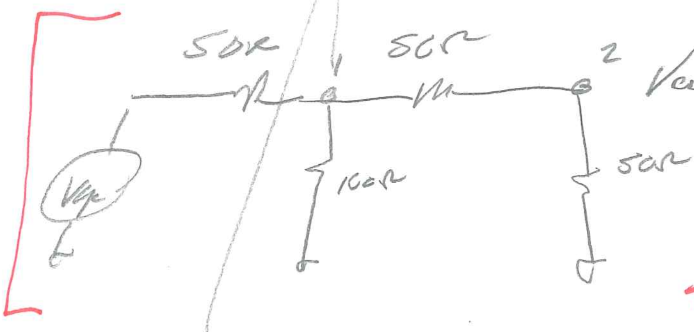
part a, 10 points

We have R1=100 Ohms and R2=50 Ohms
Given a 50 Ohm impedance reference, Find the four S-parameters of this network



S11=___0.___, S12=___1/2___ S21=___1/2___, S22=___1/4.___

1 [$Z_i = 100\Omega \,//\, (100\Omega) = 50\Omega.$]



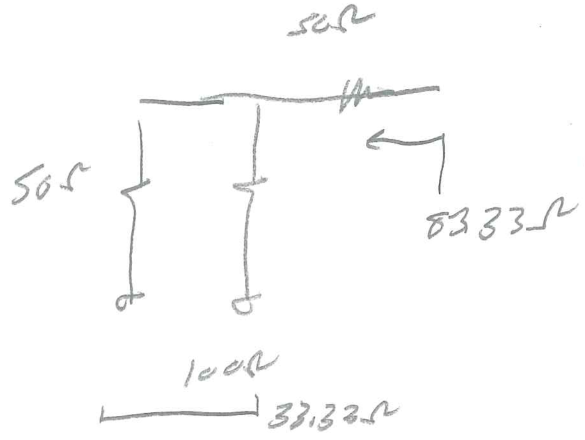
2 [$\frac{V_{out}}{V_{gs}} = \frac{50\Omega}{50\Omega+50\Omega} \cdot \frac{Z_i}{Z_i+50\Omega}$

$= \frac{1}{2} \cdot \frac{1}{2} = 1/4$]

2 [$S_{21} = 2\frac{V_o}{V_{gs}}\Big|_{Z_L=Z_S=Z_0} = 1/2$]

1 [Since $Z_i\big|_{Z_L=Z_0} = 50\Omega$, $S_{11}=0$.]

$S_{12} = S_{21}$ reciprocity.
$= 1/2$] 2



2 [$S_{22} = \frac{83.33/50-1}{83.33/50+1} = \frac{5/3-1}{5/3+1} = \frac{5-3}{5+3}$

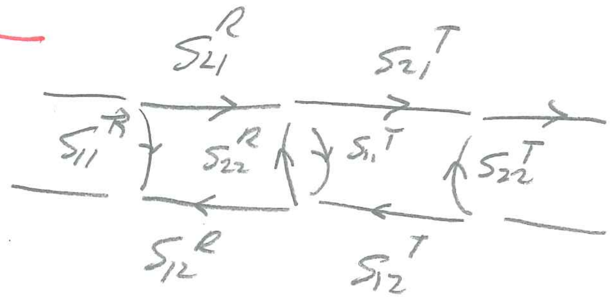
$= \frac{2}{8} = 1/4.$

part b, 10 points

We have a transistor having S11=0.5, S12=0.1, S21=3, S22=0.5. We connect the network of part a to the transistor's INPUT. Please find S21 of the combined network

S21 of the combined network = 1.714

5



4

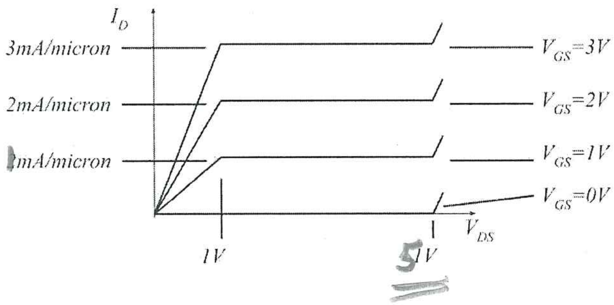
$$S_{21}\text{overall} = \frac{S_{21}^R S_{21}^T}{1 - S_{22}^R S_{11}^T}$$

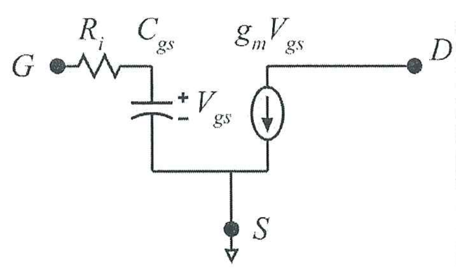
1

$$= \frac{(1/2)(3)}{1-(1/4)(1/2)} = \frac{3/2}{1-1/8} = \frac{3/2}{7/8}$$

$$= \frac{3 \cdot 4}{7} = \frac{12}{7} = 1.714$$

Problem 4, 15 points (145A), 25 points (218A)
Power amplifier design (the graph is updated)





A FET has the common-source characteristics above, normalize to FET gate width. You can infer gm from the data above. We have Ri=1/gm, and f_τ is 200 GHz. The signal frequency is 50 GHz

part a, 5 points
Assuming TEMPORARILY a 1 micron FET width, give the following.

gm= 1 mS, Cgs = 0.7956 fF Ri= 1kΩ

2 $\left[g_m = \dfrac{3mA/\mu m \cdot 1\mu m}{3V} = \dfrac{3mA}{3V} = 1 mS \right.$

2 $\left[f_T = \dfrac{g_m}{2\pi C_{gs}} \rightarrow C_{gs} = \dfrac{g_m}{2\pi f_T} = 0.795 fF \right.$

1 $\left[R_i = 1/g_m = 1k\Omega \right.$

f_T = 50 GHz

part b, 10 points

When we attempt to design impedance-transformation networks, we have found that the minimum load impedance we can synthesize is 10 Ohms.

What FET width should we select ? Wg= 133.3 µm

What maximum undistorted output power will we obtain ? Pout= 200 mW

What input power does this require ? Pin= 9.38 mW

1 $5V - 1V = 4V$ swing

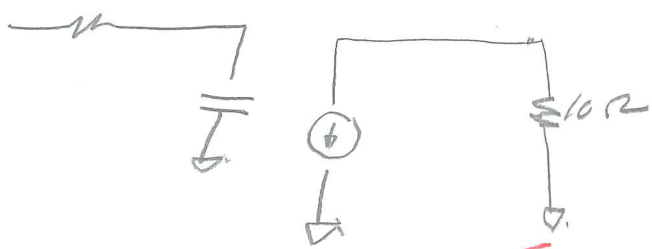
1 $R_L = 10\Omega \rightarrow I_{max} = \frac{4V}{10\Omega} = 400mA$

1 max current $= 3mA/\mu m \rightarrow$ width $= \frac{400mA}{3mA/\mu m}$

$= 133.3\ \mu m$

2 $\Delta V = 4V$. $\Delta I = 400mA$.

$P_{max} = 1/8 \cdot \Delta V \cdot \Delta I = \frac{4V \cdot 400mA}{8} = \underline{200mW}$



$g_m = 1mS/\mu m \cdot 133\mu m$

$= 133\ mS$

1 $R_i = 1/g_m = 7.5\Omega$

$C_{gs} = \frac{g_m}{2\pi f_T} = 0.1057pF$

$$P_{out} = I_{out}^2 \cdot R_L$$

$$I_{in} = I_{out} \cdot \frac{2\pi f C_{gs}}{g_m} = I_{out} \cdot \frac{f}{f_T}$$

3

$$P_{in} = I_{in}^2 \cdot R_i = I_{out}^2 \cdot \left(\frac{f}{f_T}\right)^2 \cdot R_i$$

$$\frac{P_{out}}{P_{in}} = \frac{R_L}{R_i} \cdot \left(\frac{f_T}{f}\right)^2 = \frac{10\,\Omega}{7.5\,\Omega} \left(\frac{200\,GHz}{50\,GHz}\right)^2$$

$$= \frac{4}{3}(4)^2 = \frac{4^3}{3}$$

1

$$P_{in} = \frac{P_{out}}{4^3/3} = \frac{200\,mW}{4^3/3} = \frac{200\,mW}{21.3} = 9.38\,mW$$