# ECE ECE145A (undergrad) and ECE218A (graduate) 

Final Exam. Monday December 5, 2021, noon -3 p.m.
Open book. You have 3 hrs .
Use all reasonable approximations ( $5 \%$ accuracy is fine.) ,
AFTER STATING and justifying THEM.
Think before doing complex calculations. Sometimes there is an easier way.

| Problem | Points Received | Points Possible |
| :--- | :--- | :--- |
| IA |  | 5 |
| IB |  | 5 |
| IC |  | 5 |
| ID |  | 5 |
| ID |  | 5 |
| IF |  | 10 |
| LG |  | $10(218 \mathrm{~A}$ only $)$ |
| 2 |  | 10 |
| 3 |  | 10 |
| AA |  | 10 |
| MB |  | $10(218 \mathrm{~A}$ only $)$ |
| SA | 5 |  |
| SB |  | $10(218 \mathrm{~A}$ only $)$ |
| total | $70(145 \mathrm{~A}), 100$ (218A) |  |

$G_{T}=\frac{\left|S_{21}\right|^{2}\left(1-\left|\Gamma_{s}\right|^{2}\right)\left(1-\left|\Gamma_{L}\right|^{2}\right)}{\left|\left(1-\Gamma_{s} S_{11}\right)\left(1-\Gamma_{L} S_{22}\right)-S_{21} S_{12} \Gamma_{s} \Gamma_{L}\right|^{2}} \quad G_{P}=\frac{1}{1-\left\|\Gamma_{i n}\right\|^{2}} \cdot\left|S_{21}\right|^{2} \cdot \frac{1-\left|\Gamma_{L}\right|^{2}}{\left|1-\Gamma_{L} S_{22}\right|^{2}}$
$G_{a}=\frac{1-\left|\Gamma_{S}\right|^{2}}{\left|1-\Gamma_{S} S_{11}\right|^{2}} \cdot\left|S_{21}\right|^{2} \cdot \frac{1}{1-\left\|\Gamma_{\text {out }}\right\|^{2}} \quad G_{\max }=\frac{\left|S_{21}\right|}{\left|S_{12}\right|} \cdot\left[K-\sqrt{K^{2}-1}\right]$ if $K>1$
$G_{M S}=\frac{\left|S_{21}\right|}{\left|S_{12}\right|}$. if $K<1 \quad K=\frac{1-\left|S_{11}\right|^{2}-\left|S_{22}\right|^{2}+|\Delta|^{2}}{2\left|S_{21} S_{12}\right|} \quad$ where $\Delta=\operatorname{det}[S]$
Unconditionally stable if : (1) $\mathrm{K}>1$ and (2) $\|\operatorname{det}[S]\|<1$


Problem 1, 30 points (145A), 40 points (218A)
Power Gain Definitions
part a, 5 points
At 100 GHz , the transistor has
$S 11=-1 / 2, S 21=-4, S 12=0, S 22=+1 / 3$, (S-parameters using $50 \Omega$ normalization)

The generator has (250/3) Ohms source impedance and 1 mW available power. The load is $(50 / 3)$ Ohms.
$\xrightarrow{\sim}$

If we directly connect the generator to the transistor input, but impedance-match the load to the transistor output, what RF power will be delivered to the load ?

$$
\begin{aligned}
& \text { RF power delivered to the load }=1 / .25 \mathrm{MW} \\
& \text { Load is metchsd, source is not } \left.\rightarrow G_{A}\right]^{\prime} \\
& \left.L_{s}=\frac{Z_{s}\left(z_{0}-1\right.}{Z_{6} / z_{0}+1}=\frac{5 / 3-1}{5 / 3+1}=\frac{5-3}{5+3}=\frac{2}{8}=1 / 4 .\right] / \\
& \left.\Gamma_{\text {at }}=S_{22}+\frac{S_{21} s_{2} C_{3}}{1-S_{11} L_{3}}=\frac{1}{3}+0=\frac{1}{3}\right]^{\prime}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{15 / 16}{1+1 / 8} \cdot 16 \cdot \frac{1}{1-19} \\
& =\frac{15}{16+2} \cdot 16 \cdot \frac{9}{6}=\frac{15}{18} \cdot 16 \cdot \frac{9}{8} \\
& \begin{array}{l}
6+2 \\
\left.P_{2}=1 \mathrm{~mW} \cdot \frac{5}{5} \cdot 2 \cdot 8 \cdot \frac{9}{8}=1 \mathrm{~mW} \cdot \frac{45}{4}=11.25 \mathrm{~mW}\right]
\end{array}
\end{aligned}
$$

part b, 5 points
At 100 GHz , the transistor has $S 11=-1 / 2, S 21=-4, S 12=0, S 22=+1 / 3$, (S-parameters using $50 \Omega$ normalization)

The generator has 50 Ohms source impedance and 1 mW available power. The load is 50 Ohms .
If we directly connect the generator and load to the transistor, what RF power will be delivered to the load?

$\rightarrow \underset{\sim}{\square}$

$$
\begin{aligned}
P_{L} & \left.=\left\|S_{21}\right\|^{2} \cdot P_{1 \cup y}\right]^{2} \\
& =16 \cdot 1 m W \\
& =16 \mathrm{~mW} \cdot]
\end{aligned}
$$

part c, 5 points
At 100 GHz , the transistor has
$S 11=-1 / 2, S 21=-4, S 12=0, S 22=+1 / 3$, (S-parameters using $50 \Omega$ normalization)

The generator has (250/3) Ohms source impedance and 1 mW available power. The load is $(50 / 3)$ Ohms.
$\rightarrow \underset{\sim}{\square}$

If we impedance-match the generator to the transistor input, but directly connect the load to the transistor output, what RF power will be delivered to the load?



$$
\begin{aligned}
& =\frac{16}{(716)^{2}}=\frac{36}{49} \cdot 16
\end{aligned}
$$



At 100 GHz , the transistor has
$S 11=-1 / 2, S 21=-4, S 12=0, S 22=+1 / 3$,
(S-parameters using $50 \Omega$ normalization)
The generator has (250/3) Ohms source impedance and 1 mW available power. The load is $(50 / 3)$ Ohms.


If we place impedance-matching networks between the generator and the transistor, and between the transistor and the load, what RF power will be deliver to the load? RF power delivered to the load $=$ $\qquad$
[ [i) because $S_{12}: S_{21}=0 ; a_{i n}=S_{11}$ d $s_{i 6}=S_{32}$
([2) because $S_{21} \cdot S_{12}=0$

$$
\begin{aligned}
& 1\left[G_{\text {max }}=\frac{1}{1-\left\|s_{11}\right\|^{2}}\left\|S_{21}\right\|^{2} \frac{1}{1-\left\|s_{22}\right\|^{2}}\right. \\
& =\frac{1}{1-1 / 1 / 2 /^{2}} \cdot 16 \cdot \frac{1}{1-1 / 1 / 3 / 1^{2}} \\
& =\frac{1}{1-1 / 4} \cdot 16 \cdot \frac{1}{1-1 / 7} \\
& =\frac{4}{3} \cdot 16 \cdot \frac{9}{8}=4 \cdot 2 \cdot 3=24 \\
& 1[\text { Ped }=24 \mathrm{~mW}
\end{aligned}
$$

part e, 5 points


If we directly connect the generator and load to the transistor, $\rightarrow$ hat RF power will be delivered to the load?
This is <s,

RF power delivered to the load $=$ < 5 mw

$$
\begin{aligned}
& 1\left[\begin{array}{rl}
S_{1}=S_{11}, \quad \text { rust } & =s_{22} \text { because } s_{11} s_{12}=6 . \\
& =1 / 3
\end{array}\right. \\
& =-1 / 2=1 / 3 \\
& 1\left[\begin{array}{rl}
R_{i j} & =5 i 1, \\
& =-1 / 2
\end{array}\right. \\
& {\left[L_{s}=\frac{z_{s}\left(z_{0}-1\right)}{z_{6} / z_{0}+1}=\frac{5 / 3-1}{5 / 3+1}=\frac{5-3}{5+3}=\frac{2}{8}=1 / 4 .\right.} \\
& E_{c}=\frac{z_{c} / z_{0}-1}{z_{1} / z_{0} \rightarrow 1}=\frac{1 / 3-1}{1 / 3+1}=\frac{1-3}{1+3}=\frac{-2}{4}=-1 / 2
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{16 \cdot\left(\frac{15}{16}\right)\left(\frac{3}{4}\right)}{\left(1+\frac{1}{8}\right)^{2}\left(1+\frac{1}{6}\right)^{2}} \\
& =\frac{15 \cdot \frac{3}{4}}{\left(\frac{9}{8}\right)^{2}\left(\frac{7}{6}\right)^{2}} \\
& =15 \cdot \frac{3}{4} \cdot \frac{64}{81} \cdot \frac{36}{49} \\
& 1\left[P_{2}=1 m w .15 \cdot \frac{3}{4} \cdot \frac{64}{81} \cdot \frac{36}{49}=6.53 m W\right.
\end{aligned}
$$

part f, 10 points
At 100 GHz , the transistor has S11 $=(1 / 2+j / 2) \leftarrow$ note the change ! $\mathrm{S} 21=-4, \mathrm{~S} 12=0, \mathrm{~S} 22=+1 / 3$, (Sparameters using $50 \Omega$ normalization)

The generator has 50 Ohms source impedance and 1 mW available power. The load is 50 Ohms.


We impedance-match the generator to the transistor input and impedance-match the load to the transistor output .

Please find the following:
Input impedance of the transistor $Z_{i n, T}=50+V^{\prime} 100 \sim$ Source impedance presented to the transistor $Z_{S, T}=\boxed{\sigma}-j 100 \Omega$. Output impedance of the transistor $Z_{\text {out }, T}=$ cc $\Omega$
Load impedance presented to the transistor $Z_{L, T}=100 \Omega$


$\left[z_{500 t}=z_{12}^{*}=50-\sqrt{2} 100 \Omega\right.$

part g, 10 points (218A only)


We are desinging a *power amplifier*. We have independently determined from $V_{\max }$, $V_{\min }, I_{\max }$, etc., that the optimum large-signal transistor load impedance is $Z_{L, T}=200$ $\Omega$ and that the maximum output power, at clipping, is 100 mW . We impedance-match on the input.
note $\sin s_{21} \neq 0$
Please find the following:


$$
0
$$




Available generator power at which the amplifier reaches clipping $=$ $\qquad$
**This will required some hard thinking**

$$
\begin{aligned}
& \text { 2[ } \Gamma_{<T}=\frac{200 / 50-1}{200100+1}=\frac{4-1}{4+1}=3 / 5 \\
& {\left[P_{i h}=s_{11}+\frac{s_{21} s_{12} L 2}{1-s_{22} s_{2}^{2}}=0+\frac{4 \cdot \frac{1}{8} \cdot \frac{3}{5}}{1-\frac{1}{3} \frac{\frac{g}{5}}{5}}\right.} \\
& v=\frac{3 / 10}{1-1 / 5}=\frac{3}{10-2}=\frac{3}{8}
\end{aligned}
$$

$$
\begin{aligned}
& 2=\frac{1}{1-\frac{9}{64}} \cdot 16 \cdot \frac{1-\frac{7}{25}}{\left(1-\frac{1}{5}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \Rightarrow \frac{64}{55} \cdot 16 \frac{1-9 / 25}{\left(\frac{4}{5}\right)^{2}} \\
& =\frac{64}{55} \cdot 16 \cdot \frac{16}{25} \cdot \frac{26}{16}=\frac{64}{55} \cdot 16 \\
& \eta P_{\text {in }}=\frac{\text { mas atpat pcuer }}{q a i n}=\frac{100 \mathrm{~mW}}{\left(\frac{64}{56} \cdot 16\right)}= \\
& =\frac{100 \mathrm{~mW}}{64 \cdot 16} \cdot 56=5.47 \mathrm{~mW}
\end{aligned}
$$




A FET in common-source mode has operating and available gain circles as shown (50 Ohm impedance normalization). Find the optimum generator and load impedances (in complex Ohms).

$$
\begin{aligned}
& \text { optimum source impedance }=\frac{10 \Omega+j 20 \Omega}{\text { optimum load impedance }=\$ 0 \Omega+j 100 \Omega}
\end{aligned}
$$

$$
V[15, \text { opt }=-1 / 2+d / 2
$$

$$
\begin{aligned}
& n\left[\Gamma_{2, a p t}=1 / 2+j / 2\right. \\
& \frac{z_{\text {cod }}}{z_{0}}=\frac{1+\Gamma}{1-\Gamma}=\frac{1+1 / 2+j / 2}{1-1 / 2-j / 2}=\frac{3+j}{1-j} \\
& =\frac{3+j}{1-j} \frac{1+i}{1+i}=\frac{3+i+3 i-1}{2}=\frac{2+4 i}{2} \\
& =1+2 j \\
& \text { Zupt }=50 \boxed{6}+j 100 \Omega
\end{aligned}
$$

Problem 4, 10 points (145A), 20 points (218A)
2-port parameters and signal flow graphs

Part a, 10 points
Amplifiers A and B have S-parameters $S_{i j}^{A}=\left[\begin{array}{cc}0 & 0 \\ 2 & 1 / 2\end{array}\right]$ and $S_{i j}^{B}=\left[\begin{array}{cc}1 / 2 & 0 \\ 2 & 0\end{array}\right]$. (S-parameters using $50 \Omega$ normalization).


$$
Z_{o, \text { line }}=50 \Omega, l_{\text {line }} / v_{\text {line }}=\tau_{\text {line }}=1 \mathrm{nS} .
$$

Compute $S_{21}^{C}$ as a function of frequency. (hint: first draw a signal flow graph)



Part b, 10 points (218a only)
Amplifiers A has S-parameters
$S_{i j}^{A}=\left[\begin{array}{ll}0 & 1 / 4 \\ 2 & 1 / 2\end{array}\right]$.
(S-parameters using $50 \Omega$ normalization). Resistors $R_{\text {in }}=50 \Omega$ and $R_{\text {out }}=50 \Omega$ are added in parallel to the input and in series to the output.



Volvgen=1/3

$$
\rightarrow S_{21}=2 / 3=S_{12}
$$


$\eta$


$$
\begin{aligned}
S_{21}^{B} & =\frac{S_{2 i}^{\text {in } S_{21}^{A} S_{21}^{\text {out }}}}{1-S_{22}^{A} S_{11}^{0}-S_{22}^{i} S_{21}^{A} S_{11}^{0} S_{12}^{A}} \\
& =\frac{(2 / 3) 2(2 / 3)}{1-(1 / 2)(1 / 3)-(-1 / 3)(2)(1 / 3)(1 / 4)} \\
& =\frac{8 / 9}{1-1 / 6+1 / 18}
\end{aligned}
$$



Problem 5, 5 points (145A), 15 point (218A) Power amplifier design
part a, 5 points
Teledyne's 250 nm node In HBT (heterejunction bipolar transistor) technology has a maximum safe current of 1 mA per micrometer of emitter finger lenght. For wide bandwidth (high fax), the maximumremitter finger length is 5.0 micrometers; set the emitter length at this value, but use multiple emitter to further increase maximum output current to some desired value. The maximum safe collector-emitter voltage is 4.5 V , and the minimum (knee) voltage is 0.5 Volts.

What is the maximum RF power per 1 micron of emitter finger length?
Power ( 1 micron )=


We seek to design a multi-finger HBT cell layout that interfaces to 50 Ohms , with some parallel inductance to tune out the HBT output capacitance.

How many 5 micrometer length emitter fingers would that cell use? $\qquad$
What is the maximum output power of that cell? 40 mW
What would be the collector efficiency?

[want som lo ad
r


$$
\begin{aligned}
& ]\left[P_{m x x}=80 \mu \mathrm{~m} \cdot 0.5 \mathrm{mw} / \mu \mathrm{m}=40 \mathrm{~mW} .\right. \\
& {\left[\sum_{c}=\frac{1}{2} \frac{V_{m a x}-V_{m i n}}{V V_{m i n}}=\frac{1}{2} \cdot \frac{4.50-0.50}{4.51+0.5 V}\right.} \\
& \\
& =\frac{1}{2} \cdot \frac{4}{5}=\frac{2}{5}=40 \% .
\end{aligned}
$$

$$
\begin{gathered}
0.5 \\
x
\end{gathered}
$$

part b, 10 points ( 218 A only)
The transistor is now modelled by the the equivalent circuit to the right.

$$
\begin{aligned}
& g_{m}=(1 \mathrm{~mA} / \mu \mathrm{m} \cdot q / k T) *(\text { emitter } \\
& \text { length }) *(\text { number emitter fingers }) \\
& \beta=25 \\
& R_{b b}=10 / g_{m}
\end{aligned}
$$



Given the 50 Ohm load, the 5 micron emitter length, and the number of emitter fingers you have found earlier, what input power is necessary to produce this maximum output power?

$2.4 \cdot$


$$
\begin{aligned}
& \frac{V_{0}}{V_{i i} \cdot \frac{R_{b e}}{R_{b s}+R_{b l}}}=g_{m} R_{l} \\
& {\left[\frac{I_{0 n t}}{V_{\text {ort }}}=\frac{R_{b e}}{V_{i n}}=\frac{1}{R_{b e}+R_{b b}} . q_{n} R_{2}\right.}
\end{aligned}
$$

$$
2\left[\frac{I_{\text {ont }}}{I_{i n}}=\frac{1}{1+\frac{1}{\beta}}=\frac{\beta}{3+1}\right.
$$

2

$$
\begin{aligned}
\frac{P_{c i t}}{P_{1 i}} & =\frac{V_{0}}{V_{i}} \cdot \frac{I_{0}}{T_{\cdot i}}=\frac{\beta}{\beta+1} \cdot q_{m} \cdot R_{i} \cdot \frac{R_{b e}}{P_{b e}+R_{2 s}} \\
& =\frac{25}{26} \cdot \frac{50 n}{0.65 \Omega} \cdot \frac{\beta / q_{m}}{\frac{P_{m}}{}+10 / q m} \\
& =\frac{25}{26} \cdot \frac{50 \Omega}{0.650} \cdot \frac{25}{25+10}= \\
& =\frac{25}{26} \frac{50}{0.65} \frac{25}{35}=52 \cdot 8
\end{aligned}
$$

$$
\left[P_{i n}=\frac{P_{0}}{P_{0} P_{L_{i}}}=\frac{40 \mathrm{~mW}}{P_{0} / P_{i L}}=0.757 \mathrm{~mW}\right.
$$

plternots method:


From basic tran sicker circuit design ( $\sigma C \in / 37 A$ )

$$
\begin{aligned}
& \frac{V_{0}}{V_{i n}}=\frac{T_{L}}{R_{L_{n} T}}=\frac{R_{L}}{1 / g_{L_{H}}+R_{b L / \beta}} \\
& \frac{I_{0}}{I \cdot i}=\frac{B}{B+1} \text { so } \\
& \frac{p_{0}}{p_{i}} \cdot \frac{\beta}{\beta+1} \frac{R_{L}}{1 q_{m}+R_{3} / \beta} \\
& =\frac{\beta}{\beta+1} \frac{q_{m} R_{2}}{1+g_{m} / R_{b b} / \beta} \\
& =\frac{\beta}{\beta+1} \cdot q_{m} R_{L} \frac{1}{1+q_{n} \beta_{b d} / \beta} \\
& =\frac{\beta}{\beta+1} q_{n} R_{2} \frac{\beta / q m}{\frac{\beta / q m}{}+R_{b \sigma}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\beta}{B+1} \cdot q_{m} R_{2} \cdot \frac{R_{b e}}{R_{b e}+R_{b i}} \\
& =\text { Same answer as above }
\end{aligned}
$$

