ECE145a / 218a
Bilateral Tuned Amplifier Design

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Stability

Unconditionally stable if $\left\|\Gamma_{in}\right\| < 1$ for all possible Γ_L.

Equivalently: unconditionally stable if $\left\|\Gamma_{out}\right\| < 1$ for all possible Γ_S.

A 2-port is unconditionally stable if the Rollet stability factor K > 1, where

$$1)\ K = \frac{1-\left\|S_{11}\right\|^2-\left\|S_{22}\right\|^2+\left\|S_{11}S_{22}-S_{12}S_{21}\right\|^2}{2\cdot\left\|S_{12}S_{21}\right\|}$$

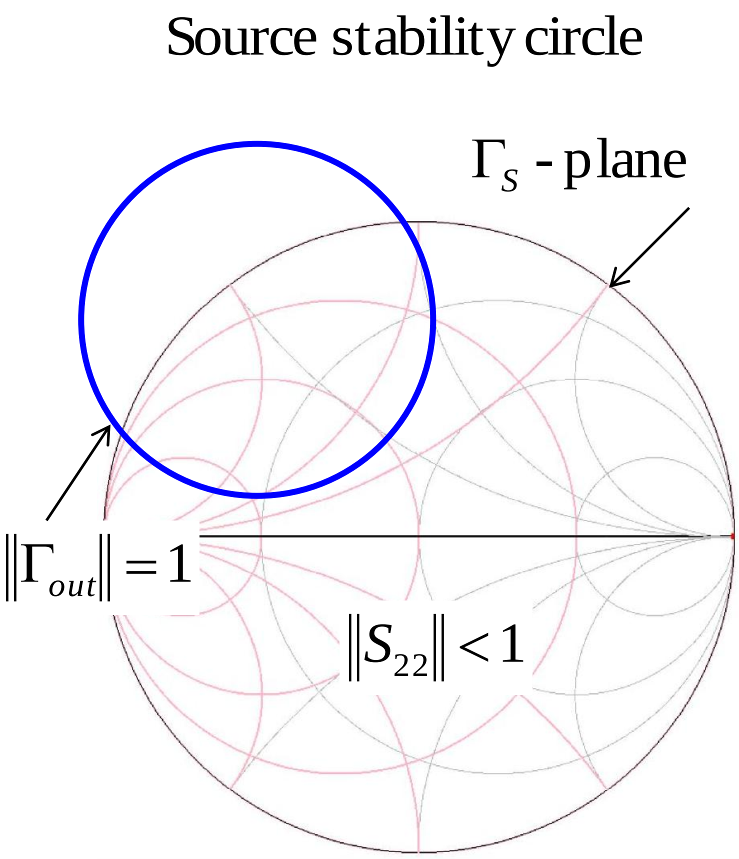
and

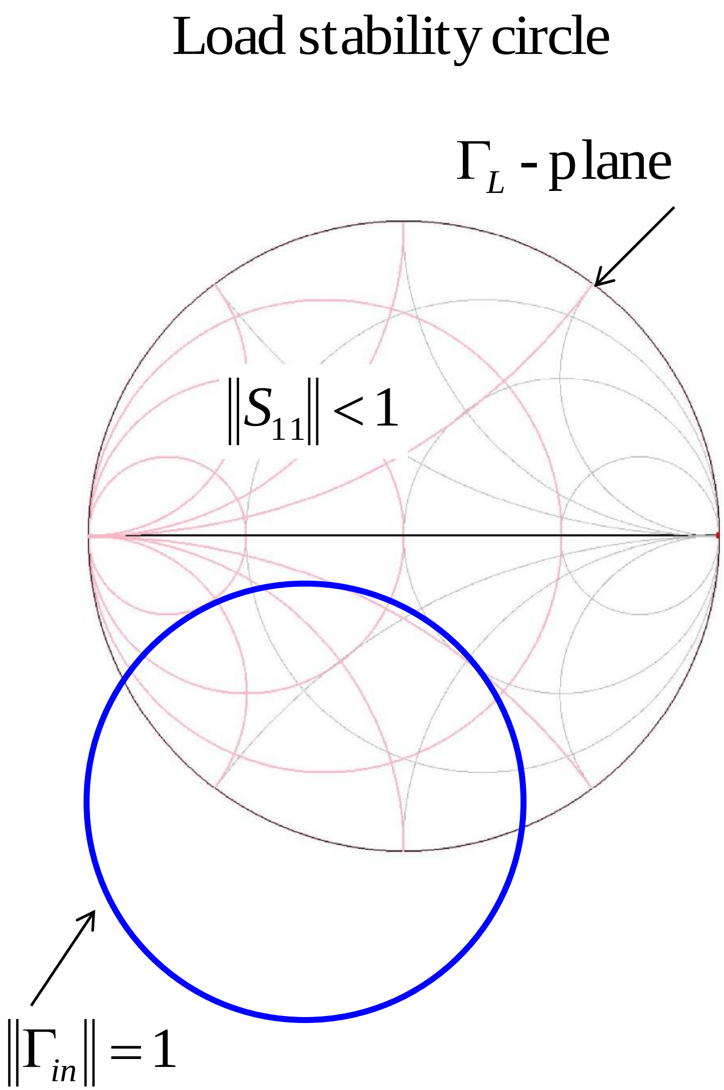
$$2a)\ \left\|S_{11}S_{22}-S_{12}S_{21}\right\| < 1.$$

An alternative 2^{nd} condition is that the stability measure B_1 be positive, where

$$2b)\ B_1 = 1+\left\|S_{11}\right\|^2-\left\|S_{22}\right\|^2+\left\|S_{11}S_{22}-S_{12}S_{21}\right\|^2$$

Potentially Unstable Amplifier

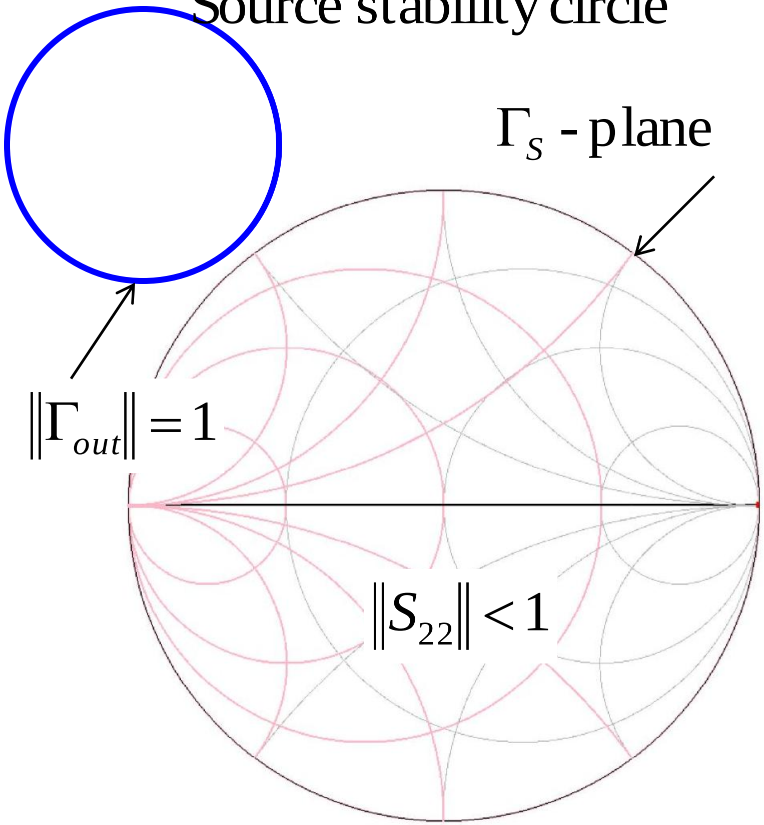




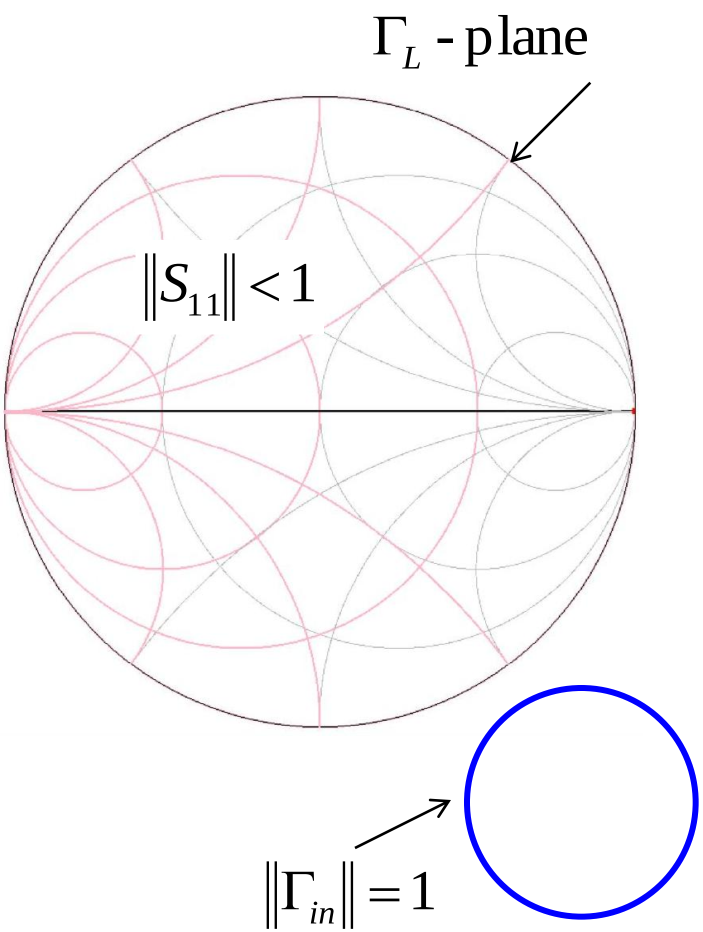
This is a test at one specific frequency; must test at all frequencies.

Unconditionally stable Amplifier

Source stability circle

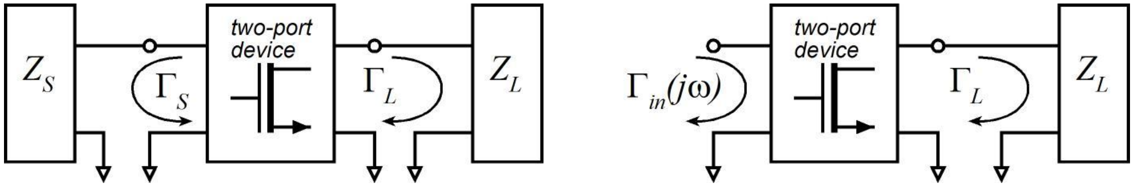


Load stability circle



This is a test at one specific frequency; must test at all frequencies.

Why Might MAG Not Exist ?



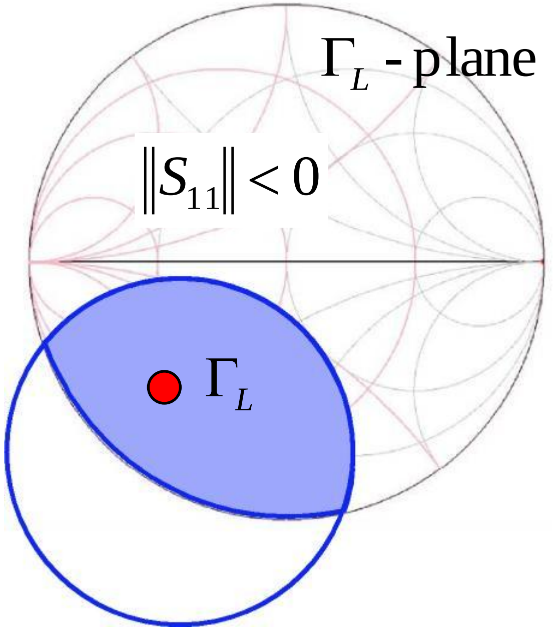
If the network is potentially unstable,
then by placing Γ_L within the load stability circle,
we force $\|\Gamma_{out}\| > 1$.

The device then has negative input resistance.

Appropriate choice of source impedance
will then cause oscillation.

If we adjust Γ_S and Γ_L to obtain
the highest possible gain,
we will instead obtain infinite gain (oscillation).

Load stability circle



The maximum available gain is infinite. This is *not* good.

Computing Maximum Available Gain

$$G_{\max} = \frac{P_{AVA}}{P_{in}} \xrightarrow{\text{iff } P_{AVA}=P_{load} \text{ and } P_{in}=P_{AVG}} \frac{P_{load}}{P_{AVG}} = G_T$$

we now need $\Gamma_L = \Gamma_{out}^*$ and $\Gamma_s = \Gamma_{in}^*$

we must simultaneously solve

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1-S_{22}\Gamma_L} = \Gamma_S^* \text{ and } \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1-S_{11}\Gamma_S} = \Gamma_L^*$$

and then substitute into

$$G_T = \frac{1-\|\Gamma_s\|^2}{\|1-\Gamma_{in}\Gamma_s\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2}$$

The calculations are long, and will not be shown.

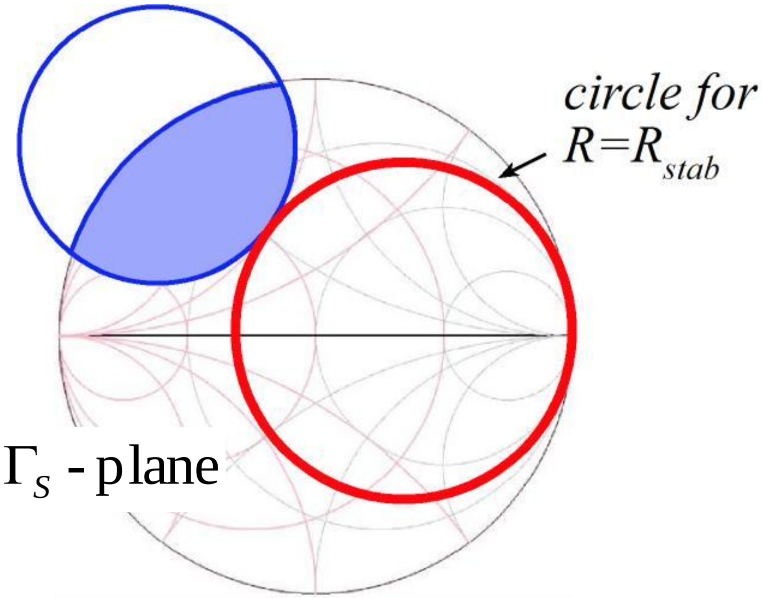
Maximum Available Gain

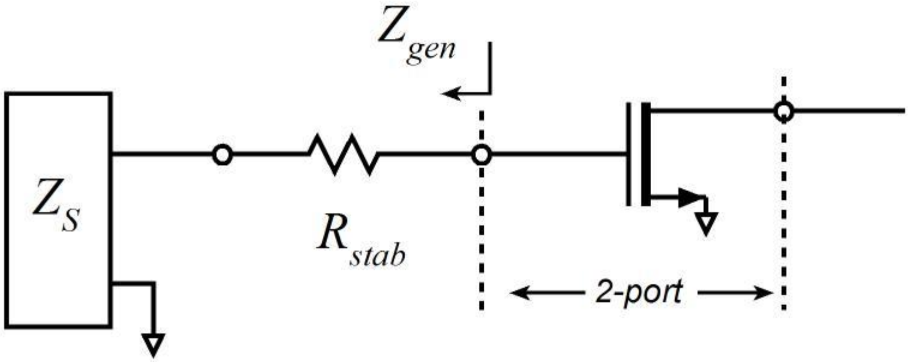
$$G_{\max} = \left\| \frac{S_{21}}{S_{12}} \right\| \cdot \left(K - \sqrt{K^2 - 1} \right)$$

$$\text{where } K = \frac{1 - \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2}{2 \cdot \|S_{12}S_{21}\|} \text{ (Rollet stability factor).}$$

Note that $G_{\max}$ is only defined for an unconditionally stable network,
i.e. $K > 1$ and $B_1 > 0$.

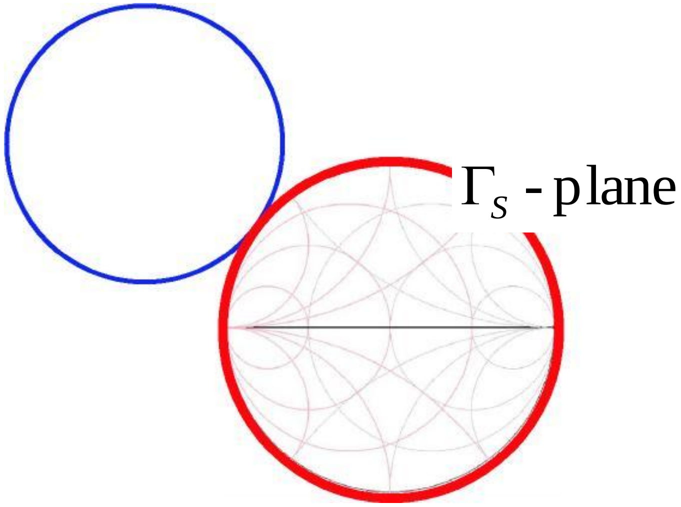
Stabilization: if device is potentially unstable

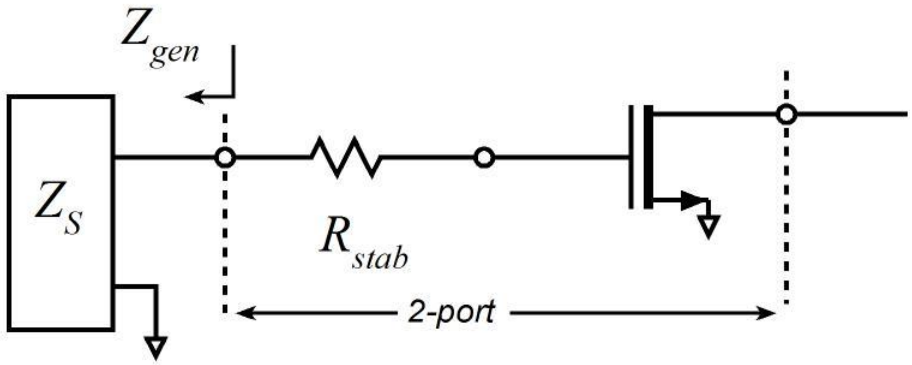




Adding series resistance R_{stab} as shown constains Z_{gen} to lie within stable region.

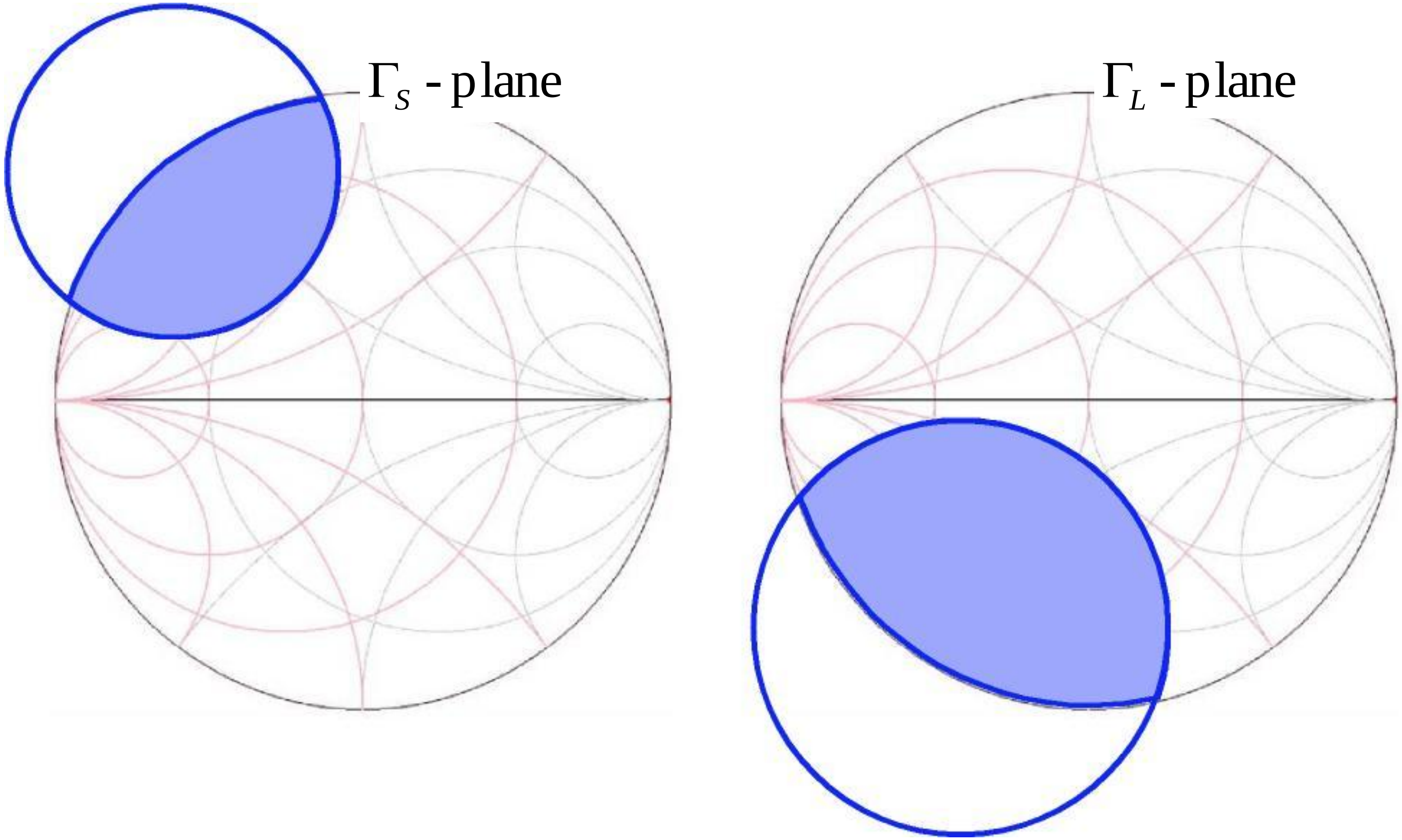
Stabilization: if device is potentially unstable





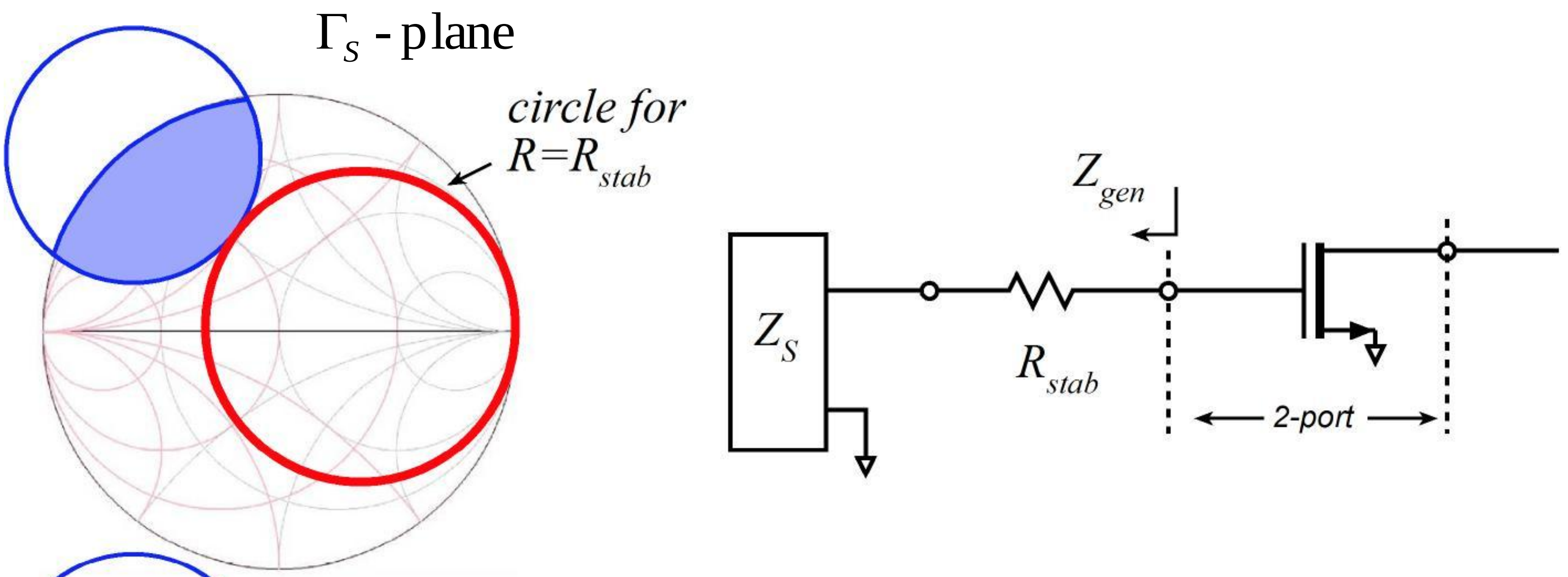
If we include R_{stab} in the 2-port being simulated in the CAD software, the stability cirle moves outwards as shown..

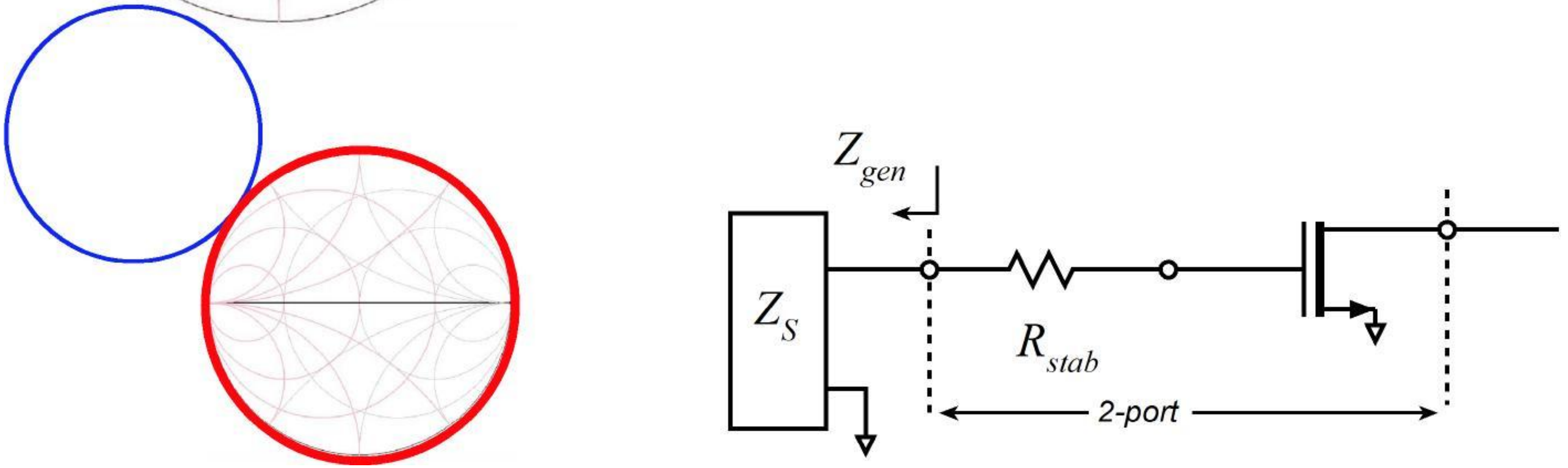
4 Obvious Stabilization Methods



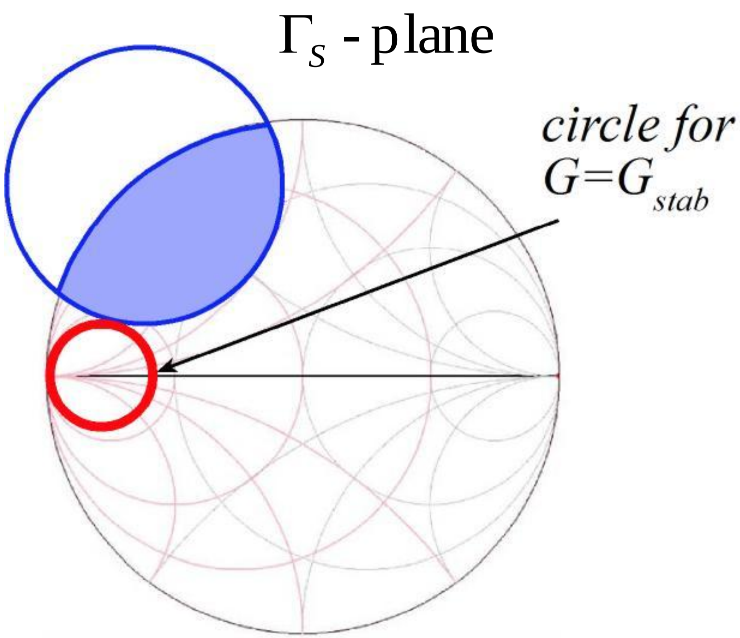
Given these stability circles, four stabilization methods are immediately apparent.

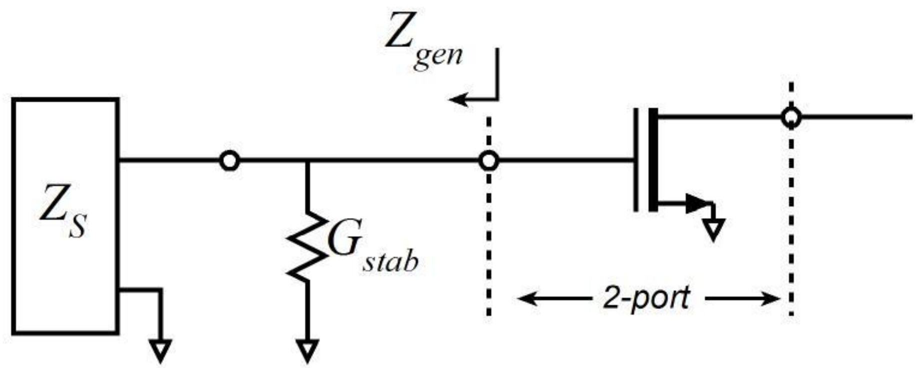
Series Stabilization On Input

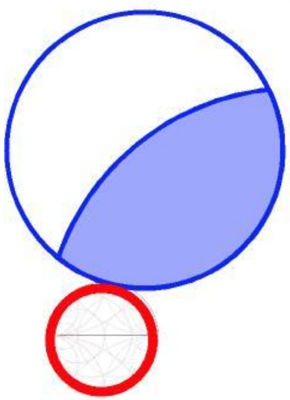


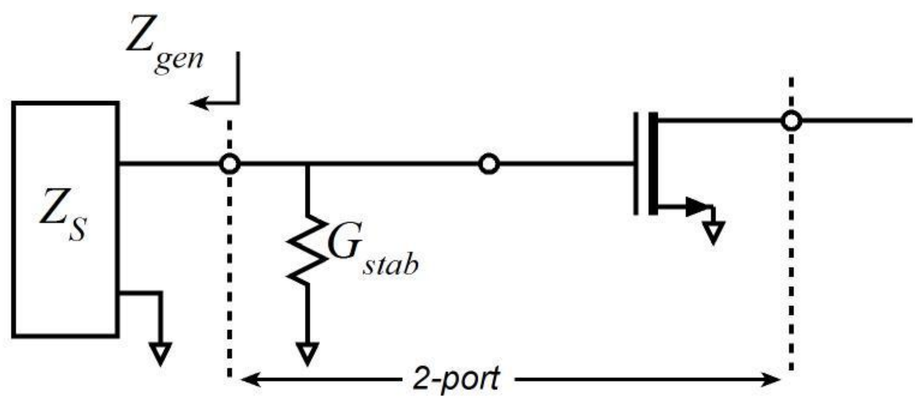


Shunt Stabilization On Input

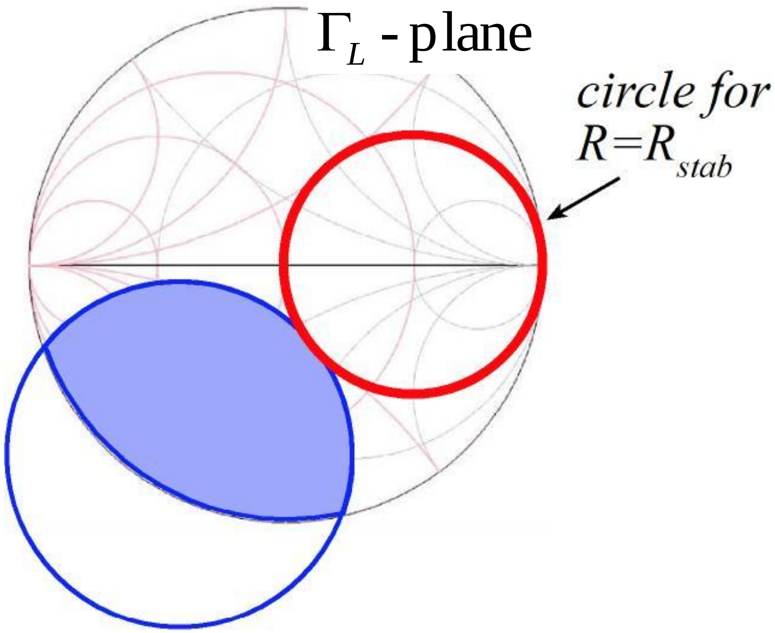


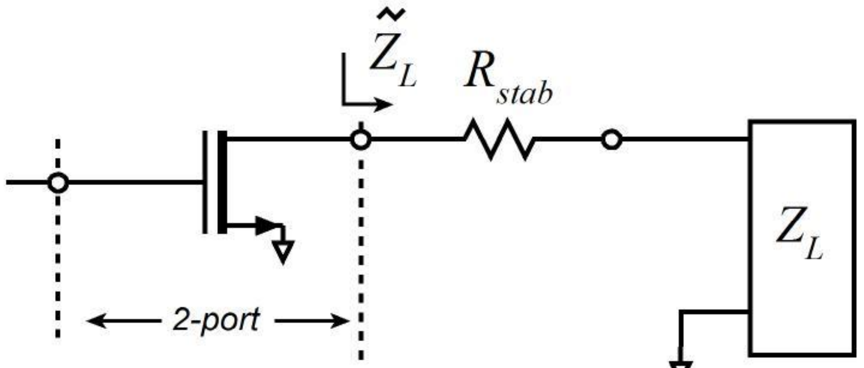


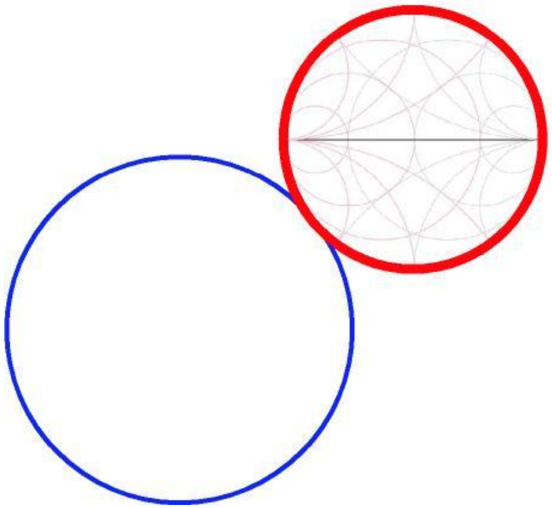


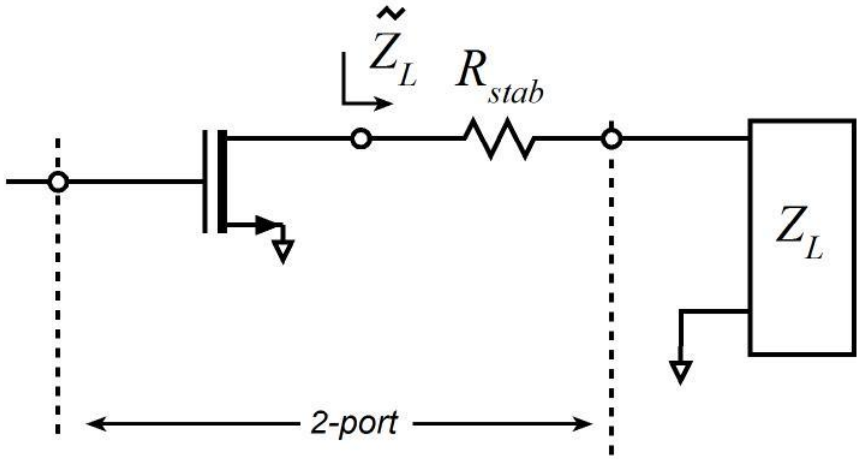


Series Stabilization On Output

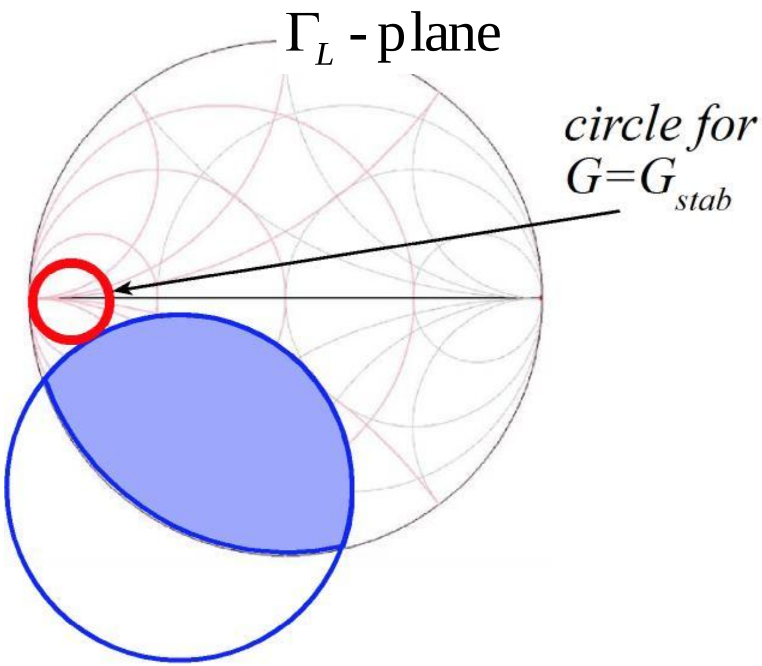


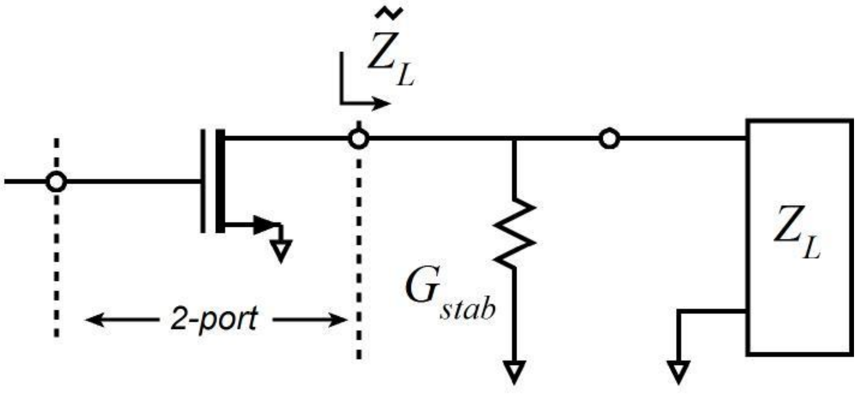


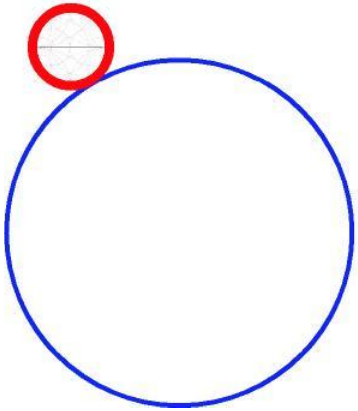


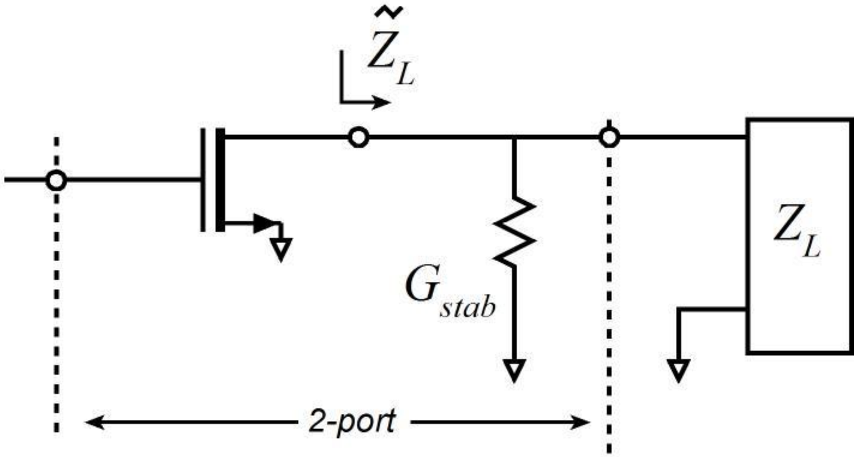


Shunt Stabilization On Output



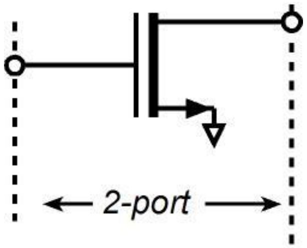






What Gain Do We Get After Stabilization ?

Before stabilization

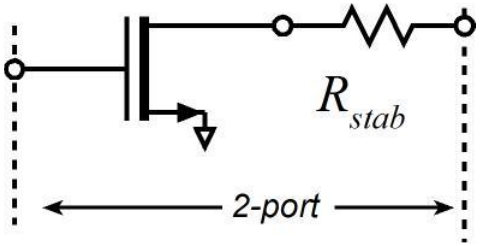


$*$ original $*$ S - parameters : S_{ij}

$$G_{\max} = \left\| \frac{S_{21}}{S_{12}} \right\| \cdot \left(K - \sqrt{K^2 - 1} \right)$$

$$= \text{undefined (unstable)}$$

After stabilization



$*$ changed $*$ S - parameters : $\tilde{S}_{ij}$

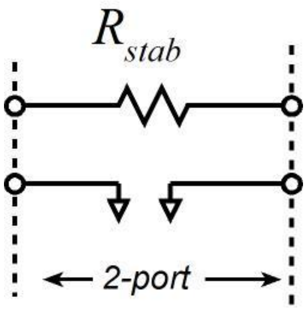
$$G_{\max\ \text{stable}} = \left\| \frac{\tilde{S}_{21}}{\tilde{S}_{12}} \right\| \cdot \left(\tilde{K} - \sqrt{\tilde{K}^2 - 1} \right)$$

but $\tilde{K} = 1$ (just stable)

$$G_{\max\ \text{stable}} = \left\| \frac{\tilde{S}_{21}}{\tilde{S}_{12}} \right\|$$

How do $\tilde{S}_{21}$ and $\tilde{S}_{12}$ compare to S_{21} and S_{12} ?

Consider First The Stabilization Network

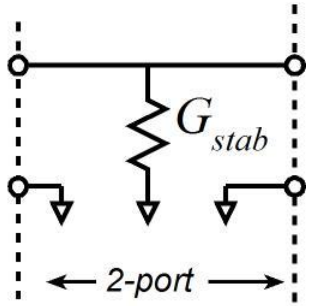


stabilizer S-parameters : S_{ij}^R

$$\left[S_{ij}^R\right]=\begin{bmatrix} S_{11}^R & S_{12}^R \\ S_{21}^R & S_{22}^R \end{bmatrix}$$

$$S_{11}^R=S_{22}^R=\frac{(R_{stab}+Z_0)-Z_0}{(R_{stab}+Z_0)+Z_0}$$

$$S_{21}^R=S_{12}^R=\frac{2Z_0}{R_{stab}+2Z_0}$$



stabilizer S-parameters : S_{ij}^G

$$\left[S_{ij}^G\right]=\begin{bmatrix} S_{11}^G & S_{12}^G \\ S_{21}^G & S_{22}^G \end{bmatrix}$$

$$S_{11}^G=S_{22}^G=\frac{(G_{stab}+Y_0)-Y_0}{(G_{stab}+Y_0)+Y_0}$$

$$S_{21}^G=S_{12}^G=\frac{2Y_0}{G_{stab}+2Y_0}$$

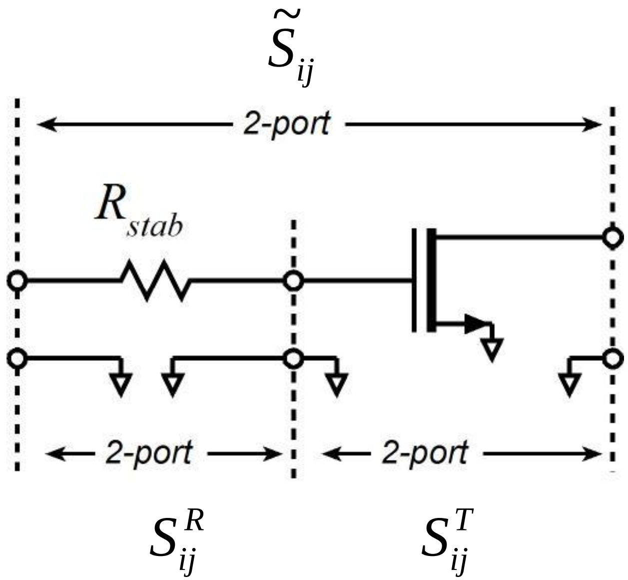
Key point is reciprocity : $S_{12}^R=S_{21}^R$ and $S_{12}^G=S_{21}^G$

What Gain Do We Get After Stabilization ?

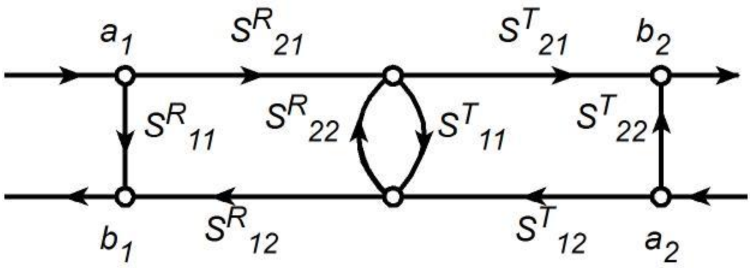
S_{ij}^{T} : transistor S-parameters

S_{ij}^{R} : stabilizer S-parameters

$\tilde{S}_{ij}$: stabilized transistor S-parameters



by inspection (Mason's gain rules) :



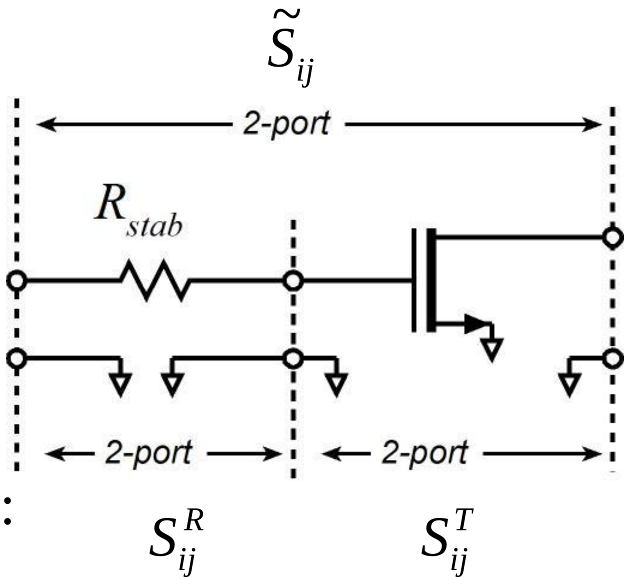
$$\tilde{S}_{21} = \frac{b_2}{a_1} = \frac{S_{21}^{R} S_{21}^{T}}{1 - S_{22}^{R} S_{11}^{T}} \quad \tilde{S}_{12} = \frac{b_1}{a_2} = \frac{S_{12}^{R} S_{12}^{T}}{1 - S_{22}^{R} S_{11}^{T}}$$

$$\frac{\tilde{S}_{21}}{\tilde{S}_{12}} = \frac{S_{21}^{R}}{S_{12}^{R}} \cdot \frac{S_{21}^{T}}{S_{12}^{T}} = \frac{S_{21}^{T}}{S_{12}^{T}} \;!!!$$

Consider More Carefully

Any 2 cascaded blocks follow this relationship :

$$\frac{\tilde{S}_{21}}{\tilde{S}_{12}} = \frac{S_{21}^{R}}{S_{12}^{R}} \cdot \frac{S_{21}^{T}}{S_{12}^{T}}$$



Passive reciprocal networks follow this relationship :

$$\frac{S_{21}^{R}}{S_{12}^{R}} = 1$$

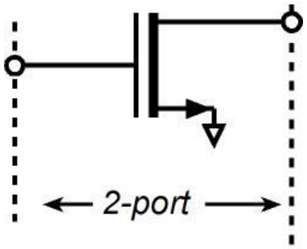
We therefore find :

$$\frac{\tilde{S}_{21}}{\tilde{S}_{12}} = \frac{S_{21}^{T}}{S_{12}^{T}}$$

Stabilizing does not change the ratio of S_{21} to S_{12}.

What Gain Do We Get After Stabilization ?

Before stabilization

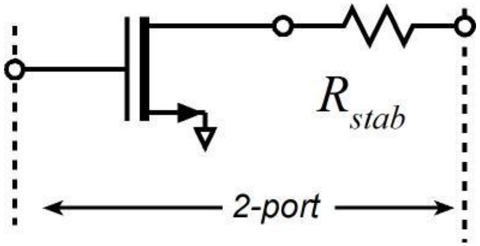


$*$ original $*$ S - parameters : S_{ij}

$$G_{\max} = \left\| \frac{S_{21}}{S_{12}} \right\| \cdot \left(K - \sqrt{K^2 - 1} \right)$$

$= \text{undefined (unstable)}$

After stabilization



$*$ changed $*$ S - parameters : $\tilde{S}_{ij}$

$$G_{\max} = \left\| \frac{\tilde{S}_{21}}{\tilde{S}_{12}} \right\| \cdot \left(\tilde{K} - \sqrt{\tilde{K}^2 - 1} \right)$$

but $\tilde{K} = 1$ (just stable)

and $\tilde{S}_{21} / \tilde{S}_{12} = S_{21} / S_{12}$

$$G_{\max\_stable} = \left\| \frac{S_{21}}{S_{12}} \right\|$$

Maximum stable gain $= G_{\max\_stable} = \|S_{21}\| / \|S_{12}\|$

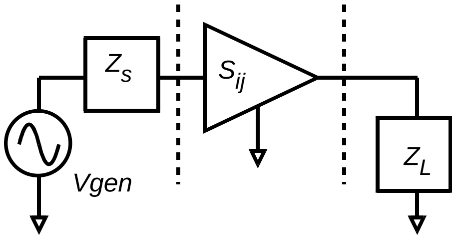
What Does Maximum Stable Gain Mean ???

The term : "Maximum stable gain" is too short to be precise.

Maximum stable gain is the maximum gain we can obtain
*if * we also guarantee that changing Z_S and Z_L
will not make the amplfier oscillate.

Design Tools: Power Gain Definitions

Transducer Gain

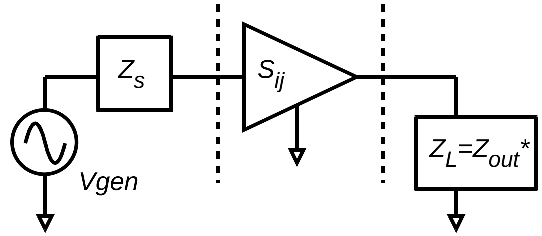


$$G_T = P_{load} / P_{av,gen}$$

$$= \frac{\text{load power}}{\text{power available from generator}}$$

$$= \text{general-case gain}$$

Available Gain

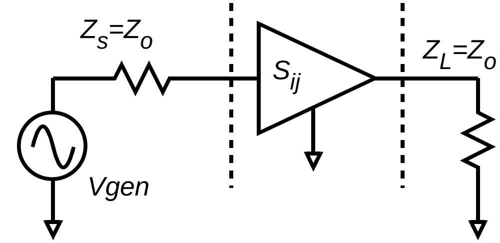


$$G_A = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power available from amplifier}}{\text{power available from generator}}$$

$$= \text{gain with output matched}$$

Insertion Gain

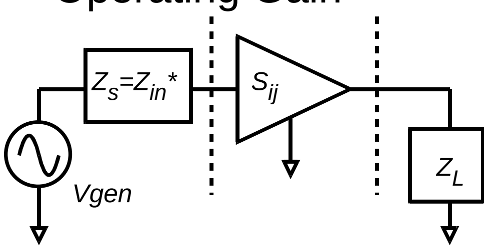


$$\left\| S_{21} \right\|^2 = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power delivered to } Z_o \text{ load}}{\text{power available from } Z_o \text{ generator}}$$

$$= \text{gain in a 50 Ohm enviroment}$$

Operating Gain

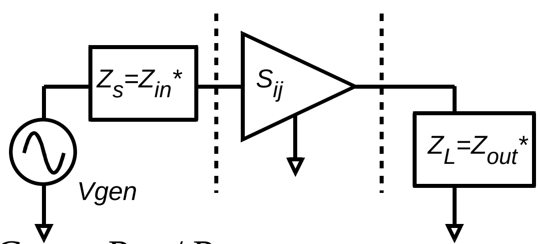


$$G_P = P_{load} / P_{gen,delivered}$$

$$= \frac{\text{load power}}{\text{power delivered from generator}}$$

$$= \text{gain with input matched}$$

Maximum Available Gain



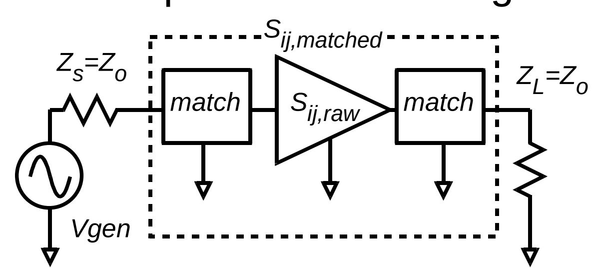
$$G_{Max} = P_{av,a} / P_{gen,delivered}$$

$$= \frac{\text{power available from amplifier}}{\text{power delivered from generator}}$$

$$= \text{gain with both ports matched}$$

...MAG may not exist...

After impedance-matching:



$$\left\| S_{21,matched} \right\|^2 = G_{\max,raw}$$

$$S_{11,matched} = S_{22,matched} = 0$$

....but only if unconditionally stable...

Unilateral Power Gain

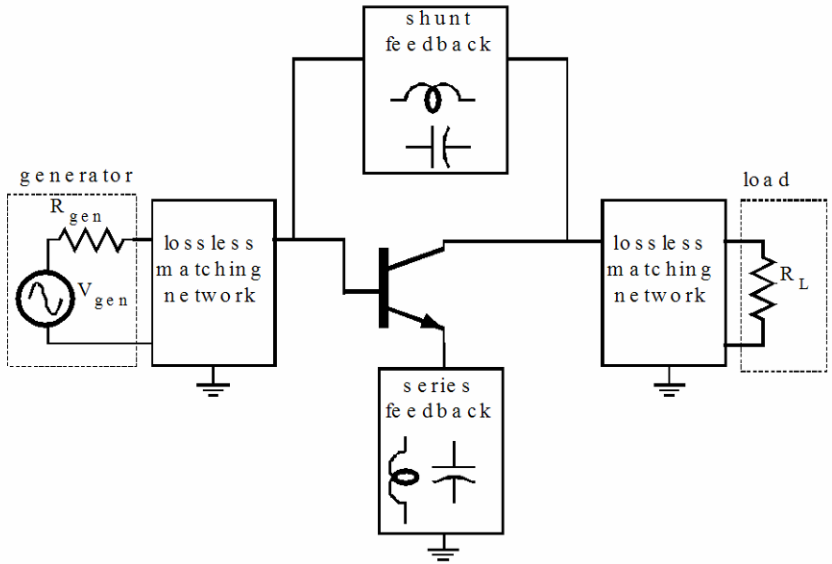
1) Cancel device feedback with external lossless feedback

$$\rightarrow Y_{12} = S_{12} = 0$$

2) Match input and output

Resulting power gain is Mason's Unilateral Gain

$$U = \frac{|Y_{21} - Y_{12}|^2}{4(G_{11}G_{22} - G_{21}G_{12})}$$



Monolithic amplifiers are not easily made unilateral

$\rightarrow$ U mostly of historical relevance to IC design

For simple BJT model, U rolls off at -20 dB/decade

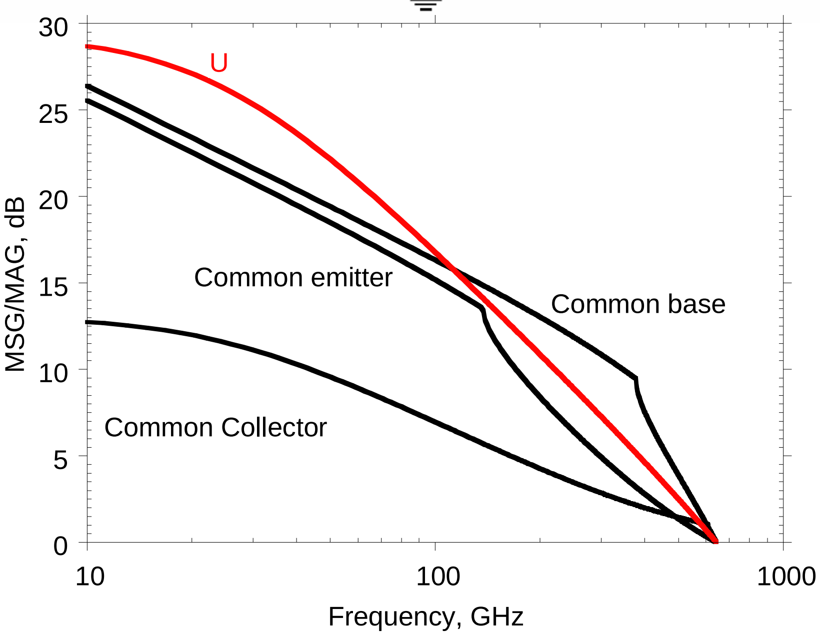
$\rightarrow$ U useful for extrapolation to find f_{max}

In III-V FETs, U shows peak from C_{ds} - R_s - R_d interaction

$\rightarrow$ U hard to use for f_{max} extrapolation

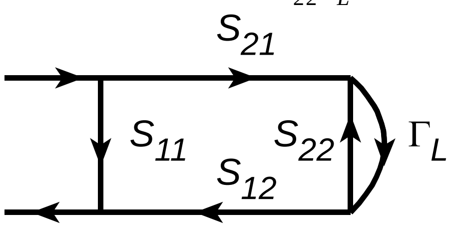
For bulk CMOS, C_{ds} is sheilded by substrate

$\rightarrow$ U should be OK for f_{max} extrapolation

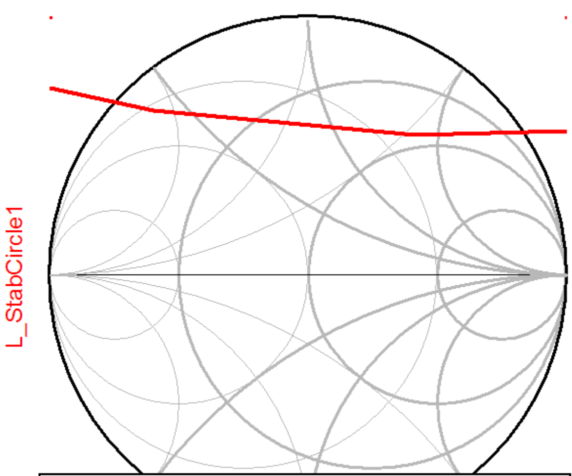


Design Tools: Stability Factors, Stability Circles

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_L}$$

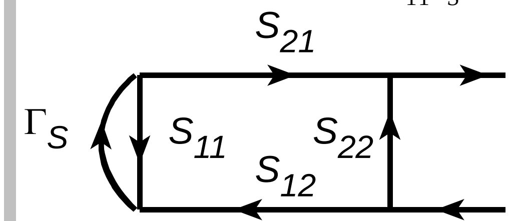


Load Stability Circle

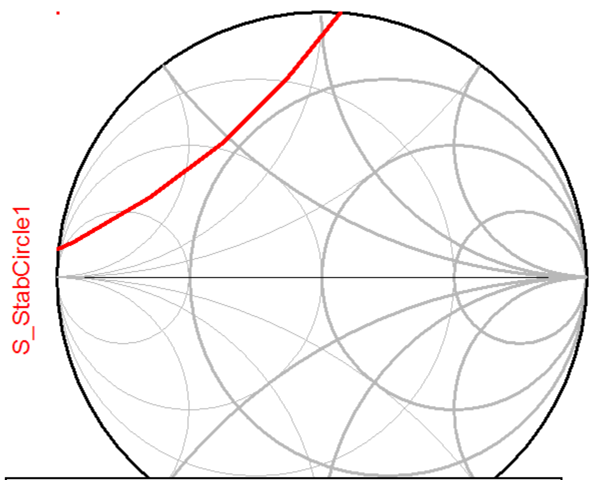


Values of Γ_L which make $\|\Gamma_{in}\| = 1 \rightarrow$ beyond lies negative R_{in}

$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{12}S_{21}}{1 - S_{11}\Gamma_S}$$



Source Stability Circle



Values of Γ_S which make $\|\Gamma_{out}\| = 1 \rightarrow$ beyond lies negative R_{out}

Unconditionally stable

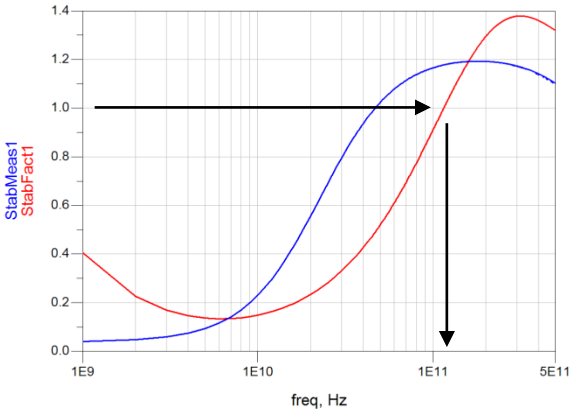
(stable with all (Γ_L, Γ_S)) if :

K = Rollet stability factor

$$= \frac{1 - |S_{11}|^2 - |S_{22}|^2 + \det^2[S]}{2|S_{21}S_{12}|} > 1$$

and B = stability measure

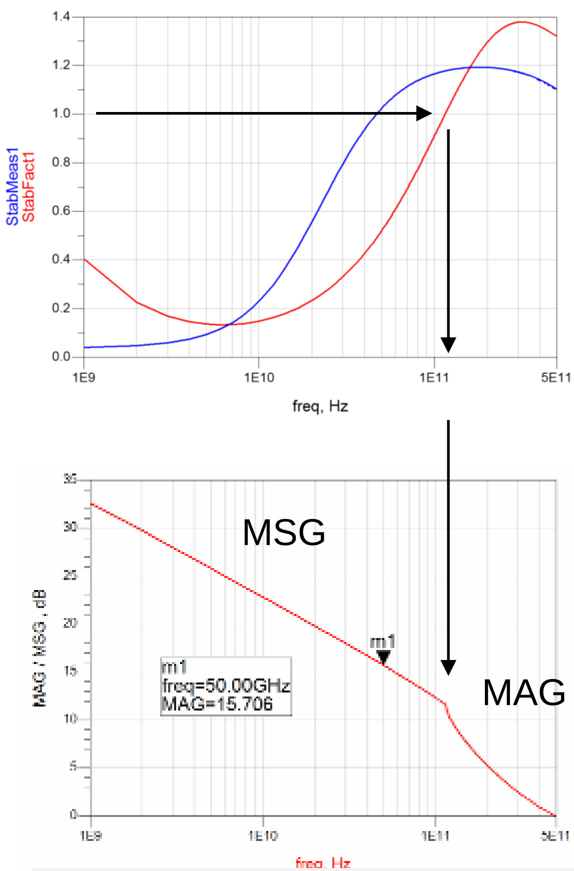
$$= 1 - |S_{11}|^2 - |S_{22}|^2 - \det^2[S] > 0$$



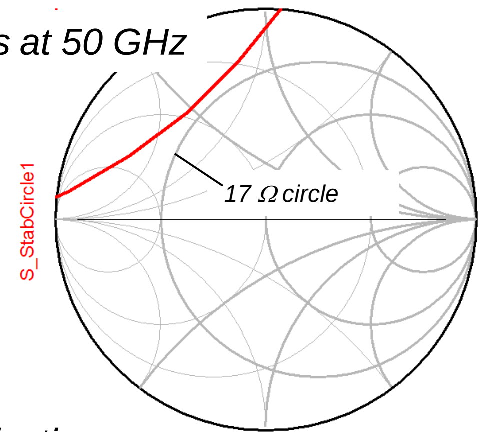
Negative port impedance → negative-R oscillator
Tuning for highest gain → infinite gain (oscillation)

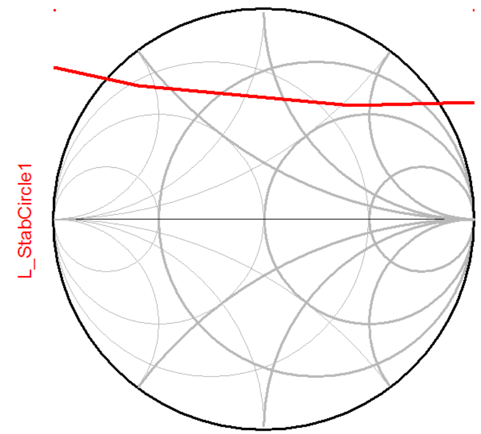
Design Tools: Maximum Stable Gain

$$\text{Maximum stable gain} = \text{MSG}$$
$$= \frac{|S_{21}|}{|S_{12}|} = \frac{|Y_{21}|}{|Y_{12}|} = \frac{|Z_{21}|}{|Z_{12}|}$$

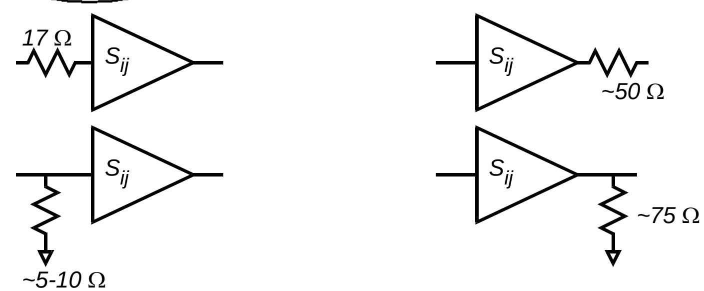


circles at 50 GHz

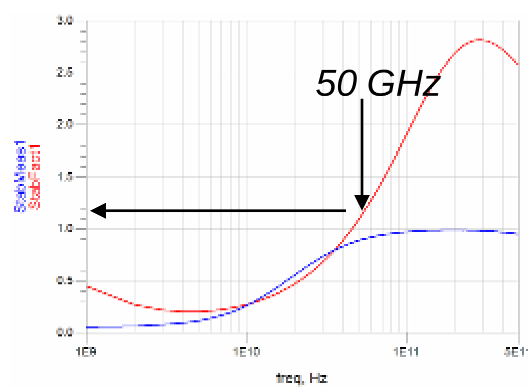


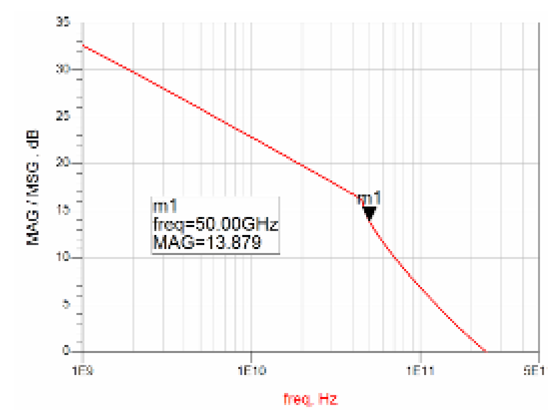


stabilization methods



results



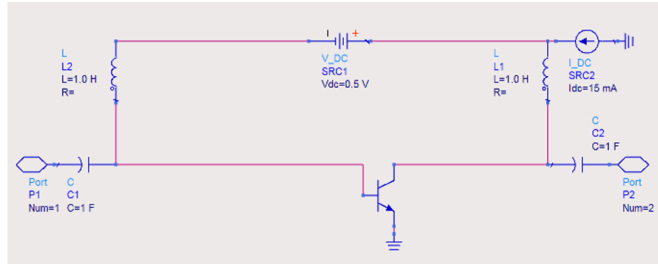


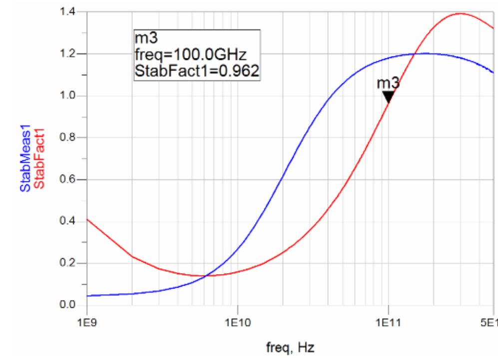
Adding series/shunt resistance excludes source or load from unstable regions → stabilizes

Design Procedure: Simple Gain-Matched Amplifier

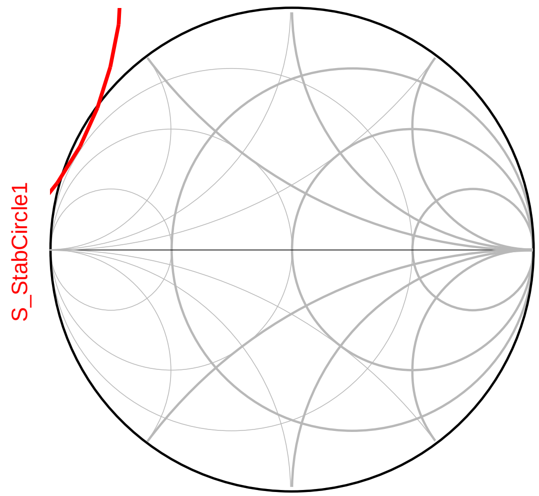
First:
stabilize at the design frequency

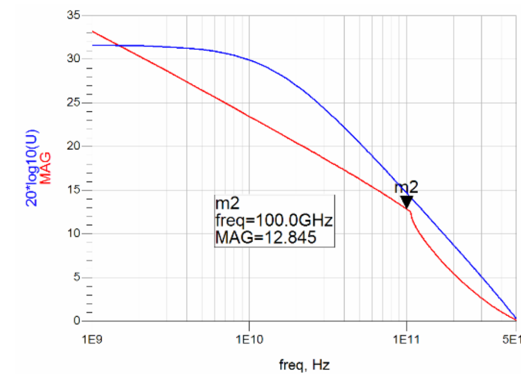
---device is potentially unstable at 100 GHz design frequency



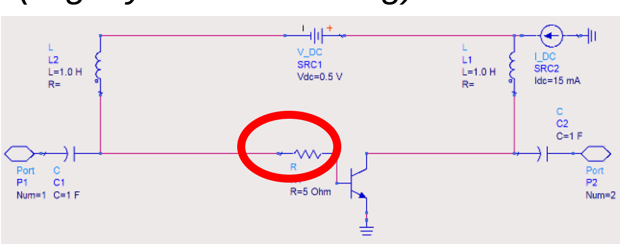


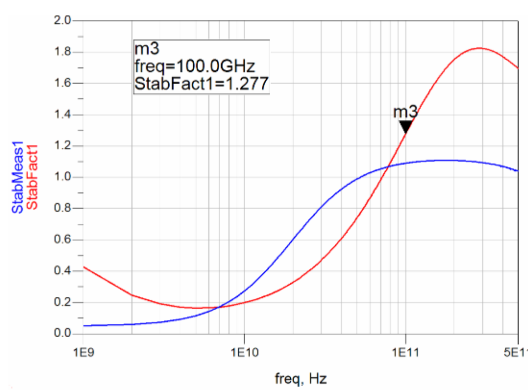
source stability circle:
~5 Ohm on input will overstabilize the device

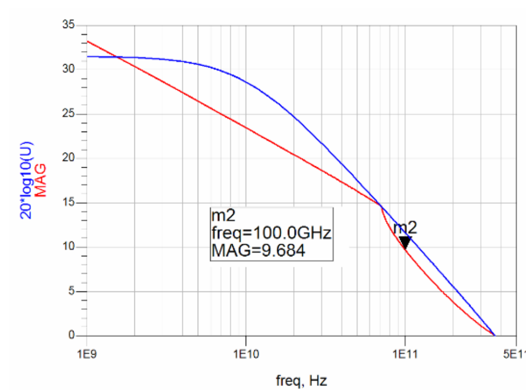




After stabilizing
(slightly over-stabilizing)





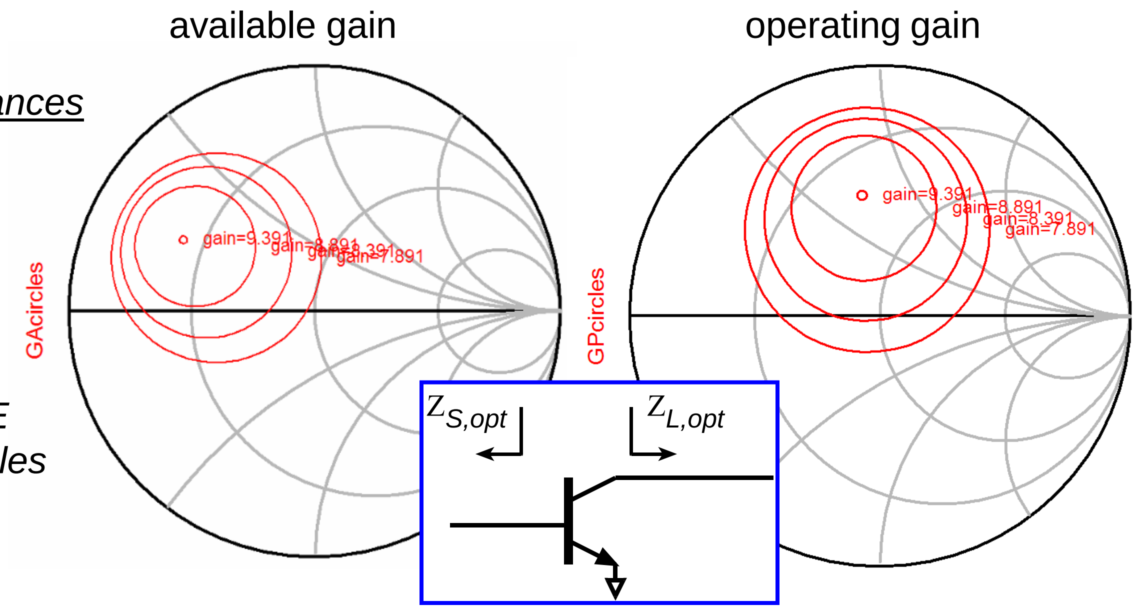


Design Procedure: Simple Gain-Matched Amplifier

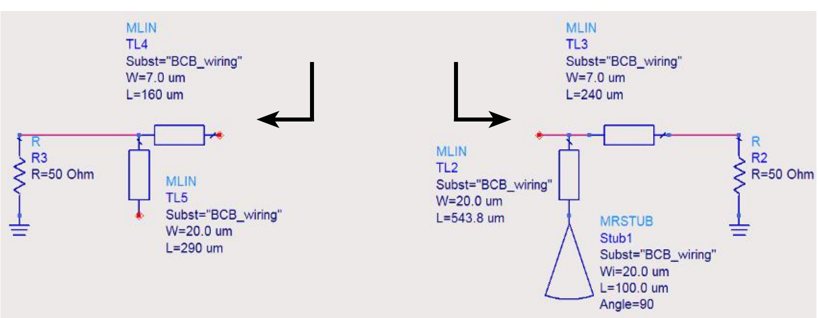
Second:
Determine required interface impedances

The Ga & Gp circles define the source & load impedances which the transistor must see

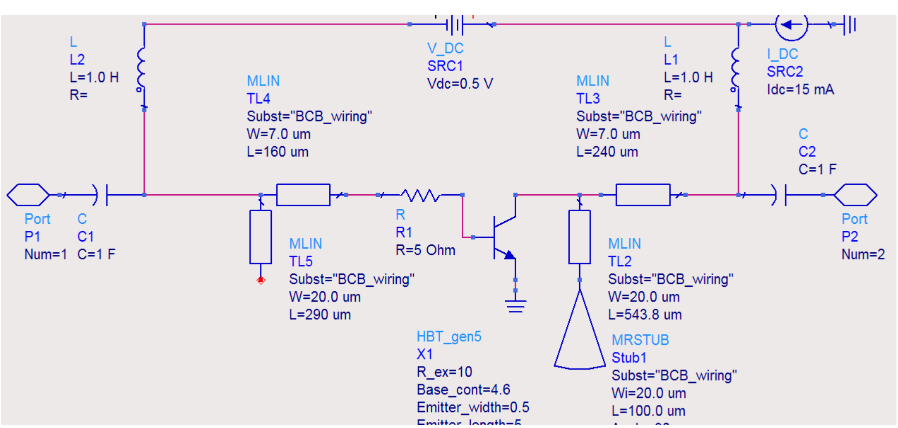
...it is necessary to OVERSTABILIZE the device to move the Ga & Gp circles towards the Smith chart center



Third:
Design Input & Output Tuning Networks
...to provide these impedances...

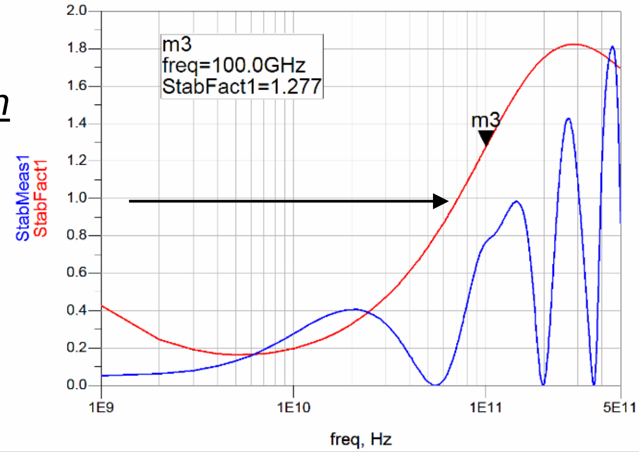


...added to device, the amplifier is not yet complete...

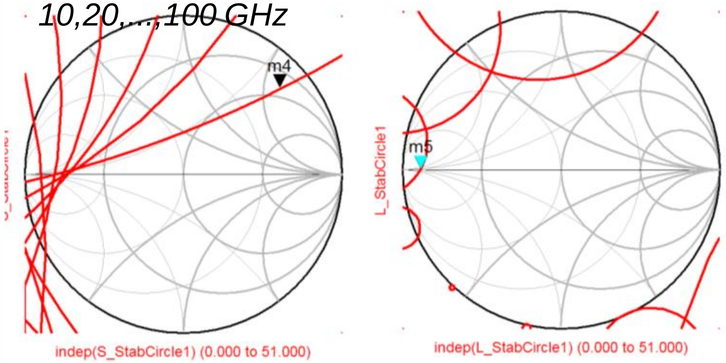


Design Procedure: Simple Gain-Matched Amplifier

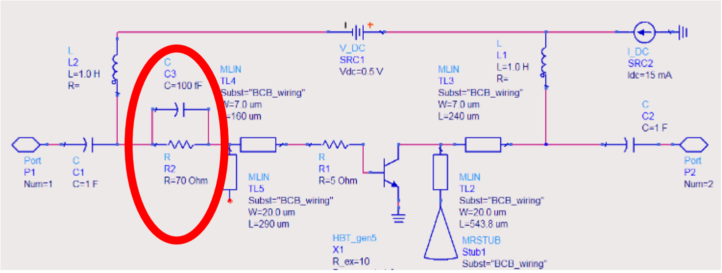
<u>Forth:</u>
<u>Add out-of-band stabilization</u>
potentially unstable below 75 GHz

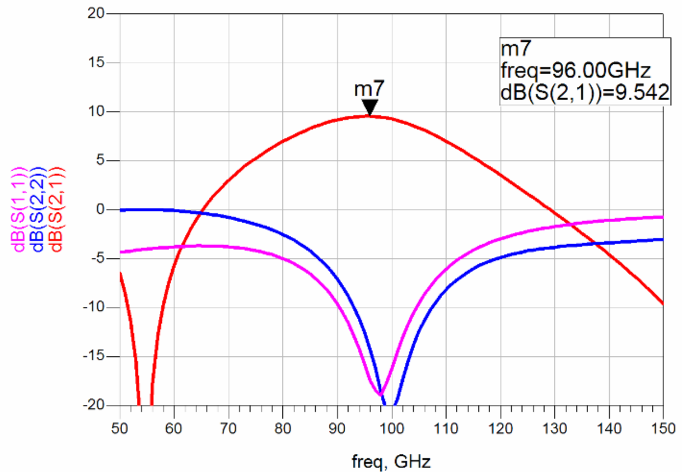


source & load stability circles & 10,20,...,100 GHz



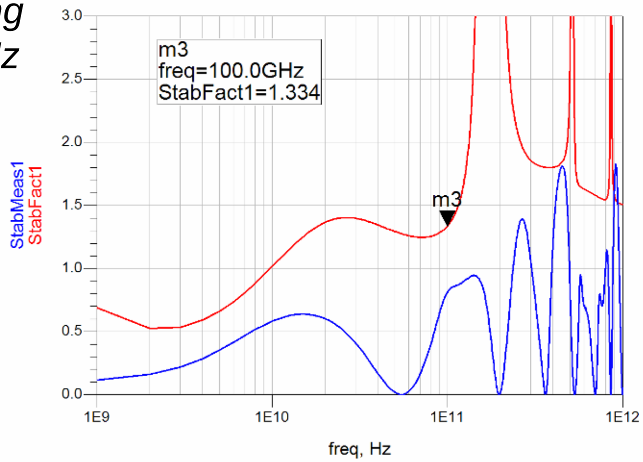
with frequency-selective series stabilization





...caused only slight mistuning & slight gain drop @ 100 GHz

...and is unconditionally stable above 10 GHz



Design Procedure: Effect of Line Losses

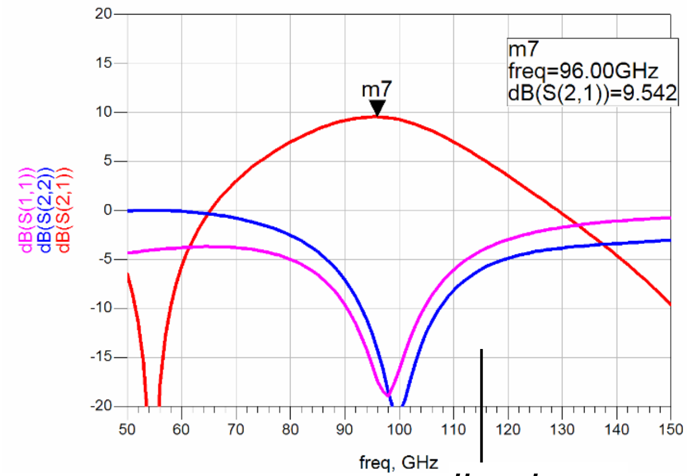
Finally:
adjusting for line losses

high line skin effect losses → reduced gain

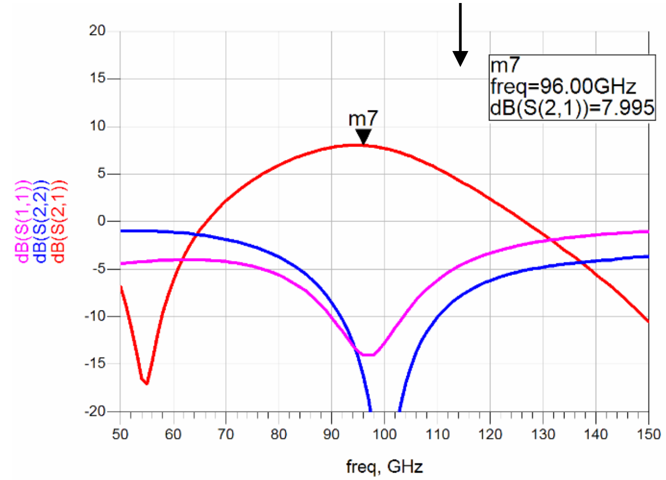
but line losses also increase stability factor

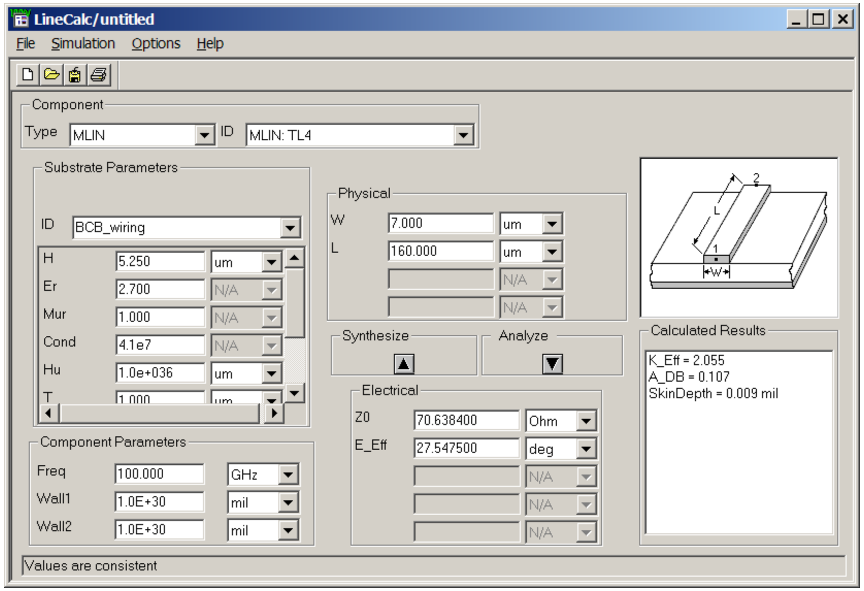
loss in gain are partly recovered
by reducing stabilization resistance &
re-tuning the design

--no analytical procedure; just component tweaking



line losses

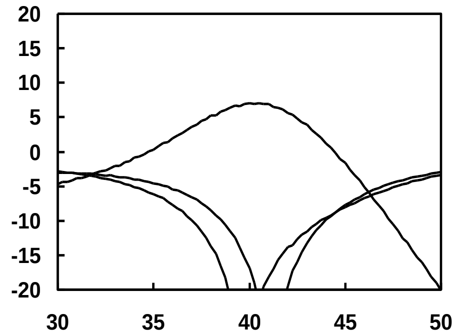


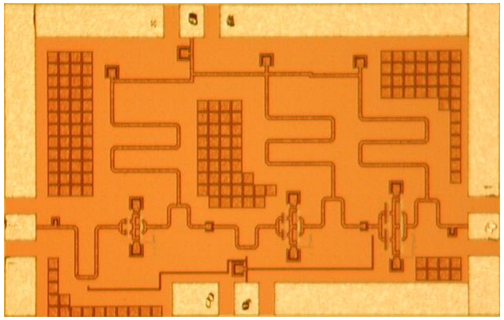


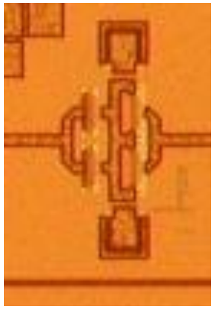
line losses have severe impact
...in VLSI wiring environment
...particularly at 50 + GHz
...particularly with high-power amplifiers

Tuned Amplifier Examples

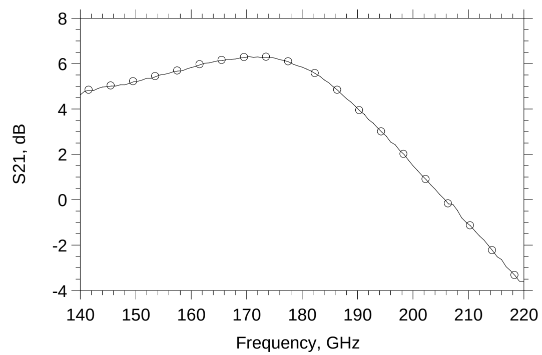
3-stage cascode in 180 nm CMOS

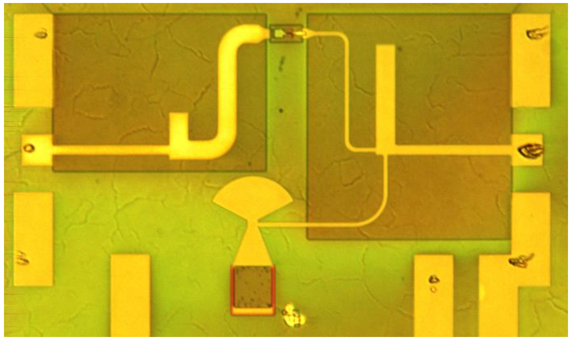


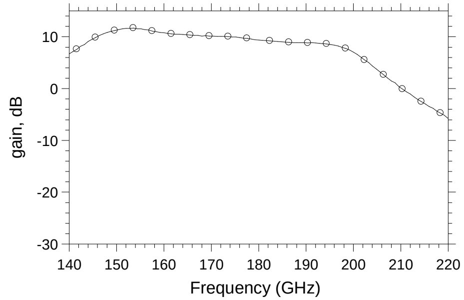


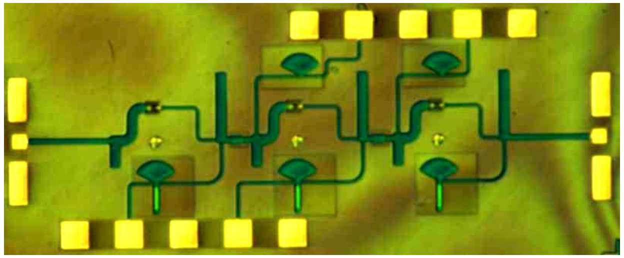


III-V HBT small-signal amplifiers









Note: simple gain-tuned amplifiers → limited applications
Transmitters need power amplifiers: need output loadline-match, not gain-match
Receivers need low-noise amplifiers: need input noise-match, not gain-match