ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. December 6, 2011

Do not open exam until instructed to.
Open notes, open books, etc
You have 3 hrs.

Use all reasonable approximations (5% accuracy is fine.),  **AFTER STATING THEM.**
**Hint: Stop and think before doing complicated calculations. For some problems, there is an easier way.**

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**Name:** ________________________

\[
G_T = \frac{|S_{21}|^2 (1-|\Gamma_x|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22}) - S_{21} S_{12} \Gamma_S \Gamma_L|^2} \\
G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2} \cdot \frac{|S_{21}|^2}{1-\Gamma_{out}} \\
G_{MS} = \frac{|S_{21}|}{|S_{12}|} \quad \text{if } K < 1 \\
K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{21} S_{12}|} \quad \text{where } \Delta = \text{det}[S] \\
G_P = \frac{1}{1-\Gamma_{in}} \cdot \frac{|S_{21}|^2 \cdot 1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2} \\
G_{max} = \frac{|S_{21}|}{|S_{12}|} \left[ K - \sqrt{K^2 - 1} \right] \text{if } K > 1
\]
Problem 1, 30 points

Gain definitions

At a signal frequency of 1 GHz, a two-port has $S_{11} = 0.7071$, $S_{12} = 0.1$, $S_{21} = 10$ and $S_{22} = 0.316$, as defined with a 50 Ohm impedance reference.

part a, 5 points

If the 2-port were directly connected to 50 Ohm load, and a 50 Ohm generator with 1 mW available power, what would be the power dissipated in the load?

First, is it potentially unstable?

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^4 + |\Delta|^2}{2 |S_{12}|^2 |S_{21}|} = \frac{1.002}{2(0.1)(10)} = 0.5$$

$\Delta = S_{12}S_{21} - S_{11}S_{22} = 1/4 = 0.7766$

Potentially unstable

Here the gain is the inserion gain $S_{21}$

$$P_{load} = \frac{1}{2} |S_{21}|^2 \times 1 \text{ mW} = 100 \text{ mW}$$
part b. 5 points

If you were to first stabilize (if necessary) and then impedance match the input and output to 50 Ohms, what would be the gain $S_{21}$ of the resulting amplifier?

\[
\text{Calculating } k \text{ is first needed.}
\]

- See previous page
- $k < 1 \rightarrow$ potentially unstable
- We must first stabilize this model.

Gain, i.e. if we stabilize the minimum amount, will be $MSG$.

Transistor S-parameters:

\[
MSG = \frac{1521^T}{15121} = 10 = 100 \quad \text{in units of } \frac{\text{Real}}{\text{Pi}}
\]

After merely a stabilizing, $S_{21}$

\[
\frac{1}{521} = \text{transfer for } MSG = 100
\]

\[
\frac{1}{521} = 10
\]
part c, 5 points

If you were to load the 2-port, without matching or stabilization elements, with a 500 Ohm load, is it possible to select a generator impedance which would cause the 2-port to oscillate?

\[
\frac{\Pi}{2} = \frac{500/s}{500/s + 1} = \frac{10-1}{10+1} = \frac{9}{11} = 0.81
\]

\[
\begin{bmatrix}
5 = \begin{bmatrix}
0.7071 & 0.1 \\
10 & 0.316
\end{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
52 = 511 + \frac{512 \cdot 521 \cdot \Pi}{1 - 522 \cdot \Pi}
\end{bmatrix}
\]

\[
= 0.7071 + \frac{10 \cdot 0.1 \cdot 9/11}{1 - 0.316 (9/11)}
\]

\[
= 0.7071 + 1.103
\]

\[
= 1.81
\]

\[
2 \left[ |\Pi_{\text{in}}| > 1, \text{ so some source impedances will result in oscillation.} \right]
\]
part d. 5 points

If you were to load the 2-port, without stabilization elements, in 50 Ohms, and then impedance-match the input to a 50 Ohm generator with 1 mW available source power, what would be the power in the load?

\[ \begin{bmatrix} 0.7071 & 0.1 \\ 10 & 0.316 \end{bmatrix} \]

\[ \text{Mold on input, not on output} \]

\[ \begin{bmatrix} \text{Operatory Gain} \\ \text{of transistor} \end{bmatrix} = \text{gain of amplifier} \]

\[ I_L = 0 \quad (50 \Omega \text{ load}) \]

\[ I_{in} = S_{11} + \frac{S_{12} S_{21} I_L^2}{1 - S_{22} I_L^2} = S_{11} \]

\[ G_P^T = \frac{1}{1 - 1 S_{11}^2} \cdot \frac{1 - 1 S_{22}^2}{1 - S_{22}^2 S_{21}^2} \]

\[ = \frac{1}{1 - 1 S_{11}^2} \cdot \frac{1 - 1 S_{22}^2}{1 - \frac{1}{4}} \]

\[ G_P^T = 1 S_{22}^2 = 200 \]

\[ \text{Power in load} = 1 \text{ mW} \cdot 200 = 200 \text{ mW} \]
part e. 5 points

If you were to take the 2-port, without stabilization elements, connect the input directly to a 50 Ohm generator with 1 mW available source power, and then impedance-match the output to a 50 Ohm load, what would be the power in the load?

Match on output, not on input.

Circuit gain \( |S_{11}|^2 = \text{transistor available gain} \)

\[
G_T = \frac{1 - |S_2|^2}{1 - |S_2|^2} \cdot |S_{21}|^2 \cdot \left( 1 - \frac{1}{P_{out}^2} \right)
\]

\[
|L_{3} = 0 (5 cm) | \Rightarrow \left[ \begin{array}{c} \mathbf{L}_{out} = S_{22}^T + \frac{S_{11}^T S_{21}^T P_{in}^0}{1 - S_{11}^T S_{12}^T} \end{array} \right] = S_{22}^T
\]

\[
G_T = 1 \cdot (100) \cdot \frac{1}{1 - |S_{21}|^2} \cdot \frac{100}{0.9} = 111.1
\]

\[
|S_{21}|^2 = G_T = 111.1
\]

Power in load = 1 mW \cdot 111.1 = 111.1 mW
part f. 5 points

If you were to take the 2-port, without stabilization elements, connect the input directly to a 500 Ohm generator with 1 mW available source power, connect the output directly to a what would be the power in the load?

\[ \frac{P_L}{P_{AVG}} = G_T = \text{transducer gain} \]

\[ I_L = 0 \quad I_3 = \frac{10 - 1}{10 + 1} = \frac{9}{11} \]

\[ G_T = \frac{15s_1}{1} \left( 1 - \left| S_{31} \right|^2 \right) \left( 1 - \left| S_{21} \right|^2 \right) \]
\[ \frac{1}{\left( 1 - \left| S_{31} \right|^2 \right) \left( 1 - \left| S_{21} \right|^2 \right) - S_{21} S_{31} S_{31} S_{21} \left| S_{31} \right|^2 \left| S_{21} \right|^2} \]
\[ = \frac{1}{s_2 \left| S_{31} \right|^2 \left( 1 - \left| S_{31} \right|^2 \right)} \]
\[ \left| S_{31} \right| = 0.7071 \]
\[ \left| S_{21} \right| = 0.421 \]
\[ = \frac{\left( 1.10 \right) (0.421)}{1} = 0.4615 \]
\[ \approx 4.6 \text{ mW} \]
\[ = 186 \text{ mW} \]

\[ P_L = 186 \text{ mW} \]
Problem 2, 10 points

Gain definitions

A bipolar transistor is shown at right. \( R_{be} = 1000 \text{ Ohms}, g_m = 100 \text{ mS}, R_{ce} = 10,000 \text{ Ohms}, C_{be} = 100 \text{ fF}. \)

The device is impedance-matched to a 50 Ohm generator and a 50 Ohm load at a 10 GHz signal frequency. Find the magnitude of \( S_{21} \) of the resulting amplifier at that frequency.

\[
\text{Device is unilateral} \rightarrow S_{12} = 0
\]

\[
\Rightarrow G_{max} = \frac{1}{1 - (521)^{-2}} = \frac{1}{1 - 1521^2}
\]

But \( S_{21} \) is hard to compute - try another approach.

If impedance matched a both ports

Power Gain = \( G_{max} = \frac{P_{out}}{P_{in}} \)

\[
\begin{align*}
9mV_{in} &= \frac{V_{be}}{V_{ce}} \\
V_{ce} &= \frac{V_{in}}{R_{ce}} + \frac{V_{be}}{R_{ce}}
\end{align*}
\]

\[
P_{in} = \frac{V_{in}^2}{R_{ce}}
\]

\[
P_{out} = (9mV_{in})^2 \cdot R_{ce}
\]

\[
\frac{P_{out}}{P_{in}} = \frac{(9mV_{in})^2 \cdot R_{ce}}{V_{in}^2 / R_{ce}} = \frac{9m^2 \cdot V_{in}^2 \cdot R_{ce}}{V_{in}^2} = \frac{9m^2 R_{ce} R_{ce}}{V_{in}^2} = \left( \frac{9mV_{in}}{V_{in}} \right)^2 \cdot R_{ce}
\]

\[\Rightarrow P_{out} = \left( \frac{9mV_{in}}{V_{in}} \right)^2 \cdot R_{ce} = \left( \frac{9mV_{in}}{V_{in}} \right)^2 \cdot 15.2 \cdot 10k \Omega = 25,000\]

\[\Rightarrow \text{CHT power gain} = 1521^2 = 25,000 \rightarrow 15 \leq 1 = 158\]
Problem 3, 15 points

Gain definitions, computation of Y-parameters

A bipolar transistor is shown at right. \( C_{cb} = 100 \, \text{fF}, \ g_m = 100 \, \text{mS} \).

It can be shown that \( S_{21} / S_{12} = Y_{21} / Y_{12} \)

---

part a, 10 points

compute \( Y_{21} \) and \( Y_{12} \)

\[
\begin{align*}
I_{out} &= g_m V_{i1} - j \omega C_{cb} V_{i2} \\
&= g_m - j \omega C_{cb} \\
Y_{21} &= g_m - j \omega C_{cb} \\
I_{in} &= -j \omega C_{cb} \cdot V_{out} \\
Y_{12} &= -j \omega C_{cb}
\end{align*}
\]
part b. 5 points

The transistor is first stabilized and then impedance-matched to 50 Ohm generator and load. Find the resulting amplifier gain magnitude $\| S_{21} \|$.

\[
\text{Amp. Gain} = m_{SG} = \frac{1}{|S_{21}^T|} = \frac{1}{|Y_{11}^T|} = \frac{1}{15_{12}^T} \frac{1}{15_{12}^T}
\]

\[
= \frac{0.15}{628 \times 10^{-3}} \left( \frac{9_m - j \omega C_3}{1 - j \omega C_3} \right) = \frac{1}{159} = (10^4 / 12\pi)
\]

\[
|S_{21}| = \sqrt{159} \approx 12.6
\]
Problem 4, 10 points
Computation of S-parameters

A bipolar transistor is shown at right. $R_{be} = 25$ Ohms, $g_m = 100$ mS, $R_{ce} = \text{infinity}$ Ohms.

Find $S_{11}$ and $S_{21}$

\[ S_{11} = \frac{2 \cdot \sqrt{2} - 1}{2 \cdot \sqrt{2} + 1} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} = -\frac{1}{3} \]

\[ S_{21} = \frac{2 \cdot \sqrt{10}}{\sqrt{2 \cdot 2 - 2}} \]

\[ = 2 \cdot \frac{25}{25 + 50} \cdot (-9 \pi Z_0) \]

\[ = 2 \cdot \frac{1}{3} \cdot (-5) \cdot \frac{V_m}{100} \]

\[ S_{21} = -\frac{10}{3} \]
Problem 5, 10 points
Stabilization

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<th>load stability circle</th>
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A MOSFET in common-source mode has \( S_{11} = 0.5 \) and \( S_{22} = 0.3 \). Source and load stability circles are as shown. Draw circuit diagrams, giving resistor values, of methods of stabilizing the transistor.

Source: the indicated circle has \( R = \frac{1 - \frac{17}{2}}{1 + \frac{17}{2}} = \frac{1 - \frac{17}{2}}{1 + \frac{17}{2}} = \frac{1}{3} \)

\( R = \frac{50\Omega}{3} = 16.7\Omega \)

the stability circle touches the \( R = 16.7\Omega \) circle

and the \( G = (1/50\Omega) \) circle

\[ \frac{50\Omega}{3} \]
\[
L_2 = 0 \text{ is stable because } |\frac{5}{\pi}| < 1
\]

From \[
L_4 = \frac{L_1}{1 - \frac{5}{\pi} L_1}
\]

So inside of stability circle \underline{is stable}

Circle tends the \underline{so a series resistance circle}

So \underline{SO \Omega \text{ in series will stabilize}}

A short-circuit on the output is potentially unstable

so parallel stabilization does not work.
Problem 6, 5 points
Gain circles

Available gain circles

Operating gain circles

At 10 GHz, a MOSFET in common-source mode operating and available gain circles as shown. Find the optimum generator and load impedances (in complex Ohms).

Available gain - source match

\[ I_s^* = -\frac{1}{2} \rightarrow Z_{opt} = Z_0 \cdot \frac{1 - j\eta}{1 + j\eta} = \frac{Z_0}{3} = 16.7 \Omega + j0 \Omega \]

Operating gain - load match

Optimum impedance, as shown, is \[ \frac{Z_L}{Z_0} = 1 + j1 \]

\[ Z_{L, opt} = Z_0 + jZ_0 = 50 \Omega + j50 \Omega \]
Problem 7, 20 points

Amplifier design

A bipolar transistor is shown at right. \( R_{be} = 25 \) Ohms, \( g_m = 100 \) mS, \( R_{ce} = 50 \) Ohms.

The amplifier is to be impedance-matched, using ONLY lumped-element **series** inductors and shunt capacitors.

\[ \begin{align*}
R_{be} & \quad g_mV_{be} & \quad R_{ce} \\
B & \quad + & \quad C \\
25 \Omega & \quad \downarrow & \quad 50 \Omega \\
V_{be} & \quad \downarrow & \quad \text{pow.} \\
E & \quad \text{pnts.}
\end{align*} \]

part a. 10 points

Design on the attached Smith chart the input matching network. Give values of all elements.

Input \( \angle \frac{\pi}{2} \) \( 2 \angle 120^\circ = \frac{25}{50\Omega} = \frac{1}{2} \)

Point \( A \) \( \Rightarrow 2\angle 0^\circ = \frac{1}{2} + j0 \)

Point \( B \) \( \Rightarrow 2\angle 30^\circ = 0.5 + j0.5 \)

\[ \text{must add series element} \frac{Z}{Z_0} = \frac{1}{2} \rightarrow Z = 0.5 \angle j0.5 \quad \text{ohms} \]

\[ L = \frac{0.5 \times 50 \Omega}{2\pi \times 10 \text{ GHz}} = 0.40 \text{ nH} \]

Point \( B \) \( \Rightarrow Y_{11} = 1.0 - j1.0 \)

Point \( C \) \( \Rightarrow Y_{11} = 1.0 + j0 \)

\[ \text{must add shunt element} \frac{1}{Y_{11}} = j1.0 - j1 \rightarrow Y_{11} = 1.0 + j0 \quad \text{ohms} \]

\[ C = \frac{1}{50\Omega \left( \frac{2}{\pi} \right) \left( 10 \text{ GHz} \right)} = 0.318 \text{ pF} \]
part b, 10 points.

Determine the magnitude of $S_{21}$ of the resulting amplifier

What is the transistor $g_m$?

Match output & compute $P_{out}/P_{in}$ because with

a matched input, $P_i = P_{out}$.

$$P_i = \frac{V_{se}^2}{R_{se}}$$

$$P_{out} = (g_m V_{se})^2 \left(\frac{1}{2}\right)^2 \cdot \frac{R_o}{R_c}$$

$$P_{o} = \left(g_m/2\right)^2 R_c e = \frac{9 m^2 R_c P_i}{4} = 3.125$$

$$\left|S_{21}^{\text{amp}}\right|^2 = \left|S_{21}^{\text{transistor}}\right|^2 = 3.125$$

$$\left|S_{21}^{\text{o.p}}\right| = \sqrt{3.125} = 1.77$$