ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 6, 2013

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.), AFTER STATING THEM.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points Received</th>
<th>Points Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>2a</td>
<td></td>
<td>10</td>
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<tr>
<td>2b</td>
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<td>2c</td>
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<td>3a</td>
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<td>4</td>
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<tr>
<td>total</td>
<td></td>
<td>100</td>
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</tbody>
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Name: ____________________________
Problem 1, 15 points

The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.
First match each Smith Chart with each circuit. Then determine as many component values as is possible (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given.
The charts all use 50 Ohm normalization:

<table>
<thead>
<tr>
<th>Points</th>
<th>Smith chart (a)</th>
<th>Circuit</th>
<th>Component values</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>(5)</td>
<td>$Z_0 = 50\Omega, \tau = 1/4,\text{ns}, R_L = 150\Omega$</td>
</tr>
<tr>
<td>2</td>
<td>Smith chart (b)</td>
<td>(5)</td>
<td>$Z_0 = 28.86\Omega, \tau = 1/4,\text{ns}, R_L = 50\Omega$</td>
</tr>
<tr>
<td>3</td>
<td>Smith chart (c)</td>
<td>(7)</td>
<td>$R_1 = 150\Omega, R_2 = 18.75\Omega$</td>
</tr>
<tr>
<td>3</td>
<td>Smith chart (d)</td>
<td>(K)</td>
<td>$R_1 = 150\Omega, R_2 = 18.75\Omega$</td>
</tr>
<tr>
<td>3</td>
<td>Smith chart (e)</td>
<td>(L)</td>
<td>$R_1 = 150\Omega, R_2 = 18.75\Omega$</td>
</tr>
<tr>
<td>2</td>
<td>Smith chart (f)</td>
<td>(G)</td>
<td>$R = 50\Omega$</td>
</tr>
</tbody>
</table>

(a) The Circuit is (5)
@ DC, 2, 4, 6 GHz, $z_{in} = 50\,\Omega \left(\frac{1.15}{0.5}\right) = 150\,\Omega$

@ 1.35 GHz, $z_{in} = 50\,\Omega \left(\frac{1.2}{0.5}\right) = 16.67\,\Omega$

-> Quarter Wave Line

$$Z_{line} = \frac{150 \times 16.67}{1.5} = 50\,\Omega$$

50\,\Omega Line, Loaded by 150\,\Omega, $\tau = 1/4\,\text{ns}$

(b) The Circuit is (5)
@ DC, 2, 4, 6 GHz, $\Gamma = 0 \Rightarrow Z_{in} = 50\,\Omega$

@ 1.35 GHz, $Z_{in} = 16.67\,\Omega$

-> Quarter Wave Line

$$Z_{line} = \sqrt{50 \cdot (16.67\,\Omega)} = \frac{50}{\sqrt{3}} = 28.86\,\Omega$$

28.86\,\Omega Line, $\tau = 1/4\,\text{ns}$, Loaded by 50\,\Omega

(c) The Circuit is (5)
@ DC $\Rightarrow \Gamma = 0.5 \Rightarrow Z_{in} = 50\,\Omega \left(\frac{1.5}{0.5}\right) = 150\,\Omega$

... capacitor behaves like an open circuit at DC, $R_1 = 150\,\Omega$

@ $f \rightarrow \infty$, $\Gamma = -0.5 \Rightarrow Z_{in} = \frac{50}{\frac{3}{2}} = 16.67\,\Omega$

... cap behaves like a short circuit as $f \rightarrow \infty$, $R_1 || R_2 = 16.67\,\Omega$

$$\frac{150 \cdot R_2}{150 + R_2} = \frac{50}{3} \,\Omega$$, solving, $R_2 = 18.75\,\Omega$
(d) The circuit is (k)

@ DC, \( \Gamma = -0.15 \), \( Z_{in} = \frac{50 \, \Omega}{3} \)

" Inductor behaves like a short ckt @ DC, \( R_1 \ || \ R_2 \to 16.67 \, \Omega \) \n
@ \( f \to \infty \), \( \Gamma = 0.15 \), \( Z_{in} = 150 \, \Omega \)

" Inductor behaves like an open ckt @ \( f \to \infty \), \( R_1 \)

\[ R_1 = 150 \, \Omega \]

\[ \Rightarrow [150 \, \Omega \ || \ R_2 = 16.67 \, \Omega \] \to \text{Solving} \]

\[ R_2 = 18.75 \, \Omega \]

(f) The circuit is (g)

@ DC

\[ Z_{\text{in}} = 0 \Rightarrow \text{Short ckt} \]

\[ R = 1 \, \text{j} \, \text{rad} \]

@ \( f \to \infty \)

\[ Z_{\text{in}} = 0 \]

\[ \Gamma_{\text{in}} = -1 \]

@ \( \omega = \frac{1}{\sqrt{LC}} \), \( Y_L + Y_C = 0 \Rightarrow Y_{\text{in}} = Y_R = \frac{1}{50} \, \text{S} \Rightarrow R = 50 \, \Omega \)

(e) The circuit is (l)

@ DC

\[ Z_{\text{in}} = R_1 \]

@ \( f \to \infty \)

\[ Z_{\text{in}} = R_1 \]

@ \( \omega = \frac{1}{\sqrt{LC}} \), \( X_L + X_C = 0 \Rightarrow R_1 \ || \ R_2 = \frac{50 \, \Omega}{3} \ [\Gamma = -0.15] \]

(At \( \Gamma = -0.15 \), \( Z_{\text{in}} = \frac{50}{3} \, \Omega \), from prev parts of problem)

\[ \Rightarrow R_2 = 18.75 \, \Omega \]
Problem 2, 35 points
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give algebraic expressions for the four Z-parameters and for the four S-parameters.

This might be a DC blocking capacitor. What value would you need for C if you wanted $S_{21} = -3$ dB at 100MHz in a 50 Ohm system?

$$\begin{align*}
Z_{11} &= \frac{V_1}{I_1} \text{ if } I_2 = 0 \\
Z_{12} &= \frac{V_1}{I_2} \text{ if } I_1 = 0 \\
Z_{21} &= \frac{V_2}{I_1} \text{ if } I_2 = 0 \\
Z_{22} &= \frac{V_2}{I_2} \text{ if } I_1 = 0
\end{align*}$$

If either $I_1$ or $I_2$ is zero i.e., one of the ports is open, current does not flow, so, accordingly $I_2$ or $I_1$ is zero. Implies, all Z-parameters $\rightarrow \infty$.

$$\begin{align*}
S_{11} &= \frac{1}{j\omega C} \\
&= \frac{1}{Z_0 + j\omega C} \\
&= \frac{1}{1 + 2j\omega Z_0 C}
\end{align*}$$

Similarly, $S_{12}$ and $S_{21}$ would be equal as the circuit is symmetric.

To compute $S_{21}$,

$$S_{21} = \frac{2V_{\text{out}}}{V_{\text{gen}}}$$

$$Z_0 = Z_L = Z_{\text{gen}}$$
\[
V_{\text{out}} = \frac{Z_0}{2Z_0 + j\omega C} \cdot V_{\text{gen}} \\
= \frac{1.9 Z_0 C}{1 + 2j\omega Z_0 C} \cdot V_{\text{gen}} \\
S_{21} = \frac{1.9 Z_0 C}{1 + 2j\omega Z_0 C} = S_{12} \quad \text{(2)}
\]

\[
\left. \left| S_{21} \right| = \frac{1}{\sqrt{2}} = \left| \frac{1}{1 + \frac{1}{j\omega (2Z_0) C}} \right| \\
\Rightarrow \sqrt{1 + \left(\frac{1}{\omega (2Z_0) C}\right)^2} = \sqrt{2} \\
\Rightarrow \frac{1}{\omega (2Z_0) C} = 1 \\
\Rightarrow C = \frac{1}{2\pi f Z_0} = \frac{1}{4\pi f Z_0}
\]

Substituting values,

\[c = 1.59 \times 10^{-11} \text{ F} \]

\[C = 15.9 \text{ F}\]
Part b. 15 points
First, compute H21 and S21, both as a function of frequency, for this network.

Second, after assuming that $g_m Z_o \gg 1$, find the frequency at which $S_{21}$ has a magnitude of 1 and compare this to the current-gain cutoff frequency. Please then comment.

\[
h_{21} = \begin{bmatrix} g_m \\ j\omega C \end{bmatrix}
\]

\[
S_{21} = \frac{2V_{out}}{V_{gen}} \bigg| Z_L = Z_0 = Z_{gen}
\]

\[
V_{in} = \frac{V_{gen}}{Z_0 + (j\omega C)} \Rightarrow \frac{V_{gen}}{1 + j\omega Z_0 C}
\]

\[
V_{out} = -\frac{g_m V_{in} Z_0}{1 + j\omega Z_0 C} \Rightarrow S_{21} = -\frac{2g_m V_{gen} Z_0}{V_{gen} (1 + j\omega Z_0 C)}
\]

\[
S_{21} = \frac{-1}{\left(\frac{1}{2g_m Z_0} + j\omega \frac{C}{2g_m}\right)} \Rightarrow |S_{21}| = 1 \quad \text{with} \quad g_m Z_0 \gg 1
\]

\[
\Rightarrow \quad \omega^2 \left(\frac{C}{2g_m}\right)^2 = 1
\]

\[
\therefore \quad \frac{1}{2g_m Z_0} \rightarrow 0
\]

[CONT'D]
\[ \omega = \frac{g_m}{C} \]

\[ f \left|_{s_1=1} \right. = \frac{2 \cdot g_m}{2\pi C} = \frac{g_m}{\pi C} = f \left|_{s_2=1} = 1 \right. \]

\[ f \left|_{h_2=1} \right. \Rightarrow 1 = \frac{g_m}{j\omega C} \Rightarrow \text{Current - Gain - Cut-off frequency} \]

\[ f \left|_{h_2=1} \right. = \frac{g_m}{2\pi C} \]

Comment: For high gain amplifiers \((g_mZ_0 >> 1)\), \( f \left|_{s_1=1} \right. \approx 2 \cdot f_C \)
Part c. 10 points
Ri=10 Ohms, Cgs=1pF, gm=100 mS, Rds=100 Ohms.

Calculate $Y_{11}$ and $Y_{21}$ at 1 GHz.

\[
Y_{11} = \frac{i_1}{v_1} \bigg|_{v_2=0}
\]
\[
Y_{21} = \frac{i_2}{v_1} \bigg|_{v_2=0}
\]

\[
y_1 = i_1 \left( R_i + \frac{1}{j\omega C_{gs}} \right)
\]
\[
y_{11} = \frac{j\omega C_{gs}}{1 + j\omega R_i C_{gs}}
\]

\[
y_{21} = \frac{g_m v_1}{v_1 (1 + j\omega R_i C_{gs})} = \frac{g_m}{1 + j\omega R_i C_{gs}}
\]

\[
I_2 = g_m v_{gs}
\]
\[
v_{gs} = \frac{v_i \left( \frac{1}{j\omega C_{gs}} \right)}{R_i + \left( \frac{1}{j\omega C_{gs}} \right)} = \frac{v_i}{1 + j\omega R_i C_{gs}}
\]

\[
Y_{11} \bigg|_{f=1GHz} = \frac{j(0.0063)}{1 + j(0.063)} S
\]
\[
Y_{21} \bigg|_{f=1GHz} = \frac{0.1}{1 + j(0.063)} S
\]
Problem 3, 35 points

Transmission-line theory

Hint: we are testing here your understanding of transmission-lines and their relationships to lumped elements. If the calculation appears to be extremely difficult, you may possibly be missing some key insight.

Part a, 10 points

You are probing a circuit using a 9MOhm oscilloscope probe (Rgen) connected to a 1MOhm oscilloscope through 1 meter of coaxial cable. The cable has 50Ohms characteristic impedance and uses Polyethylene, with a dielectric constant of 2.25, to separate the conductors.

What is the -3dB frequency of this test setup?

\[ v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s} \]

\[ \tau = \frac{L}{v} = \frac{1 \text{ m}}{2 \times 10^8 \text{ m/s}} = 0.5 \times 10^{-8} \]

\[ \tau = 5 \text{ ns} \]

\[ L = \tau Z_0 = (5 \text{ ns}) (50 \text{ Q}) \]

\[ L = 250 \text{ nH} \]

\[ C = \frac{\tau}{Z_0} = \frac{5 \text{ ns}}{50 \text{ Q}} \]

\[ C = 100 \text{ pF} \]

\[ t_{LR} = \frac{L}{R_{gen} + R_L} \]

\[ t_{LR} = \frac{250 \text{ nH}}{9 \text{ M} \Omega + 1 \text{ M} \Omega} \]

\[ t_{LR} = 25 \text{ fs} \]

\[ t_{RC} = c \left[ R_{gen} \parallel R_L \right] \]

\[ t_{RC} = 100 \text{ pF} \left[ 1 \text{ M} \Omega \parallel 9 \text{ M} \Omega \right] \]

\[ t_{RC} = 100 \text{ pF} \left[ 0.9 \text{ M} \Omega \right] \]

\[ t_{RC} = 90 \text{ ns} \]

Comparing, \( t_{RC} \gg t_{LR} \), effect of inductors can be neglected.
This is an RC circuit with \( f_{3dB} = \frac{1}{2\pi \cdot R \cdot C} \).

\[ f_{3dB} = \frac{1}{2 \pi (90 \mu S)} \]

\[ f_{3dB} = 1176.6 \text{ kHz} \]
Part b, 10 points
You are connecting a high-current driver to pulse a solid-state laser. The generator is a 1 V step-function with 0.1 Ohm output impedance. The load is 0.9 Ohm.
Generator and load are connected with a signal conductor (dark grey) of 1 cm width and 10 cm length, separated by 1 mm from a ground-plane/ground-return conductor.
Find the pulse-response 10%-90% rise time.

Approximate the effect of fringing fields at the edges of the conductors by assuming that the effective microstrip line width is the physical width plus twice the conductor-ground spacing.

1. \( l = 10 \text{ cm} \); \( W = 1 \text{ cm} \); \( H = 1 \text{ mm} \); Assuming the same dielectric, \( \varepsilon_r = 2.25 \)

2. \( Z_0 \) given that \( W_{eff} = W + 2H \)

\[
Z_0 = \frac{120\pi}{2.25} \left( \frac{1 \text{ mm}}{1 \text{ cm} + 2 \text{ mm}} \right) = 20.944 \Omega = Z_0
\]

\( \tau = \frac{l}{V} = \frac{10 \text{ cm}}{c/\sqrt{\varepsilon_r}} = 0.1 \text{ m} \times \frac{1.5}{3 \times 10^8 \text{ m/s}} = 0.5 \text{ ns} \)

\( \tau = 0.5 \text{ ns} \)

3. \( C = \frac{\pi}{4} \frac{1}{Z_0} \)

\[
C = 0.5 \text{ ns} \times \frac{20.944 \Omega}{20.944 \Omega} = 23.87 \text{ pF}
\]

\[
C = 23.87 \text{ pF}
\]

\[ L = \tau Z_0 \]

\[
L = 0.5 \text{ ns} \times 20.944 \Omega = 104.72 \text{ nH}
\]

\[
L = 104.72 \text{ nH}
\]
\[ T_{RC} = 2.3 \times 10^{-8} \text{F} \left\{ 0.1 \quad | \quad 0.9 \right\} \mu \text{s} \]
\[ T_{HR} = \frac{10 \times 4.72 \mu \text{H}}{1.5} \]
\[ T_{HR} = 10.472 \mu \text{s} \]

\[ T_{HR} \gg T_{RC} \]

Effect of capacitor can be neglected.

\[ I(t) = (1 - e^{-t/T_{HR}}) \]
\[ V_{out}(t) = (0.9 \text{\mu A}) I(t) \]
\[ V_{out} = (0.9 \text{\mu A}) \left[ 1 - e^{-t/T_{HR}} \right] \]

Time taken for response from 10% to 90% Rise Time: 
\[ 0.09 = 0.9 \left( 1 - e^{-t_1/T_{HR}} \right) \]
\[ 0.81 = 0.9 \left( 1 - e^{-t_2/T_{HR}} \right) \]

\[ e^{t_1/T_{HR}} = 0.12 \]
\[ e^{t_2/T_{HR}} = 0.09 \]
\[ e^{-(t_1-t_2)/T_{HR}} = 0.9 \]

Solving

Rise Time \approx 23.0 \mu \text{s} \]

Alternately,

Rise Time = 2.2 T_{HR}
Part c, 15 points
You are working with a Duriod board
(dielectric constant of 2.4), 0.5 mm thick.
Line 1 is 2 mm wide and 2 cm long.
Line 2 is 2 mm wide and 5 mm long.

First: find the characteristic impedance and propagation delay of both lines.

Second: assuming that the lines are both short in comparison with a quarter-wavelength, draw a lumped-element equivalent circuit, calculating all element values.

Approximate the effect of fringing fields at the edges of the conductors by assuming that the effective microstrip line width is the physical width plus twice the conductor-ground spacing.

**First:** Line 1 \( \Rightarrow W = 2\text{mm}; L = 2\text{cm}; H = 0.5\text{mm}; \varepsilon_r = 2.4 \)

Again, \( Z_0 = \frac{\eta_0}{\sqrt{\varepsilon_r}} \left[ \frac{H}{W+2H} \right] = \frac{120\pi}{\sqrt{2.4}} \left[ \frac{0.5\text{mm}}{2\text{mm}+1\text{mm}} \right] \)

\[ Z_0 = 40.155 \Omega \]

\[ \tau_1 = \frac{L_1}{V_c} = \frac{2\text{cm} \cdot \sqrt{\varepsilon_r}}{c} = \frac{2\text{cm} \cdot \sqrt{2.4}}{3 \times 10^8 \text{m/s}} = 103.28 \text{ps} \]

**Second:** Line 2 is similar to Line 1, \( Z_0 = 40.155 \Omega \)

\[ \tau_2 = \frac{L_2}{V_c} = \frac{5\text{mm} \cdot \sqrt{2.4}}{3 \times 10^8 \text{m/s}} = 2.581 \text{ps} \]

\[ L_1 \approx 4.19 \mu \text{H} \]

\[ C_1 = \frac{\tau_1}{Z_0} = \frac{103.28 \text{ps}}{40.155 \Omega} = 2.55 \text{pF} \]

\[ L_2 = 1.05 \mu \text{H} \]

\[ C_2 = \frac{\tau_2}{Z_0} = \frac{2.581 \text{ps}}{40.155 \Omega} = 0.0636 \text{pF} \]
Drawing the lumped equivalent,
Problem 3, 15 points

Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 10 GHz signal frequency. Design a lumped-element matching network which converts this impedance to \(50 + 250 \text{Ohms}\) at 10 GHz. Give all element values.

\[ Z_b = 2.0 + j 2.5 \]
\[ Z_b = 0.2 + j 0.24 \]
\[ Z_b = 795.8 \Omega \]

\[ C_s = 1.98 \mu F \]

\[ (25\mu F) \]

\[ Z_{s1} = 708.23 \Omega \]

\[ Z_{a} = 0.2 - j 1065 \]

\[ Z_{a} = 50 + (0.2 - j 1065) \]
\[ = [10 - j 132.5] \Omega \]

\[ Z_L = 144.5 = j 10 L \]

with \( \omega = 2\pi (10 \text{GHz}) \), \( L = 308.23 \mu H \)
\[ Y_B = 2.0 - j2.5 \]

Shunt capacitor \[ Y_c = j2.5 \implies Z_c = \frac{j}{2.5} \implies Z_c = \frac{50}{12.5} = \frac{1}{10C_{shunt}} \]

\[ C_{shunt} = \frac{2.5 \times 50}{2 \pi (106.4 \times 10^3) \times 50} = 795.8 \text{ fF} \]

\[ Z_{in} = 25 \Omega \]

MATCHING, N/W

(Second Element)

(Circuit)