ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. Tuesday, December 8, 12-3 p.m.

Do not open exam until instructed to.
Open notes, open books, etc. You have 3 hrs.
Use all reasonable approximations (5% accuracy is fine.)
AFTER STATING and justifying THEM.
Think before doing complex calculations. Sometimes there is an easier way.

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Name: Student

\[
G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s \Gamma_L)(1-\Gamma_L S_{22}) - S_{21} S_{12} \Gamma_s \Gamma_L|^2} \quad G_T = \frac{1}{1-\|\Gamma_s\|^2} \cdot \frac{|S_{21}|^2}{|1-\Gamma_L S_{22}|^2} \\
G_a = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} \cdot \frac{|S_{21}|^2}{1-\|\Gamma_{out}\|^2} \quad G_{\text{max}} = \frac{|S_{21}|}{|S_{12}|} \left[ K - \sqrt{K^2 - 1} \right] \text{if } K > 1 \\
G_{\text{res}} = \frac{|S_{21}|}{|S_{12}|} \cdot \text{if } K < 1 \quad K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{21} S_{12}|} \quad \text{where } \Delta = \det[S] \\
\]

Unconditionally stable if: (1) \( K > 1 \) and (2) \( \|\det[S]\| < 1 \)
Problem 1, 12 points
Two-port properties, Gain relationships

Part a, 5 points
Transistor cutoff frequencies
\( C_{gs} = 100 \text{ fF}, \ g_m = 100 \text{ mS} \)
\( R_{ds} = 100 \text{ Ohms}, \ R_i = 10 \text{ Ohms} \),

Find \( f_c \) and \( f_{\text{max}} \).

Can work either by formula or by derivation.

By derivation:

\[
\begin{align*}
\text{Compute } & \rho_{st}/\rho_{ni} \to \\
\rho_{st} & = \frac{f_T}{\pi} = \frac{2516 \text{ kHz}}{2 \sqrt{g_{os}R_i}}
\end{align*}
\]

Similarly:

\[
\begin{align*}
\eta_T & = \frac{f_T}{2 \pi g_{os}} = 1596 \text{ kHz}
\end{align*}
\]
part b, 7 points

Find the short-circuit current gain and the maximum available power gain at 60 GHz

\[
\begin{align*}
4.5 & \quad \text{[1]} \\
\beta_1 & = \frac{159 \text{Cl}_d}{65 \text{Cl}_d} = 2.65 \rightarrow 5.5 \text{dBm} \quad (20 \text{mW})
\end{align*}
\]

\[
\begin{align*}
3.5 & \quad \text{[2]} \\
\text{an. laterl - so mag} = a & = \left(\frac{131 \text{Cl}_d}{65 \text{Cl}_d}\right)^2 = 17.5 \\
& \quad \rightarrow 22 \text{dBm} \quad (10 \text{mW})
\end{align*}
\]
Problem 2, 12 points
Potentially unstable amplifier design

part a, 7 points

At a design frequency of 1 GHz, a common-source FET has source and load stability circles as below.

<table>
<thead>
<tr>
<th>Source stability circle</th>
<th>Load stability circle</th>
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<tbody>
<tr>
<td><img src="image1" alt="Source stability circle" /></td>
<td><img src="image2" alt="Load stability circle" /></td>
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Given that $S_{11}=0.5$ and $S_{22}=1.1$ at 1GHz, draw two stabilization circuits in the boxes below, giving element values.

**Solution 1**

- $Z_{in} = 75\Omega$
- $M = \frac{5\Omega + 5\Omega\cdot 5\Omega}{1 - 5\Omega\cdot 5\Omega}$

**Solution 2**

- $Z_{in}$
- $M = \frac{5\Omega + 5\Omega\cdot 5\Omega}{1 - 5\Omega\cdot 5\Omega}$

- **Source stability circle**:
  - Given $Z_{in} = 0$ gives $Z_{in} = 5\Omega$, so Center of circle is unstable.
  - Must add series resistance on input of 75\Omega, $1+0.2 = 75\Omega$

- **Load stability circle**:
  - $Z_{in} = 0$ gives $Z_{in} = 5\Omega$, so Center of Smith chart is still unstable
  - Must add parallel resistance on output of $5\Omega$, $1+0.5 = 92.8\Omega / 1-0.3$
part b, 5 points

A FET has available and operating gain circles as below at 1 GHz.

\[
\begin{array}{c|c}
\text{available gain} & \text{operating gain} \\
\hline
\frac{\pi}{2} \quad \text{Volts} & \frac{\pi}{2} \quad \text{Volts} \\
\hline
\end{array}
\]

Assuming a 50Ohm impedance normalization, what are the optimum generator and load impedances?

\[
Z_{\text{gen, opt}} = \frac{25(1+j) \Omega}{150+10\Omega} \\
Z_{\text{l, opt}} = 150 + 10\Omega
\]

\[
G_A = \frac{P_{\text{AVA}}}{P_{\text{AVG}}} = \frac{P_{L}}{P_{AVC}} = G_T
\]

So \( G_A = G_T(\pi) \)

Optimum \( Z_s, G_A = \frac{1-j}{50\Omega} \)

Optimum \( Z_L = \frac{50\Omega}{1+j} = 50\Omega \frac{1+j}{1-j} \)

25. \text{Red Circle}

\[
G_p = \frac{P_{\text{in}}}{P_{\text{out}}} = \frac{P_{L}}{P_{AVC}} = G_T
\]

So \( G_p = G_T(\pi) \)

Optimum \( I_z = 0.5 \)

Optimum \( Z_L = 50\Omega \frac{1+0.5}{1-0.5} = 50\Omega \frac{1.5}{0.5} = 150 + 150\Omega \)
Problem 3, 35 points
Power gains and stability

The transistor has S11=0, S12=0.1, S21=8, S22=0.5

part a, 5 points

If the load impedance is an open-circuit, what is the input reflection coefficient?

\[ \Gamma_i = \frac{1}{1 + \frac{S_{11}S_{22}}{1 - S_{22}}} \]

\[ = \frac{1}{1 + \frac{1 \times 0.5}{1 - 0.5}} = \frac{1}{1 + \frac{0.8}{0.5}} = 0.8 \]

Since \( \Gamma_i > 1 \), the circuit is potentially unstable.

So, without having to compute \( S_1, S_1 \), we can see that the circuit is potentially unstable.
part b. 7 points

Is it necessary to stabilize the device before simultaneous input and output matching to it? Assuming that you have stabilized, if necessary, or have not stabilized (if not necessary), what power gain will you obtain after matching on both input and output?

Unconditionally Stable? **NO**

Power gain after simultaneous matching = \( 80:1 \).

From part a, with \( |Z_i| \leq 1 \), we have \( |Z_o| > 1 \), so potentially unstable.

Since it is potentially unstable, the power gain after stabilizing and matching is the NEC = \( \frac{1521}{1512} \cdot \frac{101}{10.11} = 80 \). (19.0 dB)
part c. 8 points

(hard thinking, ok math): Can you determine from the S-parameters above what values of source reflection coefficient would lead to potential instability? Can you determine the necessary value of parallel input stabilization resistance?

\[ \begin{align*}
I_{\text{cut}} &= S_{21} + \frac{S_{12} S_{21} L^2}{1 - S_{11} L^2} = 0.5 + 0.8125 \\
\text{set} \quad |P_{\text{cut}}| = 1, \quad \text{so} \quad [P_{\text{cut}} = e^{j\theta}] \quad \boxed{0!} \\
L^2 &= 1.25 e^{j\theta} - 0.625 \\
0.625, -1.875
\end{align*} \]

**Center of**
\[
I = -0.625 \\
t = 0.625
\]

**Circle, in the \( \pi \)-plane, is the region of stability.**

\[ \text{So, we need to keep} \quad \theta \quad \text{below 0.625} \]
\[ \frac{1 + 0.625}{1 - 0.625} = 2.172. \]

**This will stabilize the amplifier.**
part d, 5 points

Without stabilizing the FET, the FET is connected to a 100 Ohm generator, with 1 mW available power, and a 100 Ohm load. Find the power in the load

\[ P_L = \frac{91 \text{ mW}}{1} \]

we are being asked for the transducer gain, since \( G_T = \frac{P_L}{P_{AVG}} \).

\[ G_T = \frac{5_2 \left( 1 - \frac{P_L}{100} \right) \left( 1 - \frac{1}{100} \right)}{1 \left( 1 - \frac{L_2}{5_2} \right) \left( 1 - \frac{L_2}{5_2} \right) - 5_2 \frac{L_2}{5_2} L_2} \]

\[ L_2 = \frac{L_2}{5_2} = \frac{100 - 50}{100 + 50} = \frac{1}{3} \]

\[ = 5_2 \left( 1 - \frac{1}{100} \right) \left( 1 - \frac{1}{100} \right) \]

\[ = \frac{82 \left[ 1 - \frac{1}{9} \right] \left[ 1 - \frac{1}{9} \right]}{1 \left( 1 - \frac{1}{3} \right) - \frac{0.8}{9} \left( 1 \right)^2} \]

\[ = \frac{58.56}{10.741} = 91.23 = \frac{P_L}{P_{AVG}} \]

\[ P_L = 91.23 \times 1 \text{ mW} = 91.23 \text{ mW} \]
part e, 5 points

Without stabilizing the FET, the FET is connected to a 50 Ohm generator, with 1mW available power, and a 50 Ohm load. Find the power in the load

\[ P_L = 64 \text{ mW}. \]

4. We are being asked for the intrinsic gain, which is 150.  

\[ 150 = 1 \text{ mW} \times (s)^2 = 64 \text{ mW} \]
\[ S_{11} = 0 \quad S_{12} = 0.1 \quad S_{21} = 8 \quad S_{22} = 0.5 \]

**Part f. 5 points**

Without stabilizing the device, the generator, with 1mW available power, is impedance-matched to the FET input, and is then connected directly to a 100 Ohm load. Find the power in the load

\[ P_L = 90.9 \text{ mW} \]

\[ G_T = \frac{P_L}{P_{avc}} \quad \frac{P_L}{P_{avc}} = G_T \quad \frac{P_L}{P_{avc}} = \frac{100}{100 + 0.8} = 0.125 \]

Note that \( |\beta| < 1 \) with this particular load. In other words, though the net is potentially unstable, the load we have been given lies within the stable region on the \( \beta \)-plane. Had this not been true, this solution would not have existed (1).

\[ P_{in} = S_{11} + S_{21} \cdot S_{22} \cdot P_L = 0.8 \cdot \frac{1}{0.8} = 0.666 \]

\[ = 0.320 \]

\[ G_D = \frac{1}{1 - \frac{1}{1.1} \cdot \frac{15 \cdot 1}{1 - 1.1 \cdot \frac{1}{1 - \frac{1}{1.1} \cdot \frac{1}{1 - 1.5 \cdot 1}}}} \]

\[ = \frac{1}{1 - (0.32)^2} = \frac{64}{1 - 0.32} = \frac{64}{0.69} = 90.9 \]

\[ P_L = 90.9 \times 1 \text{ mW} = 90.9 \text{ mW} \]
Problem 4, 22 points
*S parameters and Signal flow graphs*

A transistor has the following s-parameters:

- S11 = 0.5
- S22 = 0.25
- S12 = 0.5
- S21 = 5

A second two-port consists of a 25 Ohm resistor between its input and output ports.
part a, 5 points

Using a 50 Ohm impedance standard, compute the four S-parameters of the resistor network.

\[
\begin{align*}
S_{11} &= \frac{1}{5} \\
S_{12} &= \frac{4}{5} \\
S_{21} &= \frac{4}{6} \\
S_{22} &= \frac{1}{6}
\end{align*}
\]
part b. 7 points

The resistor network is connected between the FET input. Compute the four S-parameters of the combined network.

\[
S_{11} = 0.555 \\
S_{12} = 0.444 \\
S_{21} = 4.141 \\
S_{22} = 0.905
\]

\[
\begin{align*}
S_{21} &= \frac{S_{21} R S_{21}^T}{1 - S_{22} S_{11}} \\
\substack{\text{help for formula} \ 1 - 0.11} &= \frac{4}{1} = 4.44 \\
\frac{1}{1 - 0.11} &= 4.44 \\
\text{help for math}
\end{align*}
\]

\[
\begin{align*}
S_{12} &= \frac{S_{12} R S_{12}^T}{1 - S_{22} S_{11}} \\
\frac{(4/5)(0.5)}{1 - (1/5) \cos(0.5)} &= 0.444
\end{align*}
\]

\[
\begin{align*}
S_{21} &= \frac{S_{21} R (1 - S_{22} R S_{11}) + S_{21} S_{11} S_{22} R}{1 - S_{22} R S_{11}} \\
\frac{(4/5)(0.5)(4/5)}{1 - 1/5 (0.5)} &= 1/5 + 0.32 \\
\frac{1}{1 - 0.11} &= 0.555 \\
\text{help for math}
\end{align*}
\]

\[
\begin{align*}
S_{22} &= \frac{S_{22}^T + S_{12} R S_{21} S_{22} R}{1 - S_{22} R S_{11}} \\
\frac{(4/5)(0.5)}{1 - 1/5 (0.5)} &= 0.25 + \frac{5(4/5)(1/5)}{1 - 1/5 (0.5)} = 0.15 + 0.5 \\
\end{align*}
\]
part c. 5 points

Y-parameters

Compute the Y-parameters of this network

\[ \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{j \omega C_{11}} + \frac{1}{j \omega C_{12}} \\ \frac{1}{j \omega C_{12}} \end{bmatrix} \]

1.25 to each parameter
part d, 5 points

Z-parameters

Compute the Z-parameters of this network

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
= \begin{bmatrix}
-i \omega L_{11} & i \omega L_{12} \\
-i \omega L_{12} & -i \omega L_{22}
\end{bmatrix}
\]

by inspection:

1.25 for each parameter.
Problem 5, 19 points
Power amplifier design

An HBT has the output characteristics as shown, with a maximum 2mA/micron collector current. The (somewhat contrived) device model is to the right, with

\[ g_m = 20.0 \text{mS/} \mu \text{m} \cdot L_E \quad R_{bb} = 20 \Omega - \mu \text{m} \cdot L_E \quad C_{be} = g_m \tau_f , \text{ where } \tau_f = 0.5 \text{ ps} , \]

\[ C_{ce} = 2 fF / \mu \text{m} \cdot L_E \]

part a, 6 points

The optimum load admittance is parallel combination of a conductance G and an inductive susceptance. Setting G to 40 milliSiemens, and setting the signal frequency to 100GHz, find (1) the appropriate HBT emitter length Le and (2) the required parallel load inductance L.

\[ \text{optimum load impedance for } \omega = \frac{3}{2\pi} \text{ is} \]

\[ Z_{in} = \frac{3}{2\pi} \text{ with parallel inductive load} \]

**\[ \text{so... we have } \omega L = 40 \text{ mH} \Rightarrow \frac{1}{2\pi} L = \frac{1}{22} \text{ mH} \]**

\[ \Rightarrow L = \frac{1500 \Omega}{25 \Omega} = \frac{6000 \Omega}{100 \Omega} = 60 \mu \text{m} \]

\[ \Rightarrow C_{ce} = 2.6 fF/\mu \text{m} \cdot 60 \mu \text{m} = 120 fF \]

\[ \text{read } 2\pi f = \frac{1}{4} \text{GHz} \Rightarrow f = \frac{1}{c(2\pi f)} = 21.1 \text{GHz} \]
part b. 5 points

What is the maximum saturated output power? What is the correct collector bias voltage and collector bias current?

\[ P_{\text{max}} = \frac{1}{8} \Delta V \cdot \Delta I = \frac{1}{8} \cdot 3V \cdot \left( \frac{2 \times 10^{-3}}{10} \cdot 60 \mu A \right) \]

3

\[ = \frac{1}{8} \cdot 3V \cdot 120 \mu A = 0.15 \text{ mW} \]

1

\[ V_{B_{12}} = \frac{1V + 4V}{2} = 2.5V \]

1

\[ I_{B_{12}} = 60 \mu A \]
part e, 8 points

After impedance-matching on the amplifier input and output, what is the amplifier power gain?

\[ R_{bs} = \frac{20 \Omega}{20} = 1 \Omega \]

\[ C_{c} = 9 \mu F \quad R_{m} = 20 \text{m} \Omega \quad C_{m} = 12 \text{m} \Omega \]

\[ = 600 \Omega \]

\[ P_{i} = (2 \sqrt{2} \cdot 25 \Omega) = (9 \mu V_{be})^{2} \cdot 25 \text{m} \Omega \]

\[ P_{i} = (\sqrt{3} \Omega) = (\omega C_{b} V_{be}) (\sqrt{3}) \]

\[ \frac{P_{o}}{P_{i}} = \left( \frac{9 \mu F}{\omega C_{b} V_{be}} \right)^{2} \cdot \frac{25 \text{m} \Omega}{1 \text{\Omega}} \]

\[ \frac{P_{o}}{P_{i}} = \left( \frac{1}{2\pi f} \right)^{2} \cdot \frac{25 \text{m} \Omega}{1 \text{\Omega}} = 10.1 \cdot \frac{25 \text{m} \Omega}{1 \text{\Omega}} = 760 \]

\[ 100 \text{dB} \]

\[ \frac{P_{o}}{P_{i}} = 760 \text{ quite unrealistic (silly transistor model).} \]