ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 14, 2012

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.), AFTER STATING THEM.

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Name: _______________________
Problem 1, 20 points
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.
Match each Smith Chart with each circuit, and give all resistor values, and all transmission line delays and characteristic impedances. The charts all use 50 Ohm normalization:

Smith chart (a). Circuit= \( E \). Component values= \( R_1 = 17 \Omega, R_2 = 433.3 \Omega \)

Smith chart (b). Circuit= \( G \). Component values= \( R_1 = 93.3 \Omega, R_2 = 17 \Omega \)

Smith chart (c). Circuit= \( P \). Component values= \( R_1 = 150 \Omega, T = 250 \mu s, Z_0 = 29 \Omega \)

Smith chart (d). Circuit= \( H \). Component values= \( R_1 = 55 \Omega, T = 250 \mu s, Z_0 = 29 \Omega \)
This is network (3)

\[ R_1 \rightarrow R_2 \rightarrow \]

At DC, \( Z = R_1 \) and \( \frac{R_2}{R_1 + 1} = 1 \rightarrow R_1 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \left( \frac{1 - 0.5}{1 + 0.5} \right) = 16.7 \Omega \)

At \( o \rightarrow o \), \( Z = R_1 + R_2 \rightarrow R_1 + R_2 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \frac{1 + 0.8}{1 - 0.8} \)

\[ R_1 + R_2 = 450 \Omega \]

\[ R_2 = 433.3 \Omega \]

This is network (4)

At DC, \( Z = R_1 \) and \( \frac{R_2}{R_1 + 1} = 1 \rightarrow R_1 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \frac{1 + 1.8}{1 - 1.8} = 450 \Omega \)

\[ R_2 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \frac{1 - 0.5}{1 + 0.5} = 17 \Omega \]

\( L \), \( R_1 = 473 \Omega \)

This is network (5)

\[ R_1 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 150 \Omega \]

It is not possible to find the \( Z_0 \) of the line

Line is \( 1/4 \) @ 15%, so \( 1/4 \) @ 46%

So \( T = (1/4) \times 25.95 \)

This is network (6)

At DC, \( R_1 = Z_0 \frac{1 + \frac{1}{L}}{1 - \frac{1}{L}} = 50 \Omega \frac{1 - 0.8}{1 + 0.8} = 5.55 \Omega \)
Smith Chart D

This is network (h)

At DC, $\omega = 0$.

At DC, $R = Z_0 \frac{1 + j\omega}{1 - j\omega}$

$= 50 \frac{1 - 0.5}{1 + 0.5}$

$= 5.55 \Omega$

At 16kHz, $\omega$ is high freq. So use shunt long @ 4kHz.

$\rightarrow \gamma = 259^\circ$.

At 16kHz, the input impedance is $Z = Z_0 \frac{1 + j\omega}{1 + j\omega} = 150 \Omega$

This is the quarter-wave condition.

So, $Z_{line} = \sqrt{R \cdot Z_0} \div \sqrt{555} \cdot 150 \Omega$

$= 28.9 \Omega$
**Problem 2, 30 points**

2-port parameters and Transistor models

**Part a, 20 points**

For the network at the right, give algebraic expressions for the four Z-parameters and for the four S-parameters.

Then using $R_1=25$ Ohms, $R_2=50$ Ohms, give numerical values.

---

![Diagram of network with ports](image)

By inspection:

\[ Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \]

\[ = \begin{bmatrix} 150 & 50 \\ 50 & 50 \end{bmatrix} \]

\[ S_{11} = \begin{bmatrix} R_1 \end{bmatrix} \]

\[ Z_{in} = 50 \text{ Ohm} \]

\[ = \frac{R_1}{Z_0 + R_1} = \frac{25}{50 + 25} = 50 \text{ Ohm} \]

\[ S_{11} = \frac{R_2}{Z_0 + R_1} - \frac{1}{Z_0} = \frac{1 - 1}{1 + 1} = 0 \]
\[ Z_{in} = Z_0 \frac{Z_0 + Z_o}{Z_0 + R_1} \]

\[ Z_{in} = Z_0 \frac{Z_0 + Z_o}{Z_0 + R_1} \]

\[ S_{21} = \frac{Z_i}{Z_o} = \frac{Z_0 || (Z_0 + R_1)}{Z_0 + R_1} = \frac{50 \Omega || 75 \Omega}{75 \Omega} = 80 \Omega \]

\[ S_{22} = \frac{Z_i}{Z_o} = \frac{Z_0 || (Z_0 + R_1)}{Z_0 + R_1} = \frac{0.6}{0.6 + 1} = -0.25 \]

\[ S_{12} = \frac{V_{in}}{V_{out}} = \frac{2Z_0}{Z_0 + R_1} \]

\[ S_{12} = \frac{2Z_0}{Z_0 + R_1} \]

\[ S_{12} = \frac{2 \left( \frac{Z_0}{Z_0 + R_1} \right) \left[ \frac{(Z_0 + R_1) || Z_o}{(Z_0 + R_1) || Z_o + Z_0} \right]}{(Z_0 + R_1) || Z_o + Z_0} \]

\[ S_{12} = \frac{2 \left( \frac{50 \Omega}{50 \Omega + 25 \Omega} \right) \left( \frac{50 \Omega + 25 \Omega}{50 \Omega + 25 \Omega + 75 \Omega} \right)}{(50 \Omega + 25 \Omega) || 75 \Omega} = \frac{1}{2} \]

\[ S_{21} = S_{21} \]

as is required.
Part b. 10 points
To the right is the equivalent circuit of a FET.

First give algebraic expressions for $S_{11}$ and $S_{21}$ as a function of frequency.

Then set the transconductance to 100 mS, $R_i = 5$ Ohms, $C_{gs} = 50$ fF, $R_{DS} = \infty$, and calculate $S_{21}$ at 10GHz.

\[
S_{11} = \left[ \begin{array}{c}
Z_{in} = R_i + \frac{1}{j\omega C_{gs}} \\
\frac{Z_{in}}{Z_0} = R_i/Z_0 + \frac{1}{j\omega C_{gs} Z_0} \\
S_{11} = \frac{Z_{in} + 1}{Z_{in} + 1/j\omega C_{gs} Z_0 + 1}
\end{array} \right]
\]

\[
S_{21} = \frac{2V_o}{Z_{in} + 1} = \frac{2 \left( \frac{1}{j\omega C_{gs}} \right)}{1/j\omega C_{gs} + R_i + Z_0}
\]

\[
= -2g_m \left( \frac{R_{ds}/Z_0}{1 + j\omega C_{gs} (R_i Z_0)} \right)
\]

\[
= \frac{-10}{1 + j0.173} = \frac{-10(1 - j0.173)}{1 + (0.173)^2}
\]

\[
S_{21} = -9.71 + j1.68
\]
Problem 3, 20 points

*Elementary impedance matching network design.*

You are going to design an impedance-matching network to match a 20 Ohm resistor to a 50 Ohm generator at a frequency of 2 GHz.

Part (a), 10 points.

*Using the impedance-admittance charts that have been passed out,* design a lumped-element matching network. Use a series inductor and a shunt capacitor. Give the circuit diagram and the element values.

At Point A:

$$\frac{1}{Z_A} = \frac{20\Omega}{50\Omega} = 0.4 = \frac{1}{Z_A}$$

At Point B:

$$\frac{1}{Z_B} = 0.4 + j0.5$$

*Added normalized reactance.*

$$jX = 0.5 \Rightarrow jX = j(0.5)\frac{20\Omega}{2\pi(26\mu)}$$

$$\Rightarrow L = 25\mu \quad \frac{j\omega L}{2\pi(26\mu)} = 1.99 \mu H$$

$$Y_B = 1.0 - j1.2$$

At Point C:

$$Y_C = 1.0 - j0$$

$$\Rightarrow -jB = j1.2 \quad \Rightarrow jB = j\frac{1.2}{50\Omega} = j0.0245$$

$$\sqrt{\frac{1}{j\omega C}} = \frac{1.2}{j\frac{1}{50\Omega}(2\pi f)} = 1.91 \mu F$$
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES
Part (b), 10 points.

Lumped inductors are not available to you. Using the impedance-admittance charts that have been passed out, design a second matching network using a series transmission-line of 100 Ohms characteristic impedance and a shunt capacitor. The propagation velocity on the transmission line is 2/3 the speed of light. Give the circuit diagram and the element values (capacitance, length of line).

From Smith chart: $A \rightarrow B$

Length of 63° length =

$$\text{Length} = \frac{63° \cdot \sqrt{\frac{218.1c}{2(109.16)}}}{360°} = 18.5 \text{ mm}$$

Point B

$$Y = 1 + j1$$

Point C

$$Y = 1 + j0$$

$$\sqrt{(1c)} = j1 \Rightarrow jB = \sqrt{\frac{1}{50\mu}} \Rightarrow jwc$$

$$\Rightarrow C = \frac{1}{w^2 50\mu} = \frac{1}{2\pi f 50\mu} = 16 \mu\text{F}$$
Problem 4, 15 points
Transmission-lines and lumped elements

Part a: 5 points

A transmission-line has propagation velocity of 2/3 the speed of light. Its characteristic impedance is 75 Ohms. It is one meter long. Find the total inductance and total capacitance in this cable.

\[ L = \frac{2\pi L_0}{v} = \frac{2\pi \cdot 75}{\frac{2}{3} \cdot c} \]

\[ = 75 \cdot \frac{1m}{2 \cdot 10^8 m/s} = 75 \cdot 5 \cdot 10^{-5} = 0.375 \mu H \]

\[ C = \frac{1}{2\pi f} = \frac{5 \cdot 10^{-5}}{75 \cdot 2\pi} = 66.6 pF \]
Part b: 5 points
The above cable is loaded in a resistor $R_1 = 10$ kOhm. Find the input impedance of the cable at a frequency of 1 MHz.

*Critical hint: use any and all reasonable approximations. Do not carry terms in the calculation which are negligible.*

\[
\begin{align*}
\tau_{line} & \quad Z_{line} \quad R_1 \\
\frac{1}{\omega L} & \quad \frac{1}{C} \\
C & = 0.37 \mu F \\
\omega & = 3.7 \times 10^7 \\
\left(\frac{1}{\omega C}\right) & = 2.37 k\Omega
\end{align*}
\]

\[
\begin{align*}
Z & = 10 k\Omega \left( 1 - j \cdot 2.37 k\Omega \right) \\
& = \frac{10^2 + (2.37)^2}{10^2} \quad \text{k}\Omega \\
& \approx 5.318 + \frac{-j224.4}{\text{Ohm}}
\end{align*}
\]
**Part b: 5 points**

You are now going to design a physical matching network for the input to a transistor amplifier. The design frequency is 10 GHz.

The series line #1 must have 100 Ohm characteristic impedance and is 1/8 (guide) wavelength long at the design frequency.

The shunt line #2 must have 50 Ohm characteristic impedance and is 0.1 (guide) wavelength long at the design frequency.

The lines are microstrip lines. The substrate is 75 micrometer thick and has a dielectric constant of 12. Use the approximations \( v = c/\sqrt{\varepsilon} \) and

\[
Z_0 = 377 \Omega \cdot (\varepsilon_r)^{-1/2} \cdot H/(H+W),
\]

where \( H \) is the substrate thickness and \( W \) the conductor width.

Compute the length and width of lines 1 and line 2

- **line 1 length (m)**: 5.0825 mm
- **line 1 width (m)**: 53.2373 μm
- **line 2 length (m)**: 0.866 mm
- **line 2 width (m)**: 88.245 μm
Short line (length) \( L \)

\[
L = \frac{c}{f} = \frac{2.4 \times 10^8 \text{ m/s}}{1.25 \times 10^3 \text{ Hz}} = 2.0 \times 10^5 \text{ mm}.
\]
Please make a well-scaled sketch (top view) the matching network below with dimensions indicated.
**Problem 5, 15 points**  
*resistive feedback amplifiers*

![Circuit Diagram](attachment:diagram.png)

A FET has a transconductance of 1 mS per micron of gate width. $C_g$ is 1 fF per micron of gate width, and $C_{gd}$ is 0.1 fF per micron of gate width.

Design a resistive feedback amplifier with 20 dB gain S21 for a 75 Ohm system using this FET. Draw the circuit diagram with all element values and determine the following:

- **FET width**: $14.67 \mu m$
- **transconductance**: $2 \cdot 14.67 \, \text{S}$
- **$C_g$**: $14.67 \, \text{fF}$
- **$C_{gd}$**: $14.67 \, \text{fF}$

amplifier 3-dB bandwidth (from nodal analysis or the time constant method).

\[
\begin{align*}
(A_n)_{d0} &= 20 \quad \Rightarrow \quad A_r = 10 \\
Z_o &= 75 \, \Omega
\end{align*}
\]

To minimize reflections, $S_{in} = 0$,

\[
\begin{align*}
R_f &= (1 + A_r)Z_o \\
R_f &= 82.5 \, \Omega \quad \text{(2)}
\end{align*}
\]

\[
\begin{align*}
g_m &= \left(1 + A_r\right) / Z_o = 0.1467 \, \text{S} \\
g_m &= 0.1467 \, \text{S}
\end{align*}
\]

\[
\begin{align*}
10^3 \cdot W_f &= 0.1467 \quad \Rightarrow \quad W_f = 14.67 \, \mu \text{m}
\end{align*}
\]
\[ C_{ge} = 1 \text{\( \mu \)} F + V_{g} = 14.67 \text{\( \mu \)} F \]
\[ C_{d} = 0.1 \text{\( \mu \)} F + V_{g} = 14.67 \text{\( \mu \)} F \]

\text{Circuit}

\[ R_{g} = 825 \text{\( \Omega \)} \]

\[ G_{i} = 75 \Omega \]

\[ G_{d} = 14.67 \text{\( \mu \)} F \]

\[ G_{e} = 0.1467 \times V_{ge} \]

By 

\[ R_{in} = \left( 75 \Omega \right)^{2} = 27.75 \Omega \]

\[ R_{22} \]

is calculated as follows. First remove \( R_{g} \).

\[ V_{ge} = I_{in} R_{g} \]

\[ V_{in} = V_{ge} + \left( j_{m} V_{ge} + I_{in} \right) \times R_{L} \]

\[ = I_{in} \left[ R_{21} + \left( j_{m} \frac{I_{in}}{V_{ge}} + 1 \right) \times R_{L} \right] \]

\[ R_{in} = \frac{V_{in}}{I_{in}} = \frac{Z_{o} + \left( 1 + j_{m} Z_{o} \right) \times Z_{o}}{R_{21}} \]

\[ R_{22} = R_{f} || R_{in} \quad \text{(Hold on to \( R_{f} \) back again)} \]

\[ R_{22} = R_{f} \left[ \frac{Z_{o} + \left( 1 + j_{m} Z_{o} \right) \times Z_{o}}{R_{21}} \right] = 825 \Omega \left[ 75 + 900 \times 1875 \right] \]

\[ R_{22} = 44.6 \Omega \]

\[ a_{1} = R_{in} C_{ge} + P_{22} C_{o} = \left( 5.581 + 6.55 \right) \text{pF} \]

\[ a_{1} = 12.181 \text{pF} \]
\[
\frac{1}{a} = \omega_{34} \Rightarrow \omega_{34} = \frac{1}{2\pi a} = 13.2 \text{ GHz}
\]

\[
\omega_{34} = 13.2 \text{ GHz}
\]