ECE 145a / 218 a, notes set 4: Impedance Matching

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Impedance-Matching: Goals
Recall: Line Reflections

At end of line:

\[ V^-(z = 0) = \Gamma_L V^+(z = 0) \text{ where } \Gamma_L = \frac{(Z_L/Z_o)-1}{(Z_L/Z_o)+1} \]

At beginning of line:

\[ V^+(z = -l) = \Gamma_s V^-(z = -l) + T_s V_{\text{gen}} \text{ where } \Gamma_s = \frac{(Z_s/Z_o)-1}{(Z_s/Z_o)+1} \]
Reflections:

\[ V^{-}(z = 0) = \Gamma_L V^{+}(z = 0) \quad \text{and} \quad V^{+}(z = -l) = \Gamma_s V^{-}(z = -l) + T_s V_{\text{gen}} \]

Waves traveling along line:

\[ V^{+}(z = -l) = V^{+}(z = 0) \cdot e^{j2\pi l/\lambda} \quad \text{and} \quad V^{-}(z = -l) = V^{+}(z = 0) \cdot e^{j2\pi l/\lambda} \]

Varying frequency, \( V^{+} \) and \( V^{-} \) will vary from in-phase to out-of-phase... and will vary from constructive to destructive interference.

\( \Rightarrow \) load voltage will vary with frequency.
Matching to Eliminate Gain Ripple

Gain ripples resulting from standing waves on line

Matching will eliminate this

One reason for impedance - matching is to eliminate gain ripple from standing waves on lines
Maximum Power Transfer Theorem

(Maximum) Power Available from the Generator:

\[ P_{\text{avg}} = \left\| V_{\text{gen}} \right\|^2 / 4 \text{Re}\{Z_{\text{gen}}\} \quad \text{(RMS quantities)} \]

Load Power: \( P_L = P_{\text{AVG}} \) iff \( Z_L = Z_{\text{gen}}^* \); \( P_L < P_{\text{AVG}} \) otherwise
Matching For Maximum Power Transfer

By adding a *lossless* matching network (no resistances) between the generator and the load, we obtain $P_L = P_{AVG}$

Another reason for impedance-matching is to increase signal power transferred.
Matching for no reflection vs. matching for max Power Transfer

1\textsuperscript{st} case: $Z_{gen} = Z_0$

$\Rightarrow Z_{out,line} = Z_0$

2\textsuperscript{nd} case: $Z_{gen} \neq Z_0$

$\Rightarrow Z_{out,line} \neq Z_0$

If $Z_{out,line} \neq Z_0$, then

matching for zero reflection
and matching for maximum power transfer
are not the same.
Direct Interstage Matching vs. Matching Each to $Z_0$

**Matching each to $Z_0$**

$$
\begin{align*}
Z_{out} & \quad Z_{out}^* \\
Z_0 & \quad Z_0 \\
\text{match} & \\
\text{Z_0 length}=l \\
\Gamma_{gen} &= 0 \\
\Gamma_L &= 0
\end{align*}
$$

**Direct Stage-Stage Matching**

$$
\begin{align*}
Z_{out} & \quad Z_{in, line} = Z_{out}^* \\
Z_{out, line} & = Z_{in, M}^* \\
Z_{in, M} & \\
\text{Z_0 length}=l \\
\text{match} \\
\Gamma_{gen} & \neq 0 \\
\Gamma_L & \neq 0
\end{align*}
$$

Note: $Z_{in, line} = Z_0 \left(1 + \Gamma_L e^{-2j\beta l}\right) / \left(1 - \Gamma_L e^{-2j\beta l}\right)$, $l$ can be any length, including zero.
Impedance-Matching: Using Agilent / ADS in "tune" mode as a study tool.
Use CAD Tool (Agilent) ADS to Explore Matching

Classic Texts: present Matching with On - Paper Smith - Chart Exercises.

Today: Matching easily presented graphically in CAD program.

First: Show how to tune networks in ADS.
Second: Illustrate matching examples.
"Match" here is a circuit, having a MOSFET and an input matching network.
Tuning Elements in ADS (2)

Use series $L$, shunt $C$ network to match $Z_{in}$ to $Z_0$.

Very simple MOSFET small-signal model.
Setting Up Element Tuning in ADS:

Double-click on $L_1$, then press Tune/Opt/Stat...:

Enable tuning, then set min, max, step values...
Setting Up Element Tuning in ADS:

$L_1$ is now a tunable element:

Do the same for $C_1$:
Setting Up Element Tuning in ADS:

Go to the main testbench and select tuning:

Then manipulate windows until you can see both the "Tune Parameters" window and the Plot window.
Setting Up Element Tuning in ADS:

Usually easiest to make the plot update after every tuning change:

Need to update Schematic after tuning: otherwise, you will lose the changes made.
Impedance-Matching: Methods/Examples
Trajectories for adding series / shunt L and C

Adding Series L or C

Adding Shunt (Parallel) L or C
1st Lumped L-C Matching Network:

Network Topology

$S_{11}$ before matching at 100 GHz
1st Lumped L-C Matching Network:

Increase $L$ until $Y_{in}/Y_0 = 1.0 + jB$: Reached when $L_1 = 112$ pH

We have moved on a constant - $r$ circle towards values of higher reactance $jx$.

Increase $C$ until $Z_{in}/Z_0 = 1.0 + j0$: Reached when $C_1 = 44$ fF

We have moved on a constant - $g$ circle towards values of higher susceptance $jb$. 
1st Lumped L-C Matching Network:

Final Values

Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)
2nd Lumped L-C Matching Network:

Network Topology

$S_{11}$ before matching at 100 GHz
2nd Lumped L-C Matching Network:

Increase $L_1$ until $Y_{in} / Y_0 = 1.0 - jB$ : Reached when $L_1 = 38$ pH

We have moved on a constant - $r$ circle towards values of higher reactance $jx$.

Increase $L_2$ from $\infty$ until $Z_{in} / Z_0 = 1.0 + j0$ : Reached when $L_2 = 56$ pH

We have moved on a constant - $g$ circle towards values of higher susceptance $jb$. 
2nd Lumped L-C Matching Network:

Final Values

Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)

More direct matching trajectory → broader bandwidth
3rd Lumped L-C Matching Network:

Matching network with values

$S_{11}$ matching trajectory at 100 GHz

$A$: original $Z_{in}$

$C$: after adding $L_2$ in parallel

$D$: after adding $C_1$ in series
3rd Lumped L-C Matching Network:

Final Values

Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)

Long matching trajectory → narrower bandwidth
4th Lumped L-C Matching Network:

Matching network with values

\[ S_{11} \] matching trajectory at 100 GHz

A: original \( Z_{in} \)

B: after adding \( L_2 \) in parallel

D: after adding \( L_1 \) in series
4th Lumped L-C Matching Network:

Final Values

Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)

short matching trajectory $\rightarrow$ wider bandwidth
Multi-Section L-C Matching Network:

Tuning with multiple series/shunt elements.

Infinite # of possible matching networks.
Lines of Constant $Q$:

$Q = \frac{\text{(maximum energy stored)}}{\text{(energy dissipated per radian)}}$

$= \frac{\|X_{\text{series}}\|}{R_{\text{series}}}$

$= \frac{\|B_{\text{series}}\|}{G_{\text{series}}}$ for simple 2 - element impedances

curves of constant $Q$
look roughly like this.

Matching networks passing through high - $Q$ points will have narrow bandwidth.
Narrowband vs. Wideband Matching Networks

4-element: wideband

2-element: narrowband
Limits to Matching Network Bandwidth

Low - $Q$ load $Z_L$

Starting point ($Z_L$) is low - $Q$
→ match can be made wide or narrow

High - $Q$ load $Z_L$

Starting point ($Z_L$) is high - $Q$
→ match cannot be made wide
Shunt-Stub Matching Networks
Trajectories for Adding a Series Line of Impedance $Z_o$

Adding Series TRX line:

Series line: $Z_{line} = Z_{system\_standard} = Z_0$

(if the standard impedance is 50Ω, the line is 50Ω)

Increasing the line length rotates the load reflection coefficient $\Gamma_{in} = \Gamma_L e^{-2jl/\lambda}$
Recall: Trajectory for adding Shunt Susceptance

Adding Shunt (Parallel) Susceptance

Susceptance can be an ideal \{L \text{ or } C\}.

Susceptance can be a shunt transmission - line stub.

Susceptance can be a shunt radial stub.
Susceptance of Shunt Stub (Open-Terminated)

Open - Terminated

\[ jB_{\text{match}} \]

\[ Y = 0 + j0 \]

Short - Terminated

\[ jB_{\text{match}} \]

\[ Y = \text{infinity} \]
Shunt-Stub Matching Network:

Series line brings load to $\frac{Y}{Y_0} = 1 \pm jB$

Shunt stub adds $Y_{stub} / Y_0 = \mp jB$

Combination brings load to $\frac{Y}{Y_0} = 1 + j0$
1st Shunt-Stub Matching Network

Network Topology

$S_{11}$ before matching at 100 GHz
1st Shunt-Stub Matching Network

Increase $TL_1$ length until $Y_{in} / Y_0 = 1.0 + jB$: Reached when $2\pi l_1 / \lambda = 68\text{ degrees}$

We have moved on a constant $-\Gamma$ circle.

Increase $TL_2$ length until $Y_{in} / Y_0 = 1.0 + j0$: Reached when $2\pi l_2 / \lambda = 65\text{ degrees}$

We have moved on a constant $-g$ circle towards values of higher susceptance $jb$. 
1st Shunt-Stub Matching Network

Final Values

Performance vs Frequency
(DC - 200 GHz frequency sweep, marker at 100 GHz)
2nd Shunt-Stub Matching Network

A shorter series line section $TL_1$ brings $Z_{\text{matched}}$ from A to B

The shunt stub must now be inductive
3rd Shunt-Stub Matching Network

Series line brings load to $Y / Y_0 = 1 \pm jB$

Microstrip Radial Shunt stub adds $Y_{stub} / Y_0 = \mp jB$

Combination brings load to $Y / Y_0 = 1 + j0$

Radial stub:
a wedge-shaped capacitor with distributed effects accurately modelled.
3rd Shunt-Stub Matching Network

Final Values

Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)
Shunt-Stub Matching Networks...

...with general line impedance
Trajectories for Adding a High-Zo Series Line

Adding Series inductance

Adding series line \( Z_{\text{line}} = Z_0 \)

Adding series line \( Z_{\text{line}} > Z_0 \)

Behavior is intermediate between series inductance & series line of impedance \( Z_0 \).
Matching Network with High-Zo Series & Low-Zo Shunt Lines

Series line is mostly inductive, shunt line is mostly capacitive.
Lumped vs. Distributed Matching Networks

\[ L_{\text{shunt}} = \tau_{\text{shunt}} Z_{\text{shunt}} \quad C_{\text{shunt}} = \frac{\tau_{\text{shunt}}}{Z_{\text{shunt}}} \quad L_{\text{series}} = \tau_{\text{series}} Z_{\text{series}} \quad C_{\text{series}} = \frac{\tau_{\text{series}}}{Z_{\text{series}}} \]

If we force \( Z_{\text{series}} \to \infty \) and \( Z_{\text{shunt}} \to 0 \), while holding \( L_{\text{series}} \) & \( C_{\text{shunt}} \) constant, then \( C_{\text{series}} = \frac{L_{\text{series}}}{Z_{\text{series}}} \to 0 \) and \( L_{\text{shunt}} = C_{\text{shunt}} Z_{\text{shunt}}^2 \to 0 \)

L - C matching network is limiting case of high - Z/low - Z network.

Distributed matching networks can be approximated by LC networks.
Trajectories for Adding a Low-Z₀ Series Line

Adding shunt capacitance

Adding series line $Z_{line} = Z₀$

Adding series line $Z_{line} < Z₀$

Behavior is intermediate between shunt capacitance & series line of impedance $Z₀$
Matching Network with High-Zo Series & Low-Zo Series Lines

Series line is mostly inductive.

Shunt line is mostly capacitive

Dotted lines show lumped $(L,C)$ trajectory
Lumped vs. Distributed Matching Networks

\[ L_{\text{high}} = \tau_{\text{high}} Z_{\text{high}} \quad C_{\text{high}} = \frac{\tau_{\text{high}}}{Z_{\text{high}}} \]

If we force \( Z_{\text{high}} \to \infty \) and \( Z_{\text{low}} \to 0 \), while holding \( L_{\text{high}} \) & \( C_{\text{low}} \) constant, then:

\[ C_{\text{high}} = \frac{L_{\text{high}}}{Z_{\text{high}}^2} \to 0 \]

and \( L_{\text{low}} = C_{\text{low}} Z_{\text{low}}^2 \to 0 \)

L - C matching network is limiting case of high - \( Z/\text{low} - Z \) network.

Distributed matching networks can be approximated by LC networks.
Lumped vs. Distributed Matching Networks

Given this mask layout...

...your eyes should see this.
Lumped vs. Distributed Matching Networks

Given this mask layout...

...your eyes should see this.
Quarter-wave and related Matching Networks...
Quarter-wave Impedance Transformer

At load: \[
\frac{Z_L}{Z_{\text{line}}} = \frac{1 + \tilde{\Gamma}_L}{1 - \tilde{\Gamma}_L}
\]

At input: \[
\frac{Z_{\text{in}}}{Z_{\text{line}}} = \frac{1 + \tilde{\Gamma}_{\text{in}}}{1 - \tilde{\Gamma}_{\text{in}}}
\]

But: \[
\tilde{\Gamma}_{\text{in}} = \tilde{\Gamma}_L e^{-j \frac{4 \pi l}{\lambda}} = -\tilde{\Gamma}_L \text{ if } l = \frac{\lambda}{4}
\]

So: \[
\frac{Z_{\text{in}}}{Z_{\text{line}}} = \left( \frac{Z_L}{Z_{\text{line}}} \right)^{-1} \text{ if } l = \frac{\lambda}{4}
\]

\[
Z_{\text{in}} Z_L = Z_{\text{line}}^2 \text{ if } l = \frac{\lambda}{4}
\]

\(\tilde{\Gamma}_L\) and \(\tilde{\Gamma}_{\text{in}}\) are reflection coefficients using \(Z_{\text{line}}\) as the impedance standard, not \(Z_0\).
Regardless of the starting point \((Z_L)\), and ending point \((Z_{in})\), the impedance trajectory contains the points \((A, B)\), with resistive input impedance such that

\[
R_{in,A} \cdot R_{in,B} = Z_{line}^2
\]
Quarter-wave Impedance Transformer: Resistive Loads

\[ R_{in} R_L = Z_{line}^2 \]

Pick line impedance to match \( Z_{in} \) to \( Z_0 \):

\[ Z_{line} = \sqrt{R_L Z_0} \]
Quarter-wave Impedance Transformer: General Loads

For a general load impedance $Z_L = 1/Y_L = R_L + jX_L$

where $Y_L = G_L \pm jB_L$, one can add a shunt

susceptance $\mp jB_L$, bringing the load impedance
to $Z'_L = R'_L = 1/G_L$

Then one adds a quarter - wavelength line of impedance

$Z_{line} = \sqrt{R_L Z_0}$

to match the input impedance to $Z_0$. 