ECE145a / 218a
Power Gain Definitions

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Power Gain Definitions: Summary

Transducer Gain
\[ G_T = \frac{P_{load}}{P_{av.gen}} = \frac{\text{load power}}{\text{power available from generator}} = \text{general-case gain} \]

Available Gain
\[ G_A = \frac{P_{av,a}}{P_{av.gen}} = \frac{\text{power available from amplifier}}{\text{power available from generator}} = \text{gain with output matched} \]

Insertion Gain
\[ \|S_{21}\|^2 = \frac{P_{av,a}}{P_{av.gen}} = \frac{\text{power delivered to } Z_o \text{ load}}{\text{power available from } Z_o \text{ generator}} = \text{gain in a 50 Ohm environment} \]

Operating Gain
\[ G_p = \frac{P_{load}}{P_{gen.delivered}} = \frac{\text{load power}}{\text{power delivered from generator}} = \text{gain with input matched} \]

Maximum Available Gain
\[ G_{\text{Max}} = \frac{P_{av,a}}{P_{gen.delivered}} = \frac{\text{power available from amplifier}}{\text{power delivered from generator}} = \text{gain with both ports matched} \]

...but only if unconditionally stable...

After impedance-matching:
\[ \|S_{21,\text{matched}}\|^2 = G_{\text{max,raw}} \]
\[ S_{11,\text{matched}} = S_{22,\text{matched}} = 0 \]
Types of 2-Ports To Consider

The 2-port might be a transistor ....neither $S_{11}$ nor $S_{22} = 0$

The 2-port might be a transistor with matching built-in $S_{11}$ & $S_{22}$ close or equal to zero

The 2-port might be a general amplifier with arbitrary $S_{ij}$

For all of these, we could choose to add *additional* *matching networks at the generator and at the load.
### Insertion Power Gain

From S-parameter properties,

\[ S_{21} = 2 \frac{V_{\text{out}}}{V_{\text{gen}}} \bigg|_{Z_{\text{gen}}=Z_L=Z_0} \]

But if \( Z_{\text{gen}} = Z_0 \rightarrow P_{\text{av,gen}} = \left| V_{\text{gen}} \right|^2 / 4Z_0 \)

And if \( Z_L = Z_0 \rightarrow P_{\text{load}} = \left| V_{\text{out}} \right|^2 / Z_0 \)

\[ \rightarrow P_{\text{load}} / P_{\text{av,gen}} = \frac{\left| V_{\text{out}} \right|^2}{4 \left| V_{\text{gen}} \right|^2} = \| S_{21} \|^2 \quad \text{but only if} \quad Z_{\text{gen}} = Z_L = Z_0 \]

hence

\[ \| S_{21} \|^2 = \frac{\text{power delivered to } Z_0 \text{ load}}{\text{power available from } Z_0 \text{ generator}} = \text{gain in a } Z_0 \text{ (50 Ohm?) environment} \]
Transducer Power Gain

\[ G_T = \frac{P_{load}}{P_{av,\text{gen}}} = \frac{\text{power delivered to load}}{\text{power available from generator}} = \text{general - case gain} \]

Amplifier input impedance might or might not be matched to \( Z_{source} (Z_{gen}) \).

Amplifier output impedance might or might not be matched to \( Z_{load} \).

Generator impedance might or might not be \( Z_o \).

Load impedance might or might not be \( Z_o \).

\( G_T \) depends upon both \( Z_s \) and \( Z_L \).
Operating Power Gain

\[ G_p = \frac{P_{load}}{P_{gen,delivered}} = \frac{\text{load power}}{\text{power delivered from generator}} \]

Note: \[ P_{load} / P_{gen,delivered} = P_{load} / P_{avg} = G_T \] iff \[ Z_{gen} = Z_{in}^* \]

hence \( G_p = \text{gain} \) with input matched

Amplifier input impedance *is* matched to \( Z_{source} (Z_{gen}) \).

Amplifier output impedance might or might not be matched to \( Z_{load} \).

Generator impedance might or might not be \( Z_o \)

Load impedance might or might not be \( Z_o \)

\( G_p \) depends upon \( Z_L \) but not upon \( Z_s \).
Available Power Gain

\[ G_A = \frac{P_{av,a}}{P_{av,gen}} = \frac{\text{power available from amplifier}}{	ext{power available from generator}} \]

Note: \( \frac{P_{av}}{P_{avg}} = \frac{P_{load}}{P_{avg}} = G_T \) iff \( Z_{load} = Z_{out}^* \)

hence \( G_a = \text{gain with output matched} \)

Amplifier input impedance might or might not be matched to \( Z_{source} \) (\( Z_{gen} \)).

Amplifier output impedance *is* matched to \( Z_{load} \).

Generator impedance might or might not be \( Z_o \)

Load impedance might or might not be \( Z_o \)

\( G_A \) depends upon \( Z_s \) but not upon \( Z_L \).
Maximum Available Power Gain

\[ G_{\text{max}} = \frac{P_{\text{av},a}}{P_{\text{gen,delivered}}} \]

\[ = \frac{\text{power available from amplifier}}{\text{power delivered from generator}}. \]

Note: \[ \frac{P_{\text{av}}}{P_{\text{gen,delivered}}} = \frac{P_{\text{load}}}{P_{\text{avg}}} = G_T \] iff \( Z_{\text{gen}} = Z_{\text{in}}^* \) and \( Z_{\text{load}} = Z_{\text{out}}^* \)

hence \( G_{\text{max}} \) = gain with both input and output matched

Simultaneous input/output matching may not be possible \( \rightarrow \) MAG may not exist.

Amplifier input impedance *is* matched to \( Z_{\text{source}} \) (\( Z_{\text{gen}} \)).

Amplifier output impedance *is* matched to \( Z_{\text{load}} \).

Generator impedance might or might not be \( Z_o \)

Load impedance might or might not be \( Z_o \)

\( G_{\text{max}} \) depends upon neither \( Z_s \) nor \( Z_L \).
MAG Does Not Always Exist

\[ S_{21} \text{ and } S_{12} \text{ represent interaction terms between input and output.} \]

If \( S_{21}S_{12} \neq 0 \), then input & output tuning become mutually interactive.

With sufficiently large \( S_{21}S_{12} \), simultaneous matching of input & output is not possible.

In this case, \( G_{\text{max}} \) no longer exists.

This condition corresponds to potential amplifier instability.

Will be covered later.
\[ \| S_{21,\text{matched}} \|^2 = \left( \frac{2V_{out}}{V_{gen}} \right)^2 = \frac{\| V_{out} \|^2}{\| V_{gen} \|^2 / 4Z_o} = P_{\text{load}} / P_{\text{avg}} \]

but \( P_{\text{load}} / P_{\text{avg}} = G_{\text{max,raw}} \)

So: \( \| S_{21,\text{matched}} \|^2 = G_{\text{max,raw}} \)

The dB insertion gain of the amplifier is the dB MAG of the transistor.

Implication: examining the transistor MAG before we design a matching network tells us the gain we should expect to obtain after matching.
Mason's Unilateral Power Gain

1) Cancel device feedback with external lossless feedback
\[ Y_{12}^{\text{overall}} = S_{12}^{\text{overall}} = 0 \]
2) Match input and output
Resulting power gain is Mason's Unilateral Gain
\[ U = \frac{1}{4} \left( \frac{Y_{12}^{\text{FET}}}{G_{11}^{\text{FET}}} - \frac{Y_{12}^{\text{FET}}}{G_{22}^{\text{FET}}} \right)^2 \]
Note carefully the difference between \( Y_{ij}^{\text{FET}} \) and \( Y_{ij}^{\text{overall}} \).

Monolithic amplifiers are not easily made unilateral
\[ \rightarrow U \text{ mostly of historical relevance to IC design} \]

For simple BJT model, \( U \) rolls off at -20 dB/decade
\[ \rightarrow U \text{ useful for extrapolation to find } f_{\text{max}} \]

In III-V FETs, \( U \) shows peak from \( C_{ds} - R_s - R_d \) interaction
\[ \rightarrow U \text{ hard to use for } f_{\text{max}} \text{ extrapolation} \]

For bulk CMOS, \( C_{ds} \) is shielded by substrate
\[ \rightarrow U \text{ should be OK for } f_{\text{max}} \text{ extrapolation} \]
**Why Ga and Gp Matter: Matching Network Design**

Circuit simulators (ADS, etc) provide contour plots of Ga vs source impedance and Gp vs load impedance.

These, the Ga & Gp circles, show the variation in transistor gain as generator and load impedance are tuned.

The center of these circles are the (generator, load) impedances required for maximum gain.

We can then separately design Input & Output Tuning Networks to provide these impedances...

...added to device, the amplifier is realized

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**Caution:** the above assumes that MAG exists; we must examine this critical point in the next lecture.
Simple Matching Example: Unilateral Device Model

Simple FET model; \( C_{gd} = 0 \rightarrow S_{12} = 0 \)

\[ \rightarrow \text{unilateral device.} \]

\[ G_{\text{max}} = \frac{P_{\text{ava}}}{P_{\text{in}}} = \frac{P_{\text{load}}}{P_{\text{in}}} \text{ because } R_L = R_{\text{out}} \]

\[ P_{\text{in}} = I_{\text{in}}^2 R_{\text{in}} \quad \text{and } V_{gs} = \frac{I_{\text{in}}}{\omega C_{gs}} \]

\[ P_{\text{out}} = I_{\text{load}}^2 R_{\text{load}} = \left( g_m V_{gs}/2 \right)^2 R_{ds} = \frac{g_m^2 \| I_{\text{in}} \|^2}{4 \omega^2 C_{gs}^2} R_{ds} \]

\[ \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{g_m^2 R_{ds}}{4 \omega^2 C_{gs}^2 R_{\text{in}}} = \frac{f_{\text{max}}^2}{f^2} \text{ where } f_{\text{max}} = \frac{g_m}{2 \pi C_{gs}} \sqrt{\frac{R_{ds}}{4 R_{\text{in}}}} \]

If we impedance - match, then \( \| S_{21} \|^2 = \frac{f_{\text{max}}^2}{f^2} \text{ at } f = f_{\text{match}} \)

while, from inspection, \( S_{21,\text{FET}} = -\frac{2 g_m (R_{DS} \| Z_0)}{1 + j \omega (R_{\text{in}} + Z_0) C_{gs}} \)