

ECE 145A / 218 C, notes set 7: Two-Port Parameters

Mark Rodwell

University of California, Santa Barbara

rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

Device Descriptions for Circuit Design

Equivalent - circuit model

- physically based

- includes dependence upon DC bias & frequency

- often includes device size dependence

- weakness : necessary simplified, hence some errors

2 - Port Model

- matrix of tabular data

- need one model for each bias point, each frequency

 - huge data sets required.

- medium for both (a) measured data and (b) E/M simulation data

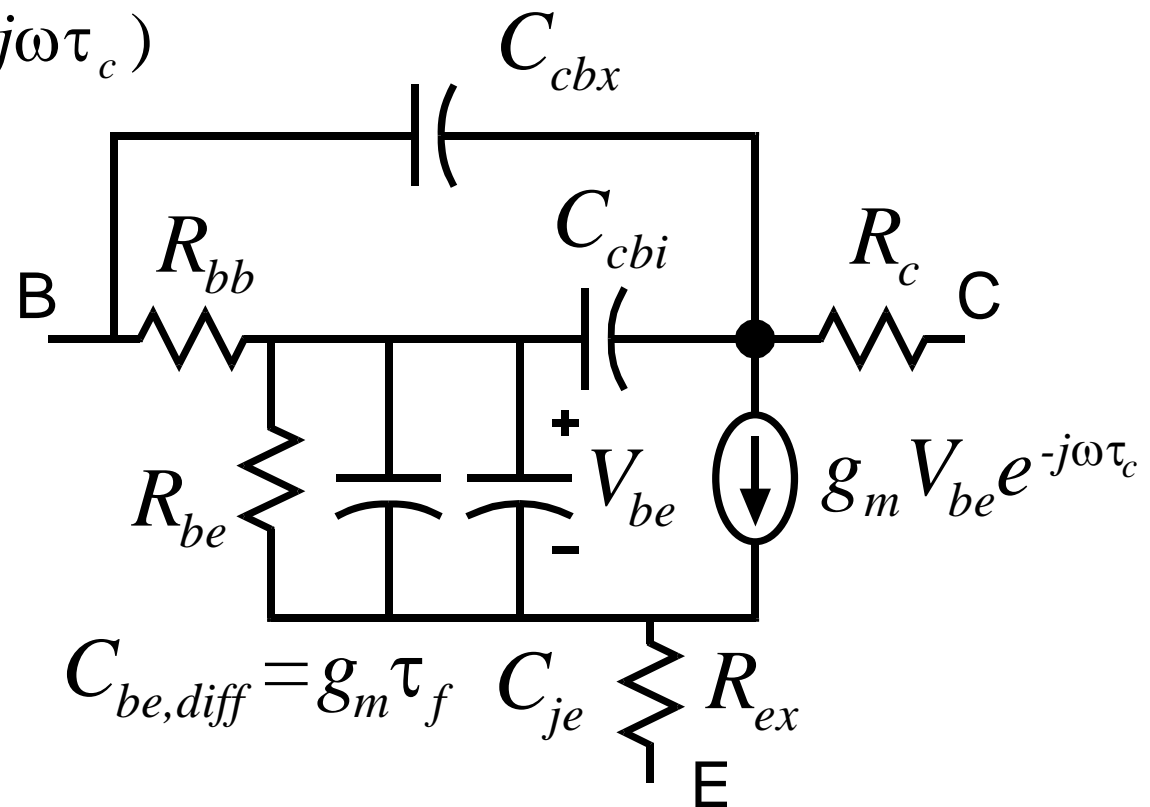
2 - port methods also useful for general network theory.

Recall: HBT equivalent-circuit model

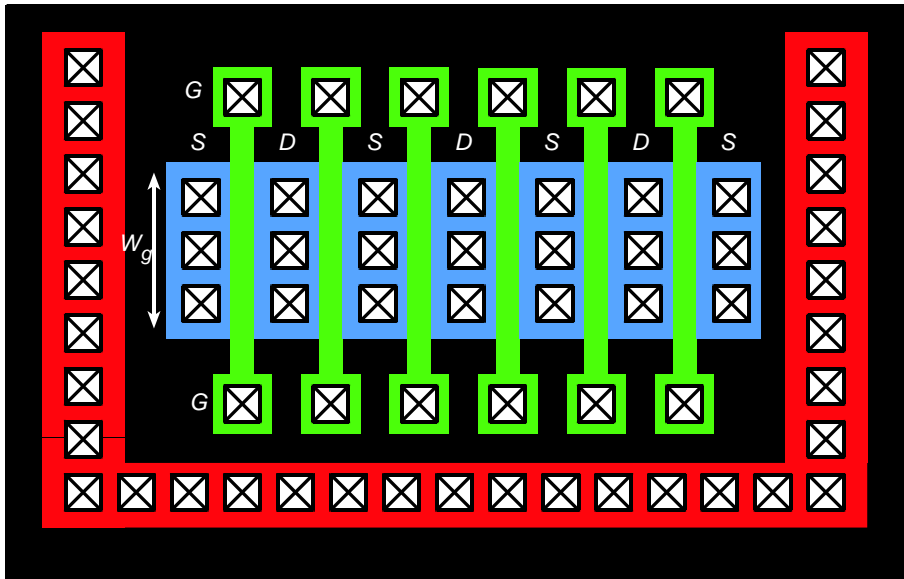
$$g_m = g_{m0} \exp(-j\omega\tau_c)$$

$$R_{be} = \beta / g_m$$

$$\tau_f = \tau_b + \tau_c$$



Recall: FET Equivalent Circuit Model



$$g_m \sim \frac{\epsilon}{T_{eq}} v_{eff} (NW_g) \text{ or } \frac{\epsilon}{T_{eq}} \mu (NW_g) (V_{gs} - V_{th})$$

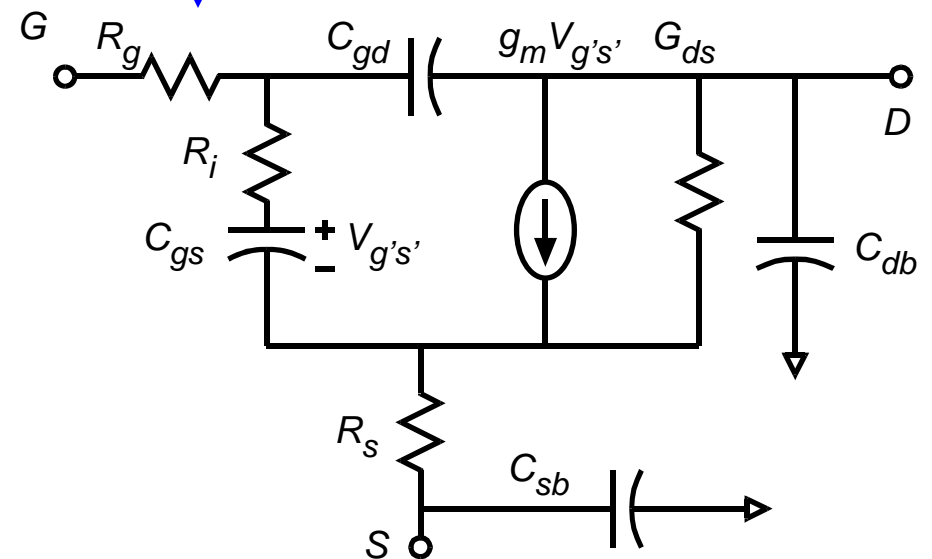
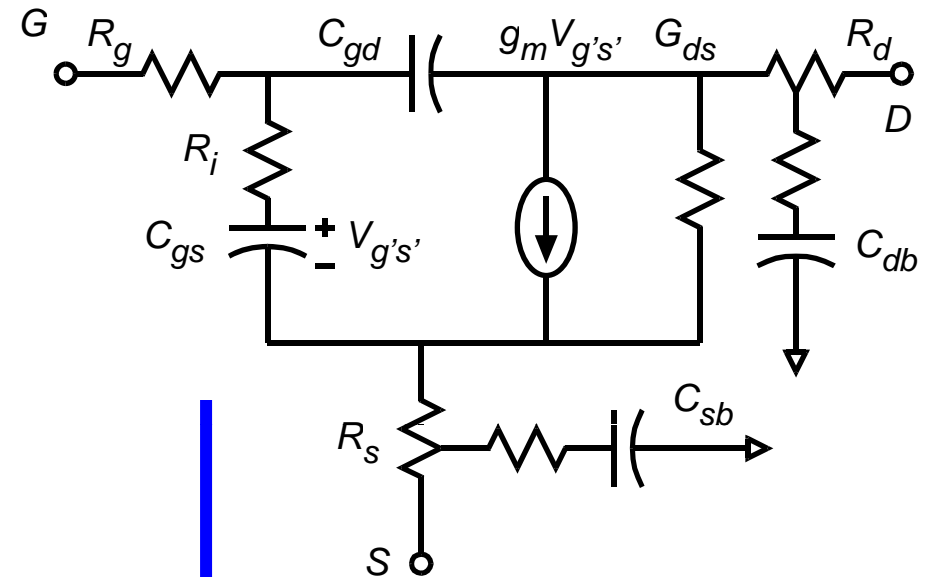
$$C_{gs} \sim \frac{\epsilon}{T_{eq}} L_g (NW_g) \quad R_g \sim \frac{\rho_s}{12L_g} \left(\frac{W_g}{N} \right) + \frac{R_{end}}{2N}$$

$$C_{gd} \propto NW_g \quad R_d \propto 1/NW_g$$

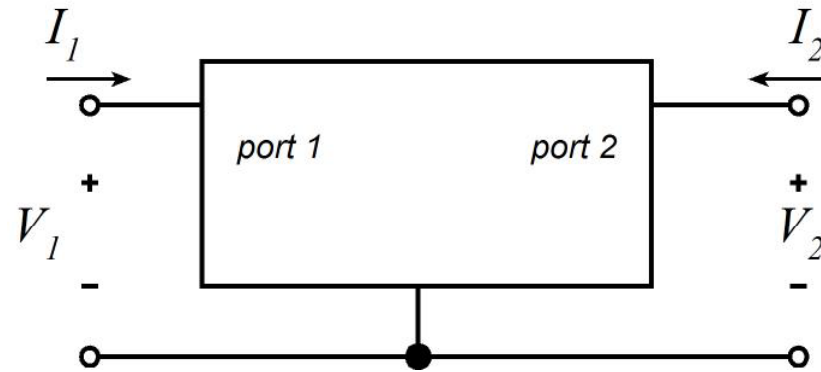
$$G_{ds} \propto NW_g \quad R_s \propto 1/NW_g$$

$$R_i \sim 1/g_m \quad C_{sb} \propto NW_g$$

$$C_{db} \propto NW_g$$



2-Port Descriptions (3-Wire Network or Device)



Box might contain : a transistor, a passive element, a subcircuit

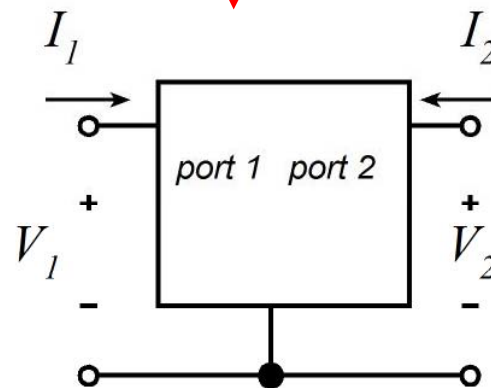
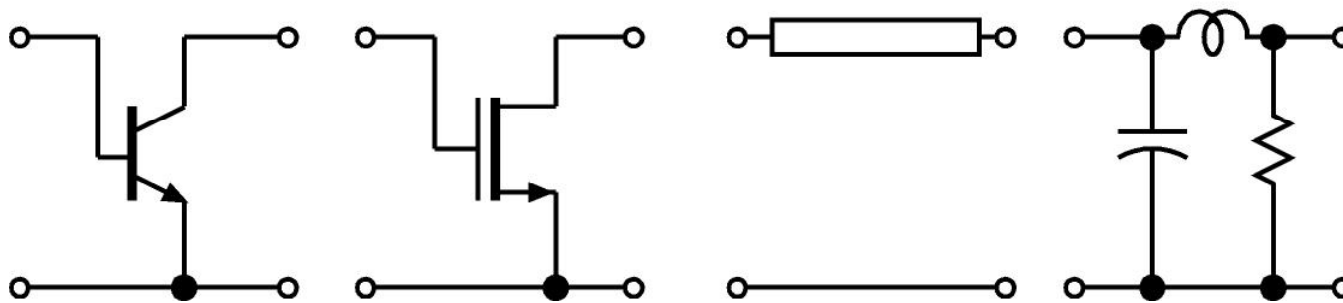
The terminal characteristics relate the variables V_1 , V_2 , I_1 , and I_2 .

There are 2 degrees of freedom.

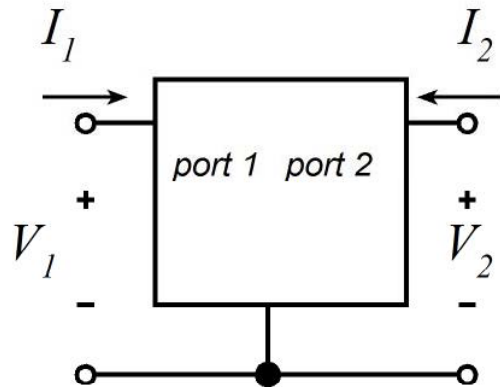
Any two variables can be set as the *independent variables*.

The remaining two variables, the *dependent variables*,
can then be written as functions of the independent variables.

Two-Port Parameters: Represent Device or Network



Admittance Parameters



Frequency - domain description :

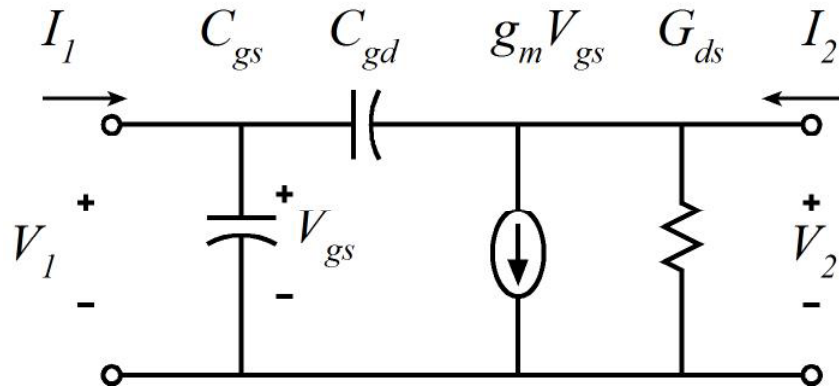
$$v_1(t) = \text{Re}\{V_1 e^{j\omega t}\} \quad , \quad i_1(t) = \text{Re}\{I_1 e^{j\omega t}\} \quad , \text{ etc.}$$

$$\begin{bmatrix} I_1(j\omega) \\ I_2(j\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(j\omega) & Y_{12}(j\omega) \\ Y_{21}(j\omega) & Y_{22}(j\omega) \end{bmatrix} \begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \end{bmatrix}$$

Currents are written as functions of voltages.

DC bias is taken as implicit.

Admittance Parameters Example: Simple FET Model

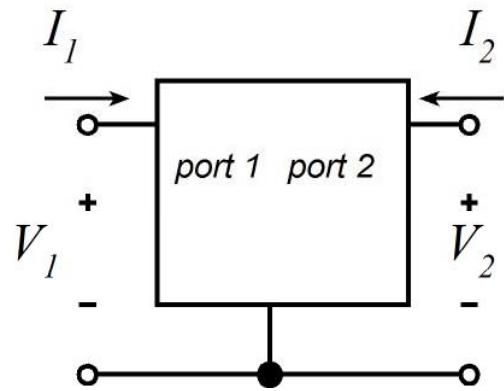


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By inspection :

$$\mathbf{Y} = [Y_{ij}] = \begin{bmatrix} j\omega(C_{gs} + C_{gd}) & -j\omega C_{gd} \\ g_m - j\omega C_{gd} & G_{ds} + j\omega C_{gd} \end{bmatrix}$$

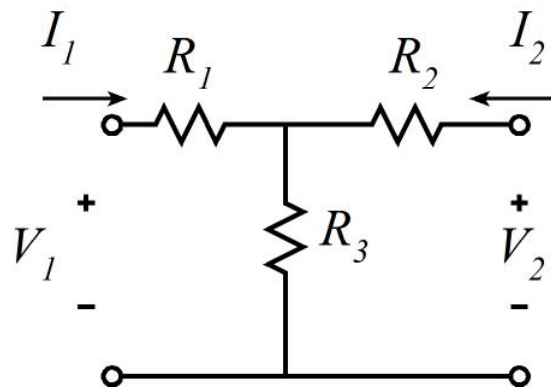
Impedance Parameters



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Voltages are written as functions of currents.

Example :

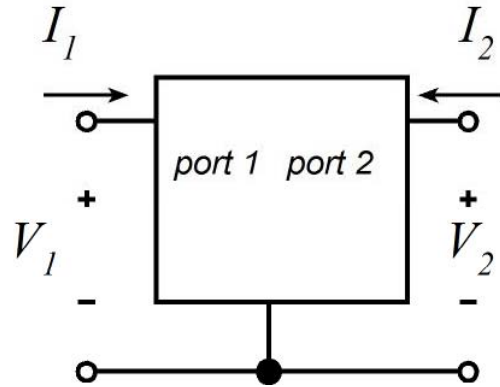


By inspection :

$$\mathbf{Z} = [Z_{ij}] = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

...easy !

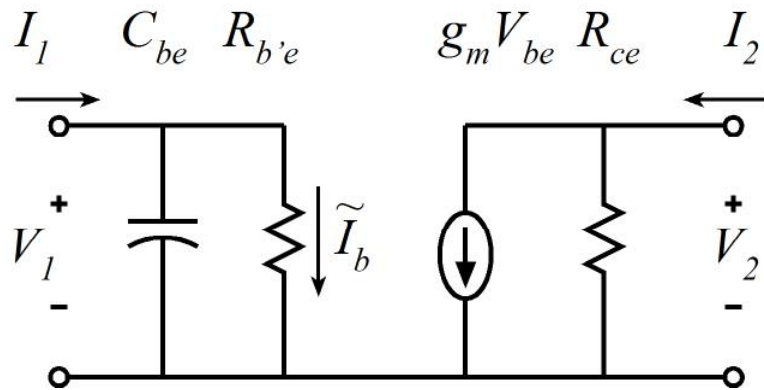
Hybrid Parameters: Old and Obscure



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

This is certainly an odd choice of independent variables.

Hybrid Parameter Example: Simple BJT Model



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\text{note : } g_m V_{be} = \beta V_{be} / R_{be} = \beta \tilde{I}_b$$

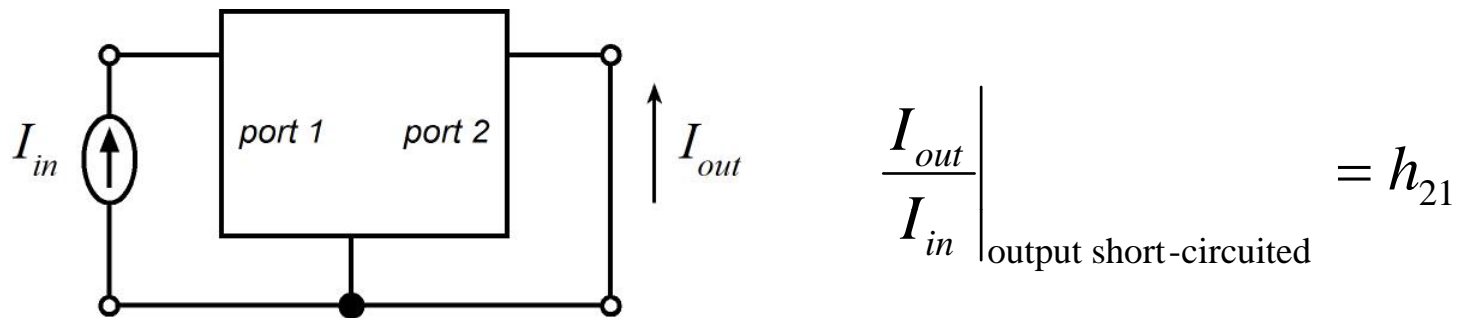
By inspection :

$$\mathbf{H} = \begin{bmatrix} \frac{1}{1/R_{be} + j\omega C_{be}} & 0 \\ \frac{1}{1 + j\omega R_{be} C_{be}} & \frac{1}{R_{ce}} \end{bmatrix}$$

This is related to short - circuit current gain.

Short-Circuit Current Gain

short - circuit current gain $\equiv \frac{I_2}{I_1} \Big|_{V_2=0} = \frac{I_2}{I_1} \Big|_{\text{output short-circuited.}}$

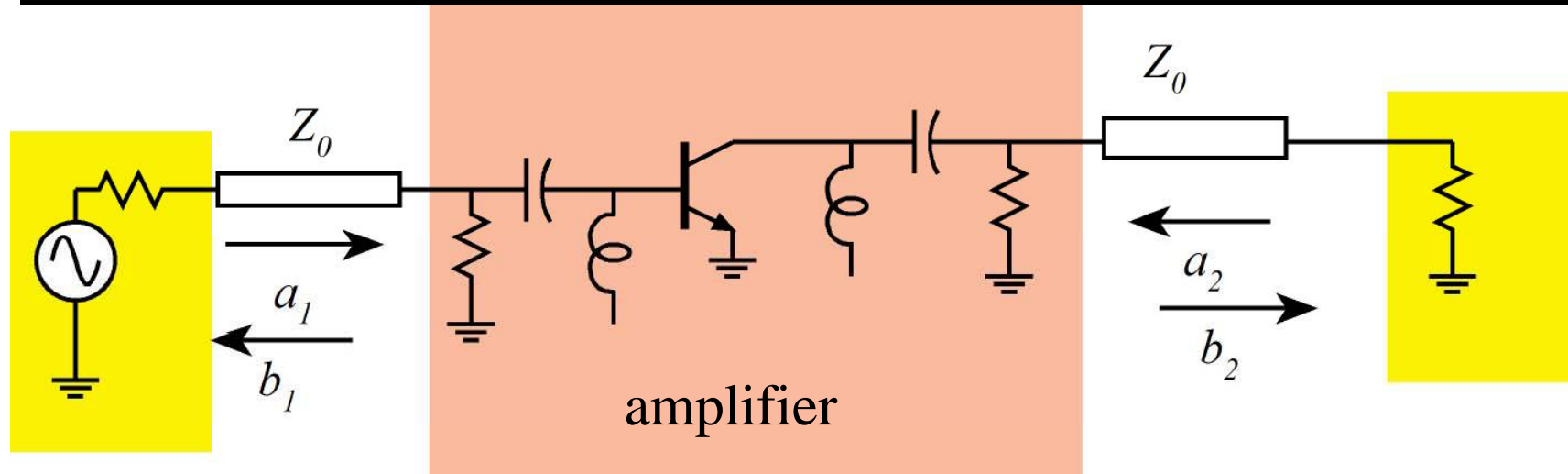


f_τ = frequency at which $\|h_{21}\|$ * extrapolates * to 1.
 = "short - circuit current - gain cutoff frequency".

For the highly simplified model on the prior page, if $\beta \gg 1$,

$$f_\tau \cong g_m / 2\pi C_{be}.$$

Definition of S-parameters... with a's and b's



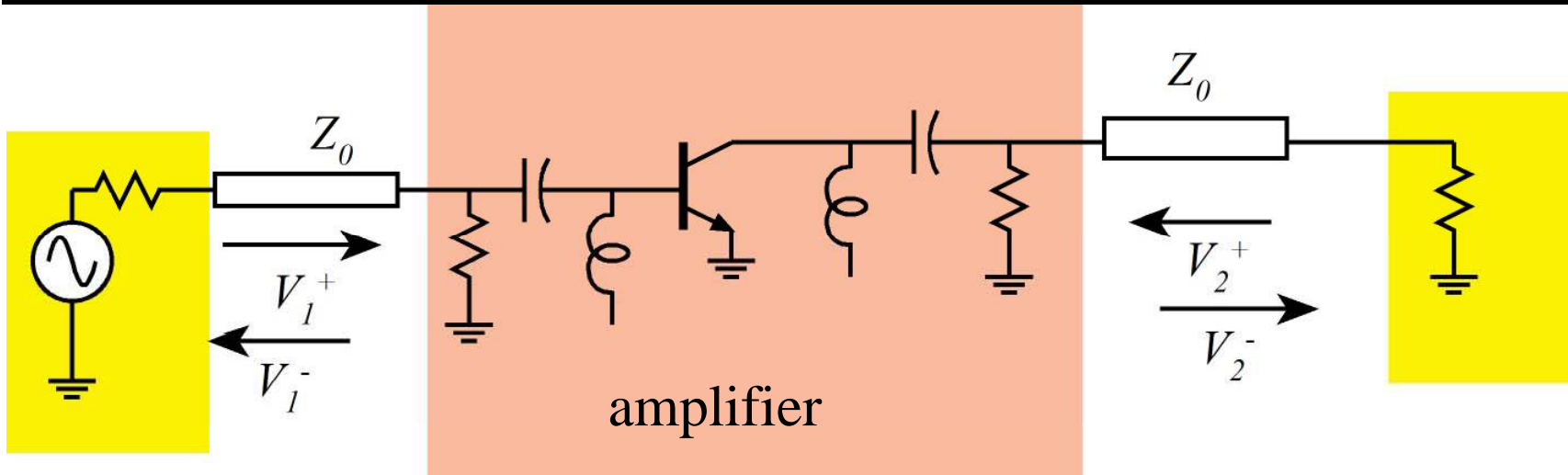
S - parameters are rigorously defined in terms of the wave amplitudes on transmission lines connected to the device under test :

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

...where the a's and b's are the wave amplitudes.

$$a = V^+ / \sqrt{Z_0} \text{ and } b = V^- / \sqrt{Z_0}$$

Definition of S-parameters... with V_+ 's and V_- 's

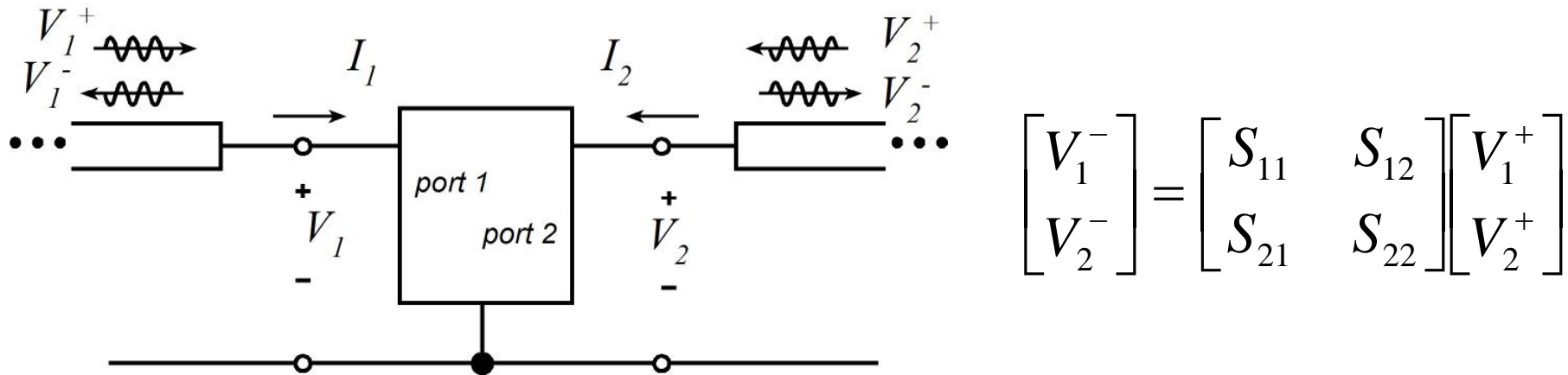


We can also write

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

"+" waves travel towards the 2 - port; "-" waves travel away.

De-Mystifying S-Parameters



At port 1, $V_1 = V_1^+ + V_1^-$ and $I_1 = (V_1^+ - V_1^-)/Z_0$

At port 2, $V_2 = V_2^+ + V_2^-$ and $I_2 = (V_2^+ - V_2^-)/Z_0$

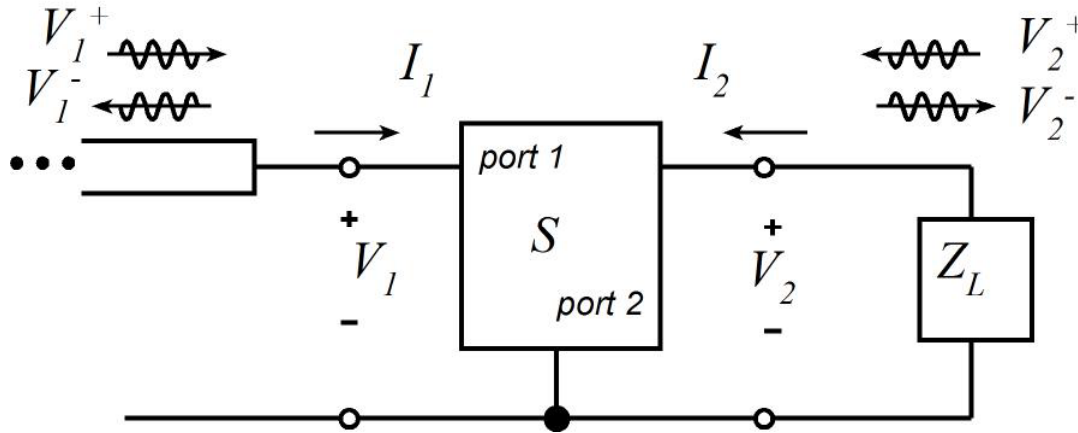
If we know the relationship between $[I_1, I_2]$ and $[V_1, V_2]$:

(**Y** or **Z** parameters),

we can calculate the relationship between $[V_1^+, V_2^+]$ and $[V_1^-, V_2^-]$:

(**S** parameters).

How to Compute S-parameters quickly: S_{11}



If $Z_L = Z_0$ then $\Gamma_L = 0$, hence $V_2^+ = 0$

$$\text{Now } S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0}, \text{ so } S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{Z_L=Z_0}$$

Defining $Z_{in}|_{Z_L=Z_0}$ as the input impedance given $Z_L = Z_0$,

$$S_{11} = \frac{Z_{in}|_{Z_L=Z_0} - Z_0}{Z_{in}|_{Z_L=Z_0} + Z_0}$$

Input impedance = Input Reflection Coefficient

Noting that
$$S_{11} = \frac{Z_{in}|_{Z_L=Z_0} - Z_0}{Z_{in}|_{Z_L=Z_0} + Z_0}$$

Note that reflection coefficient (S_{11}) is a method of specifying input impedance.

$\left\{ \begin{array}{c} S_{11} \\ Z_{in}|_{Z_L=Z_0} \end{array} \right\}$ is the input $\left\{ \begin{array}{c} \text{reflection coefficient} \\ \text{impedance} \end{array} \right\}$

given that the load $\left\{ \begin{array}{c} \text{reflection coefficient} \\ \text{impedance} \end{array} \right\}$ is $\left\{ \begin{array}{c} 0 \\ Z_0 \end{array} \right\}$.

Output impedance = Output Reflection Coefficient

The same analysis & comments clearly applies to S_{22} .

$$\text{By symmetry, } S_{22} = \frac{Z_{out}|_{Z_{gen}=Z_0} - Z_0}{Z_{out}|_{Z_{gen}=Z_0} + Z_0}$$

Note that reflection coefficient (S_{22}) is a method of specifying output impedance.

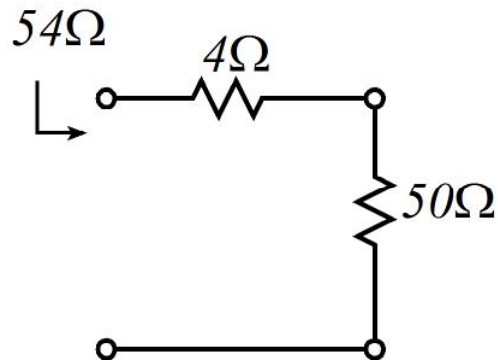
$\left\{ \begin{array}{c} S_{22} \\ Z_{out}|_{Z_{gen}=Z_0} \end{array} \right\}$ is the output $\left\{ \begin{array}{c} \text{reflection coefficient} \\ \text{impedance} \end{array} \right\}$

given that the generator $\left\{ \begin{array}{c} \text{reflection coefficient} \\ \text{impedance} \end{array} \right\}$ is $\left\{ \begin{array}{c} 0 \\ Z_0 \end{array} \right\}$.

Computing S_{11} : Example



Given $Z_0 = 50\Omega$, what is S_{11} ?



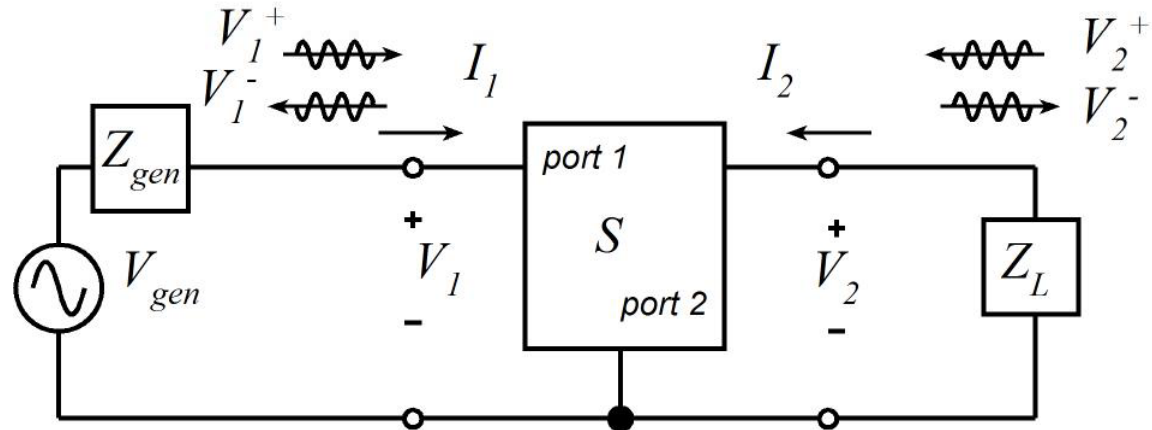
$$Z_{in}|_{Z_L=Z_0} = 54\Omega$$

$$\text{So } S_{11} = \frac{Z_{in}|_{Z_L=Z_0} - Z_0}{Z_{in}|_{Z_L=Z_0} + Z_0} = \frac{54\Omega - 50\Omega}{54\Omega + 50\Omega} = \frac{4}{104}$$

by similar arguments, $S_{22} = 4/104$

Computing S_{21}

$$\text{Set : } Z_{gen} = Z_L = Z_0$$



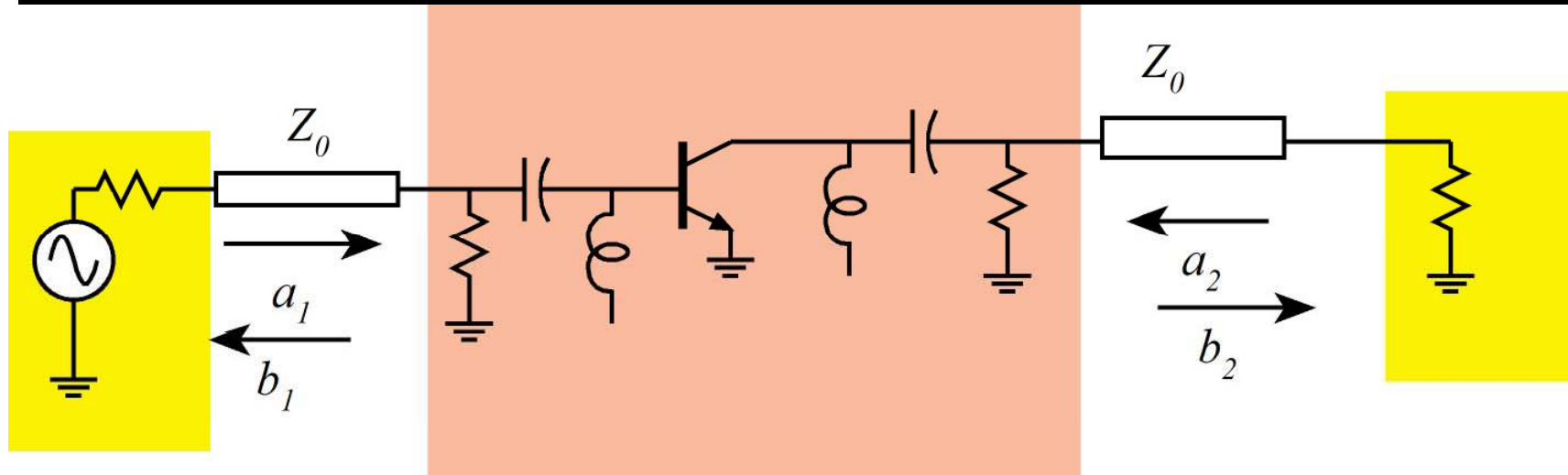
Given $Z_{gen} = Z_0$ we have $\Gamma_s = 0$ and $T_s = Z_0 / (Z_0 + Z_{gen}) = 1/2$,

hence $V_1^+ = T_s V_{gen} + \Gamma_s V_1^- = V_{gen} / 2$.

Given $Z_L = Z_0$ we have $\Gamma_L = 0$, hence $V_{out} = V_2 = V_2^+ + V_2^- = V_2^-$

$$\text{So } S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = \left. \frac{V_2^-}{V_1^+} \right|_{Z_L=Z_0} = \left. \frac{V_2}{V_1^+} \right|_{Z_L=Z_0} = \left. \frac{V_{out}}{(V_{gen} / 2)} \right|_{Z_L=Z_0=Z_{gen}} = \left. \frac{2V_{out}}{V_{gen}} \right|_{Z_L=Z_0=Z_{gen}}$$

Relating amplifier Gains to S-parameters...S21



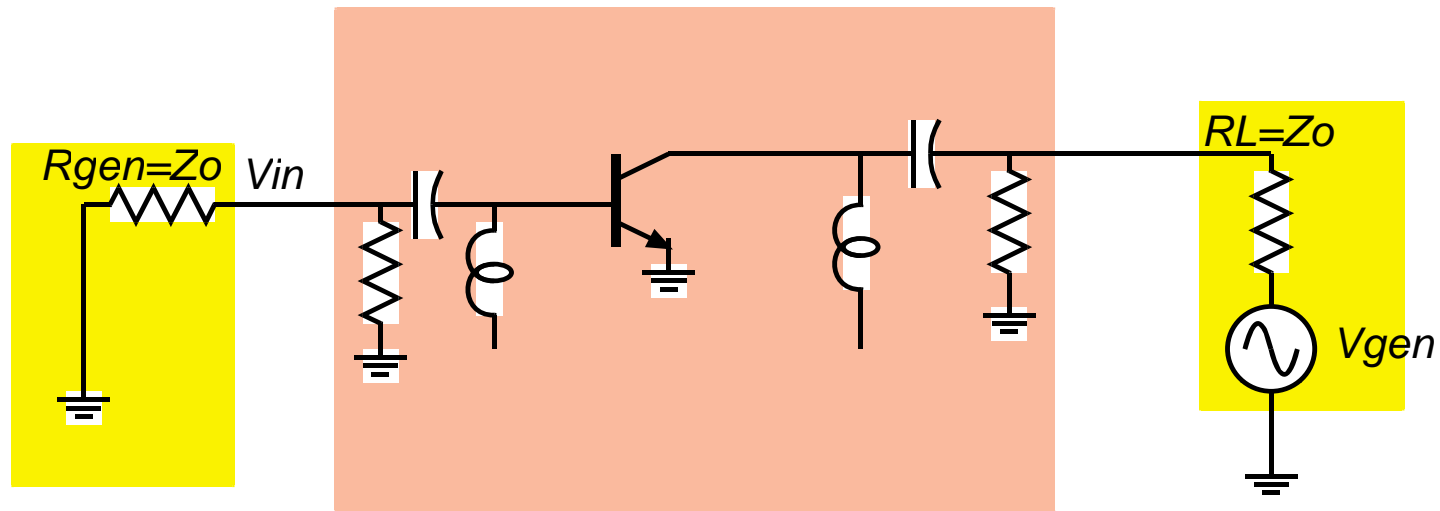
These relationships allow us to develop a simpler way of finding the S - parameters :

$$S_{21} = 2 \frac{v_{out}}{v_{gen}} \Big|_{Z_{generator} = Z_{load} = Z_0}$$

..which is simply **how much bigger** the signal became upon **insertion** of the amplifier in the 50 Ohm system.

S_{21} is called the **insertion gain**.

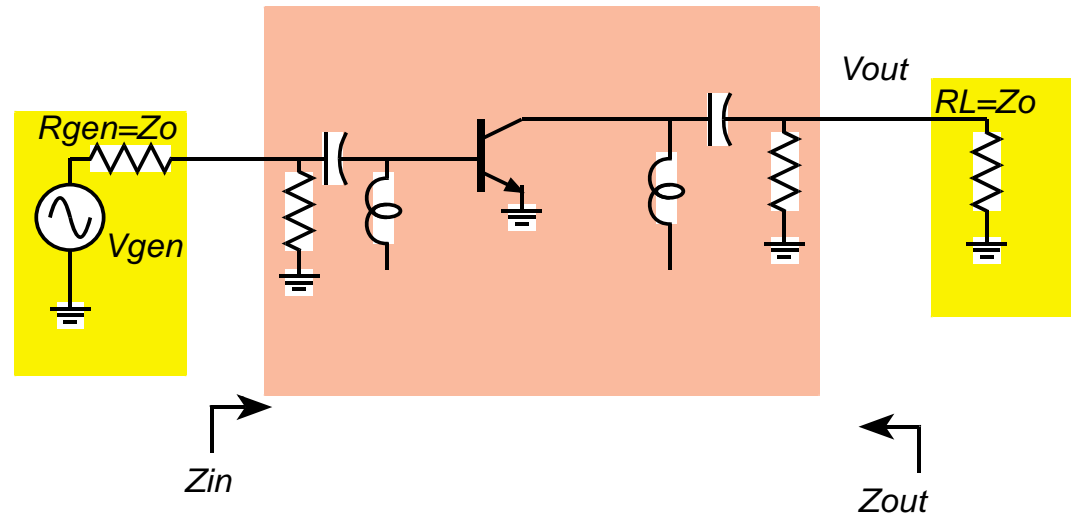
Relating amplifier Gains to S-parameters...S12



By symmetry

$$S_{12} = 2 \frac{v_{in}}{v_{gen}} \Big|_{Z_{generator} = Z_{load} = Z_0}$$

Relating amplifier Gains to S-parameters...S11 and S22



S11 and S22 can be directly related to input and output impedances

$$S_{11} = \frac{(Z_{in} / Z_0) - 1}{(Z_{in} / Z_0) + 1}, \text{ where } Z_{in} = Z_{in} \Big|_{Z_{load} = Z_0}$$

$$S_{22} = \frac{(Z_{out} / Z_0) - 1}{(Z_{out} / Z_0) + 1}, \text{ where } Z_{out} = Z_{out} \Big|_{Z_{generator} = Z_0}$$

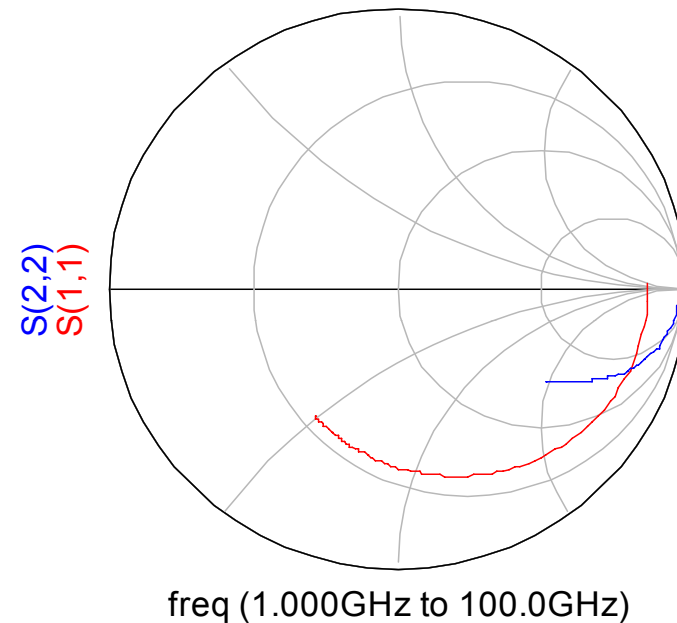
...in practice, we do not need to plug into the formulas : knowing the Z_{in} tells us the S_{11} , because the formula is one - to - one and is neatly represented by the Smith chart

Relating amplifier Gains to S-parameters...S11 and S22

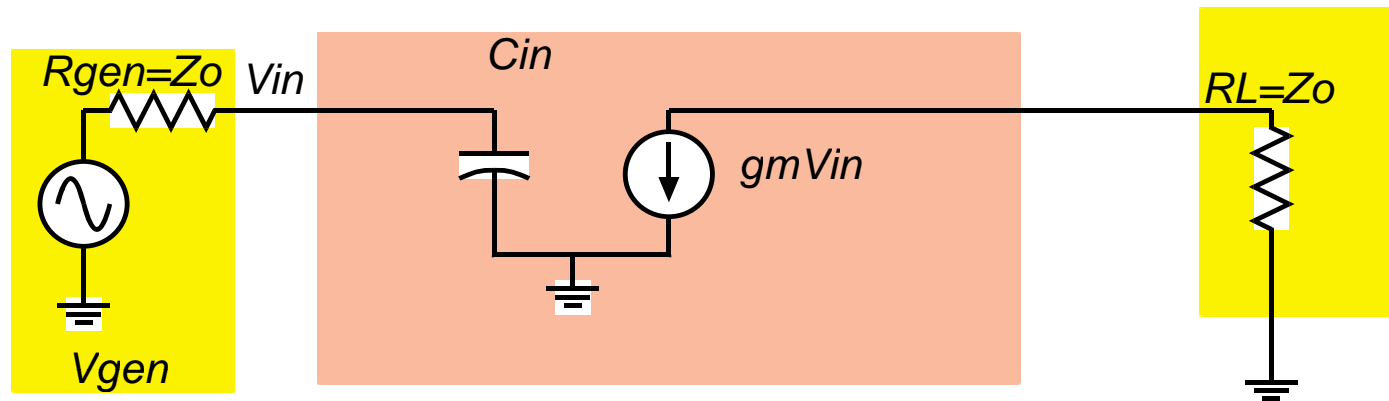
$$S_{11} = \frac{(Z_{in} / Z_0) - 1}{(Z_{in} / Z_0) + 1}, \text{ where } Z_{in} = Z_{in} \Big|_{Z_{load} = Z_0}$$

$$S_{22} = \frac{(Z_{out} / Z_0) - 1}{(Z_{out} / Z_0) + 1}, \text{ where } Z_{out} = Z_{out} \Big|_{Z_{generator} = Z_0}$$

we do not need to plug into the formulas : the Smith chart is a plot of this formula, so the Smith chart plots S_{ij} and Z_{in} at the same time



Example of working with S-parameters



$$S_{21} = \frac{-g_m Z_o}{1 + j\omega C_{in} Z_o}$$

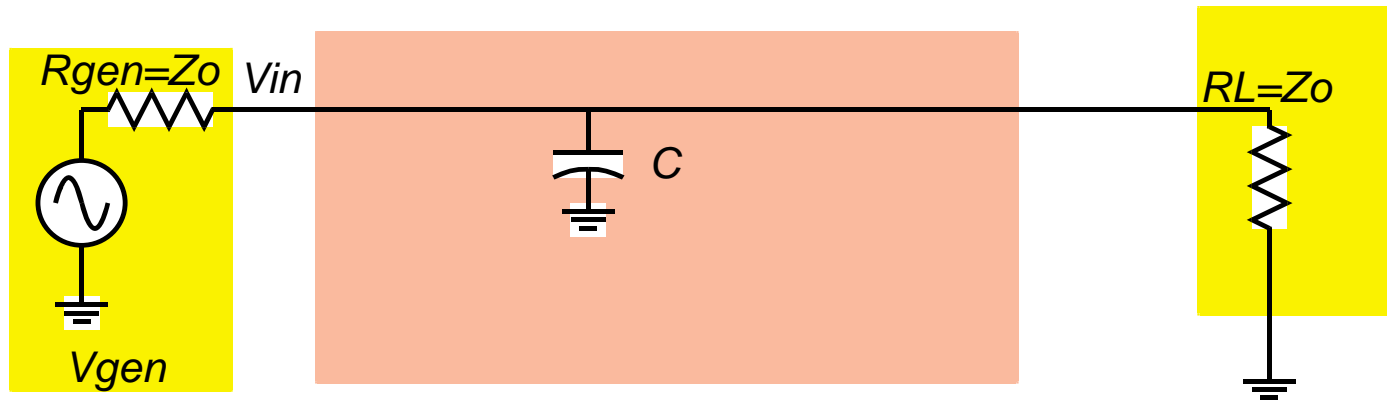
$$S_{11} = \frac{(Z_{in} / Z_0) - 1}{(Z_{in} / Z_0) + 1}, \text{ where } Z_{in} = 1 / j\omega C_{in}$$

$$S_{22} = \frac{(Z_{out} / Z_0) - 1}{(Z_{out} / Z_0) + 1}, \text{ where } Z_{out} = \text{infinity}$$

$$S_{12} = 0$$

...easy !!!

Example of working with S-parameters



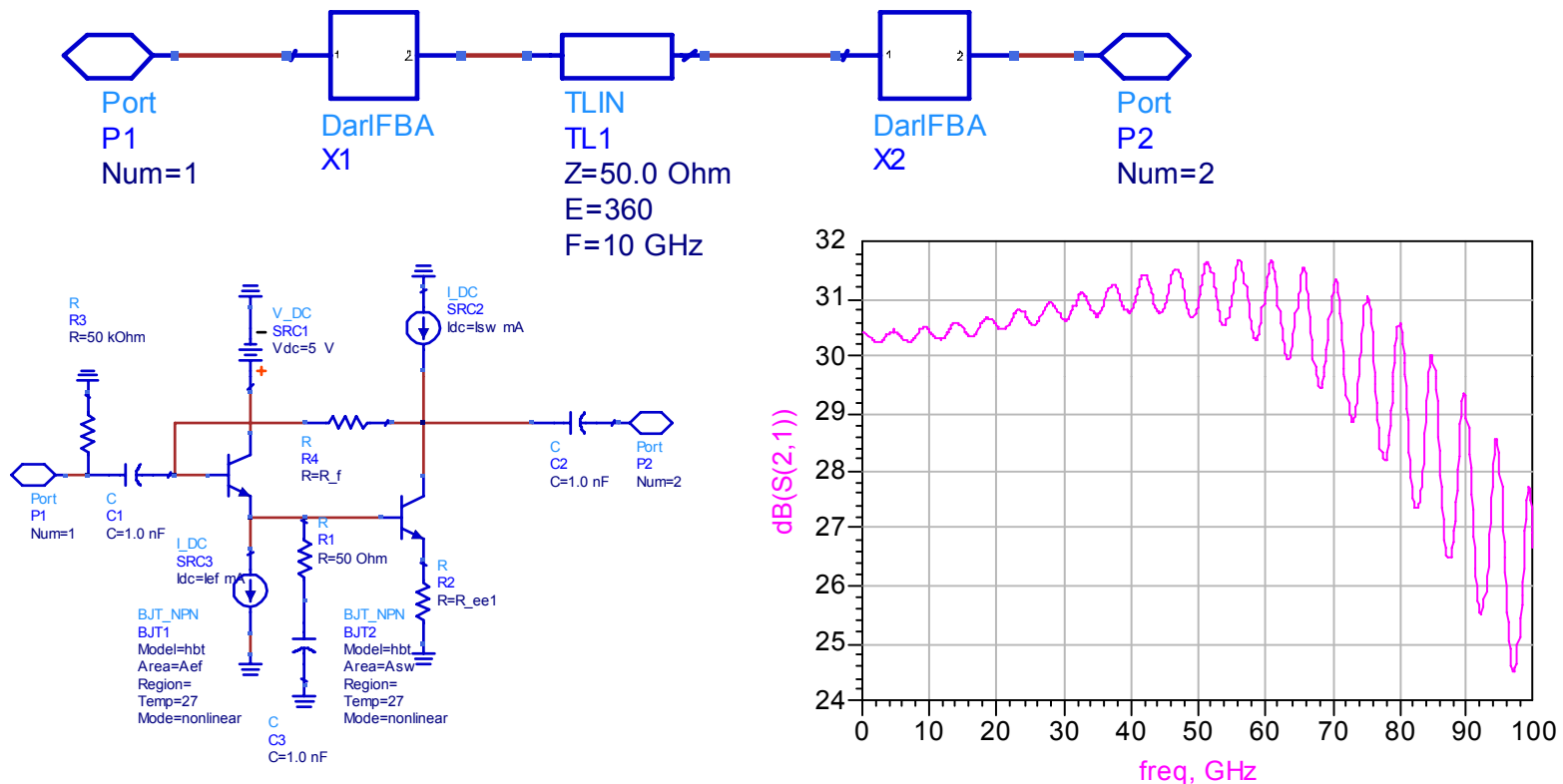
$$S_{21} = \frac{\left(Z_0 \parallel \frac{1}{j\omega C} \right)}{Z_0 + \left(Z_0 \parallel \frac{1}{j\omega C} \right)} = S_{12} = \frac{1}{1 + j\omega C Z_0 / 2}$$

$$S_{11} = \frac{(Z_{in} / Z_0) - 1}{(Z_{in} / Z_0) + 1} = \frac{j\omega C Z_0 / 2}{1 + j\omega C Z_0 / 2}, \text{ where } Z_{in} = \left(Z_0 \parallel \frac{1}{j\omega C} \right)$$

$$S_{22} = \frac{(Z_{out} / Z_0) - 1}{(Z_{out} / Z_0) + 1}, \text{ where } Z_{out} = \left(Z_0 \parallel \frac{1}{j\omega C} \right)$$

...this illustrates the importance of " $Z_{in} |_{Z_L=Z_0}$ ", etc

Why do we care about impedances matched to 50 Ohms?



Standing waves on transmission lines cause gain/phase ripples of the form $(1 - \Gamma_S \Gamma_L \exp(-j2f\tau))^{-1}$. Either we must have short transmission lines, or the lines must be well - terminated