ECE145a / 218a
Bilateral Tuned Amplifier Design: Stability

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Stability

Instability: non-zero output with zero input.

Stability theory: many equivalent versions:

Classical control system theory

Bode & Nyquist methods: find phase margin.
Find real part of closed-loop poles.
Do any lie in right half of s-plane?

Network theory:

Analyze circuit by nodal analysis.
Find poles in $V_{out}(s)/V_{gen}(s)$.
Do any lie in right half of s-plane?

Impedance viewpoint

Does $Z_{in}(j\omega)$ have a negative real part?

Reflection (S) viewpoint

Is the magnitude of $\Gamma_{in}$ greater than 1?
Stability: LaPlace Transform / Eigenvalue Method

Physical system (circuits, etc.) in small - signal limit → transfer function.

\[
\frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} \text{ or } \frac{b(s)}{a(s)} \text{ or } \frac{V_{\text{in}}(s)}{I_{\text{in}}(s)} \text{ etc } = H(s)
\]

\[
H(s) = c_1 \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots} = c_2 \frac{(s - s_{z1})(s - s_{z2})(s - s_{z3})\ldots}{(s - s_{p1})(s - s_{p2})(s - s_{p3})\ldots}
\]

Impulse response:

\[
h(t) = k_1 \exp(s_{p1} t) + k_2 \exp(s_{p2} t) + k_3 \exp(s_{p3} t) + \ldots
\]

Poles are (generally) complex:

\[
s_{pi} = \sigma_{pi} + j\omega_{pi}
\]

If any poles lie in right half of s - plane (\(\sigma_{pi}\) positive)

then \(k_i \exp(s_{pi} t)\) will grow without limit.

→ unstable system.
Unstable system if any pole has positive real part

\[ s_{pi} = \sigma_{pi} + j\omega_{pi}; \text{ negative } \sigma_{pi} \]
\[ \rightarrow \exp(s_{pi}t) \text{ decays.} \]
\[ \rightarrow \text{stable system.} \]

\[ s_{pi} = \sigma_{pi} + j\omega_{pi}; \text{ positive } \sigma_{pi} \]
\[ \rightarrow \exp(s_{pi}t) \text{ grows.} \]
\[ \rightarrow \text{unstable system.} \]
Impedances:

\[ I(t) = I_0 e^{st} \]

\[ V(t) = V_0 e^{st} \]

\[ Z = \frac{1}{sC} \]

\[ Z = sL \]

\[ Z = R \]

This assumes zero initial conditions.
Consider a 1-port device having a negative real part to $Z_{in}(j\omega)$.

We connect an external load to consider stability of the combined system.
Network Theory: One-Port Potential Instability.

Nodal analysis: \[ \frac{V_{in}}{R_s + 1/ sC} + \frac{V_{in}}{R_{in} + sL} = 0 \]

Hence \( V_{in} = 0 \) (stable) or \[ \frac{1}{R_s + 1/ s_p C} + \frac{1}{R_{in} + s_p L} = 0 \]

\[ R_s + 1/ s_p C + R_{in} + s_p L = 0 \quad \rightarrow \quad s_p^2 LC + s(R_{in} + R_s)C + 1 = 0 \]

\[ s_{p1,2} = -\left(\frac{R_{in} + R_s}{2L}\right) \pm \sqrt{\left(\frac{R_{in} + R_s}{2L}\right)^2 - \frac{1}{LC}} \]

\( (R_{in} + R_s) \) positive \( \rightarrow \) \( \text{Re}\{s_{p1,2}\} < 0 \rightarrow \text{stable} \)

\( (R_{in} + R_s) \) negative \( \rightarrow \) \( \text{Re}\{s_{p1,2}\} > 0 \rightarrow \text{unstable} \)
Ideas: Unconditional stability, Potential instability

\[(R_{in} + R_s) \text{ positive} \rightarrow \text{Re}\{s_{p1,2}\} < 0 \rightarrow \text{stable}\]
\[(R_{in} + R_s) \text{ negative} \rightarrow \text{Re}\{s_{p1,2}\} > 0 \rightarrow \text{unstable}\]

If \( R_{in} \) is positive, no (positive) value of \( R_s \) produces an unstable system
\[\rightarrow \text{ device is unconditionally stable}.\]

If \( R_{in} \) is negative, some (positive) values of \( R_s \) produces an unstable system
\[\rightarrow \text{ device is potentially stable} \]
Unconditional stability, Potential instability

From impedance viewpoint:

A one-port is unconditionally stable if:
\[
\text{Re}\{Z_{in}(j\omega)\} > 0 \quad \text{for all frequencies.}
\]

Alternatively, a one-port is unconditionally stable if:
\[
\text{Re}\{Y_{in}(j\omega)\} > 0 \quad \text{for all frequencies.}
\]

If \[
\text{Re}\{Z_{in}(j\omega)\} < 0 \quad \text{for some frequencies,}
\]
then the device is potentially unstable.
Potential instability: Reflection Viewpoint

From reflection viewpoint:

\[ \Gamma_{in}(j\omega) = \frac{Z_{in}(j\omega) - Z_0}{Z_{in}(j\omega) + Z_0} \]

A one-port is unconditionally stable if:

\[ \|\Gamma_{in}(j\omega)\| < 1 \] for all frequencies.

If \( \|\Gamma_{in}(j\omega)\| > 1 \) for some frequencies, then the device is potentially unstable.
Stability of a Two-Port: Look at $\Gamma_{\text{in}}$

There are two degrees of freedom: $\Gamma_S$ and $\Gamma_L$.

Think of it as a 1-port:

$$\Gamma_{\text{in}} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$

The device is unconditionally stable if $\|\Gamma_{\text{in}}(j\omega)\| < 1$ for all load reflection coefficients $\Gamma_L$ such that $\|\Gamma_L\| \leq 1$

Why do we not consider $\|\Gamma_L\| > 1$?
Stability of a Two-Port: Look at $\Gamma_{\text{out}}$ Instead.

There are two degrees of freedom: $\Gamma_S$ and $\Gamma_L$.

Think of it as a 1-port:

$$\Gamma_{\text{out}} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

The device is unconditionally stable if $\left\|\Gamma_{\text{out}}(j\omega)\right\| < 1$ for all load reflection coefficients $\Gamma_S$ such that $\left\|\Gamma_S\right\| \leq 1$

Why do we not consider $\left\|\Gamma_S\right\| > 1$?
**Input Stability Circle**

\[ \Gamma_{out} = S_{22} + \Gamma_s \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_s} \]

values of \( \Gamma_s \) which give \( ||\Gamma_{out}|| = 1 \)

\( \Gamma_s = 0 \) so \( \Gamma_{out} = S_{22} \)
Is the inside or the outside of the circle stable?

\[
\Gamma_{out} = S_{22} + \Gamma_s \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_s}
\]

Values of \( \Gamma_s \) which give \( \|\Gamma_{out}\| = 1 \)

\( \Gamma_s \)-plane

\( \Gamma_s = 0 \) so \( \Gamma_{out} = S_{22} \)

If \( \|S_{22}\| < 1 \), then the center of the Smith chart is stable.

If \( \|S_{22}\| > 1 \), then the center of the Smith chart is unstable.
\[ \Gamma_{\text{in}} = S_{11} + \Gamma_L \frac{S_{21} S_{12}}{1 - S_{22} \Gamma_L} \]

\[ \Gamma_L - \text{plane} \]

\[ \Gamma_L = 0 \text{ so } \Gamma_{\text{in}} = S_{11} \]

values of \( \Gamma_L \) which give \( \| \Gamma_{\text{in}} \| = 1 \)
Is the inside or the outside of the circle stable?

\[ \Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \]

\[ \Gamma_L = 0 \text{ so } \Gamma_{in} = S_{11} \]

Values of \( \Gamma_L \) which give \( \|\Gamma_{out}\| = 1 \)

If \( \|S_{11}\| < 1 \), then the center of the Smith chart is stable.

If \( \|S_{11}\| > 1 \), then the center of the Smith chart is unstable.
Potentially Unstable Amplifier

Source stability circle

\[ \| \Gamma_{out} \| = 1 \]
\[ \| S_{22} \| < 1 \]

Load stability circle

\[ \| \Gamma_{in} \| = 1 \]
\[ \| S_{11} \| < 1 \]

This is a test at one specific frequency; must test at all frequencies.
Unconditionally stable Amplifier

Source stability circle

\[ \|\Gamma_{out}\| = 1 \]

\[ \|S_{22}\| < 1 \]

Load stability circle

\[ \|\Gamma_{in}\| = 1 \]

\[ \|S_{11}\| < 1 \]

This is a test at one specific frequency; must test at all frequencies.
Is this possible ????

Source stability circle

\[ \|\Gamma_{out}\| = 1 \]

\[ \|S_{22}\| < 1 \]

Load stability circle

\[ \|S_{11}\| < 1 \]

\[ \|\Gamma_{in}\| = 1 \]

Stop and think clearly...
We need check only one stability circle

If there is no $\Gamma_L$ for which $\|\Gamma_{in}\| > 1$,
then no combination of $\Gamma_S$ and $\Gamma_L$ can cause oscillation.

If there is no $\Gamma_S$ for which $\|\Gamma_{out}\| > 1$,
then no combination of $\Gamma_S$ and $\Gamma_L$ can cause oscillation.

If one stablity circle passes the stablity test, then so must the other.
"Safe" and "Unsafe" Impedances

Source stability circle

\[ \|\Gamma_{out}\| = 1 \]

\[ \|S_{22}\| < 1 \]

\( \Gamma_S \) outside the shaded region gives \( \|\Gamma_{out}\| < 1 \), cannot oscillate with any \( \Gamma_L \). 

\( \Gamma_S \) inside in the shaded region gives \( \|\Gamma_{out}\| > 1 \), might oscillate given with wrong \( \Gamma_L \).
"Safe" and "Unsafe" Impedances

\[ \Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \]

\[ \| S_{11} \| < 1 \]

\[ \| \Gamma_{in} \| = 1 \]

\[ \Gamma_L \text{ outside the shaded region gives } \| \Gamma_{in} \| < 1, \text{ cannot oscillate with any } \Gamma_S. \]

\[ \Gamma_L \text{ inside in the shaded region gives } \| \Gamma_{in} \| > 1, \text{ might oscillate given with wrong } \Gamma_S. \]
Stable Interfaces to a Potentially Unstable Device

Source stability circle

Load stability circle

If either $\Gamma_S$ or $\Gamma_L$ lies outside the danger zones, the circuit will be stable.
Stability Factors

\[ \Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \quad \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S} \]

If there is no \( \Gamma_L \) for which \( \|\Gamma_{in}\| > 1 \), the network is unconditionally stable.
If there is no \( \Gamma_S \) for which \( \|\Gamma_{out}\| > 1 \), the network is unconditionally stable.

A 2-port is unconditionally stable if the Rollet stability factor \( K > 1 \), where

1) \( K = \frac{1 - \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2}{2 \cdot \|S_{12}S_{21}\|} \)

and

2a) \( \|S_{11}S_{22} - S_{12}S_{21}\| < 1 \).

An alternative 2\textsuperscript{nd} condition is that the stability measure \( B_1 \) be positive, where

2b) \( B_1 = 1 + \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2 \)