

ECE145a / 218a
Bilateral Tuned Amplifier Design: Stability

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Stability

Instability : non - zero output with zero input.

Stability theory : many equivalent versions :

Classical control system theory

Bode & Nyquist methods : find phase margin.

Find real part of closed - loop poles.

Do any lie in right half of s - plane ?

Network theory :

Analyze circuit by nodal analysis.

Find poles in $V_{out}(s)/V_{gen}(s)$.

Do any lie in right half of s - plane ?

Impedance viewpoint

Does $Z_{in}(j\omega)$ have a negative real part ?

Reflection (S) viewpoint

Is the magnitude of Γ_{in} greater than 1 ?

Stability: LaPlace Transform / Eigenvalue Method

Physical system (circuits, etc.) in small - signal limit \rightarrow transfer function.

$$\frac{V_{out}(s)}{V_{gen}(s)} \text{ or } \frac{b(s)}{a(s)} \text{ or } \frac{V_{in}(s)}{I_{in}(s)} \text{ etc} = H(s)$$

$$H(s) = c_1 \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots} = c_2 \frac{(s - s_{z1})(s - s_{z2})(s - s_{z3})\dots}{(s - s_{p1})(s - s_{p2})(s - s_{p3})\dots}$$

Impulse response :

$$h(t) = k_1 \exp(s_{p1}t) + k_2 \exp(s_{p2}t) + k_3 \exp(s_{p3}t) + \dots$$

Poles are (generally) complex :

$$s_{pi} = \sigma_{pi} + j\omega_{pi}$$

If any poles lie in right half of s - plane (σ_{pi} positive)

then $k_i \exp(s_{pi}t)$ will grow without limit.

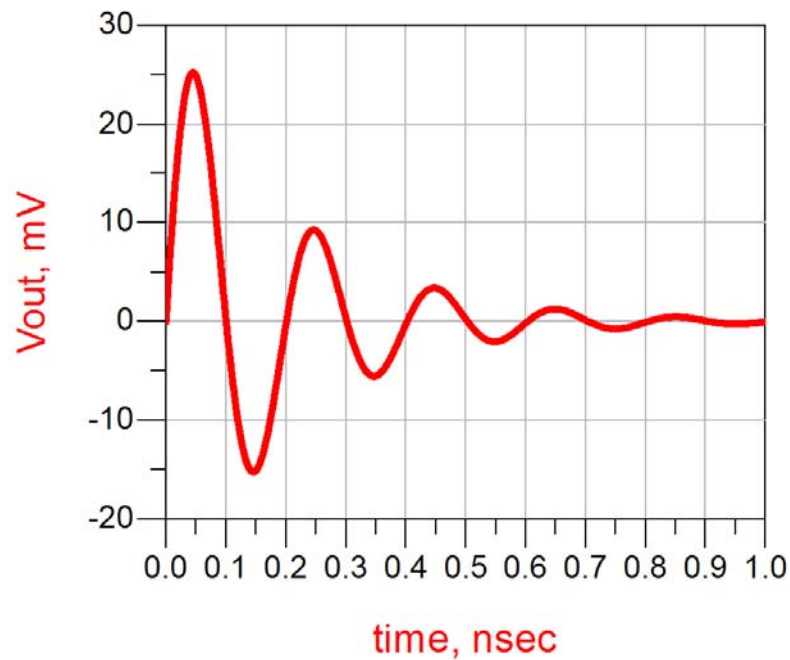
\rightarrow unstable system.

Unstable system if any pole has positive real part

$$s_{pi} = \sigma_{pi} + j\omega_{pi}; \text{ negative } \sigma_{pi}$$

→ $\exp(s_{pi}t)$ decays.

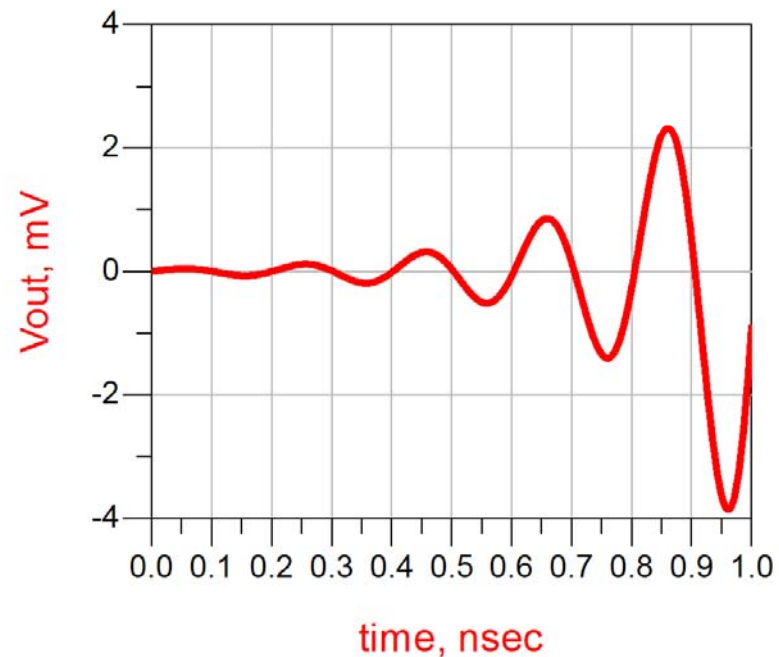
→ stable system.



$$s_{pi} = \sigma_{pi} + j\omega_{pi}; \text{ positive } \sigma_{pi}$$

→ $\exp(s_{pi}t)$ grows.

→ unstable system.



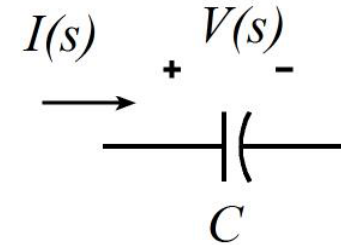
Stability from Network Viewpoint

Impedances :

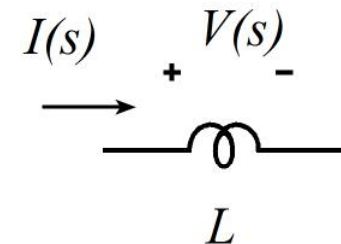
$$I(t) = Ie^{st}$$

$$V(t) = Ve^{st}$$

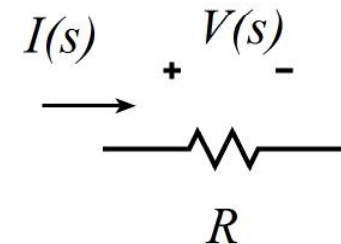
$$Z = 1/sC$$



$$Z = sL$$

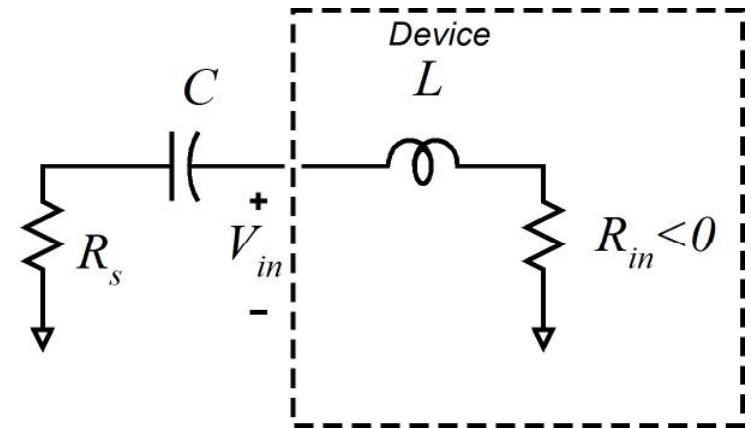


$$Z = R$$



This assumes zero initial conditions

Network Theory: One-Port Potential Instability.



$$\text{Nodal analysis: } \frac{V_{in}}{R_s + 1/sC} + \frac{V_{in}}{R_{in} + sL} = 0$$

$$\text{Hence } V_{in} = 0 \text{ (stable) or } \frac{1}{R_s + 1/s_p C} + \frac{1}{R_{in} + s_p L} = 0$$

$$R_s + 1/s_p C + R_{in} + s_p L = 0 \rightarrow s_p^2 LC + s(R_{in} + R_s)C + 1 = 0$$

$$s_{p1,2} = \left(\frac{R_{in} + R_s}{2L} \right) \pm \sqrt{\left(\frac{R_{in} + R_s}{2L} \right)^2 - \frac{1}{LC}}$$

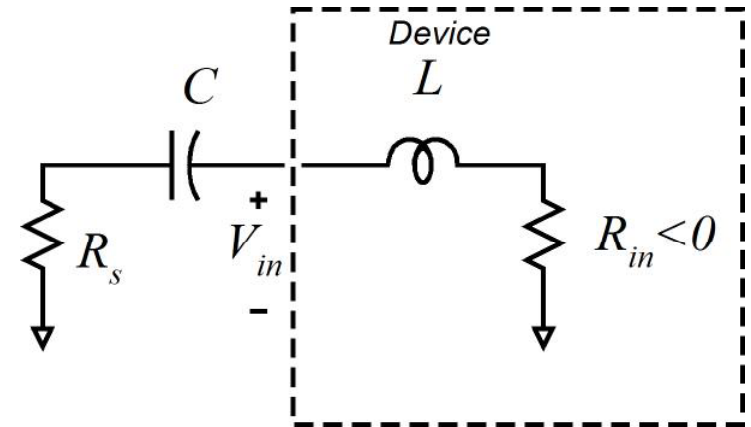
$(R_{in} + R_s)$ positive $\rightarrow \text{Re}\{s_{p1,2}\} < 0 \rightarrow \text{stable}$

$(R_{in} + R_s)$ negative $\rightarrow \text{Re}\{s_{p1,2}\} > 0 \rightarrow \text{unstable}$

Ideas: Unconditional stability, Potential instability

$(R_{in} + R_s)$ positive $\rightarrow \text{Re}\{s_{p1,2}\} < 0 \rightarrow$ stable

$(R_{in} + R_s)$ negative $\rightarrow \text{Re}\{s_{p1,2}\} > 0 \rightarrow$ unstable

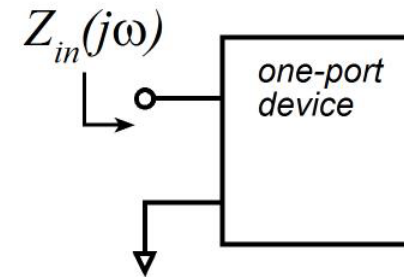


If R_{in} is positive, no (positive) value of R_s produces an unstable system
 \rightarrow device is unconditionally stable.

If R_{in} is negative, some (positive) values of R_s produces an unstable system
 \rightarrow device is potentially stable

Unconditional stability, Potential instability

From impedance viewpoint :



A one - port is unconditionally stable if :

$$\operatorname{Re}\{Z_{in}(j\omega)\} > 0 \quad \text{for all frequencies.}$$

Alternatively, a one - port is unconditionally stable if :

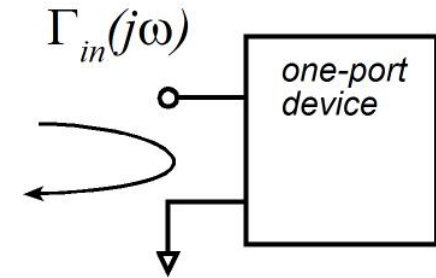
$$\operatorname{Re}\{Y_{in}(j\omega)\} > 0 \quad \text{for all frequencies.}$$

If $\operatorname{Re}\{Z_{in}(j\omega)\} < 0$ for some frequencies,
then the device is potentially unstable.

Potential instability: Reflection Viewpoint

From reflection viewpoint :

$$\Gamma_{in}(j\omega) = \frac{Z_{in}(j\omega) - Z_0}{Z_{in}(j\omega) + Z_0}$$



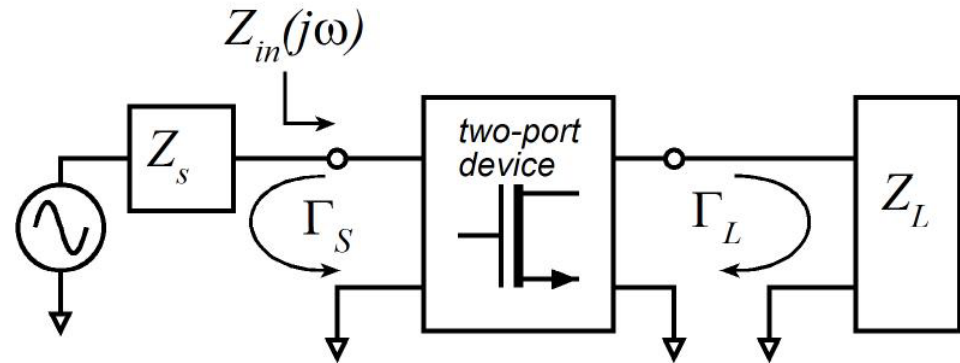
A one - port is unconditionally stable if :

$$\|\Gamma_{in}(j\omega)\| < 1 \quad \text{for all frequencies.}$$

If $\|\Gamma_{in}(j\omega)\| > 1$ for some frequencies,
then the device is potentially unstable.

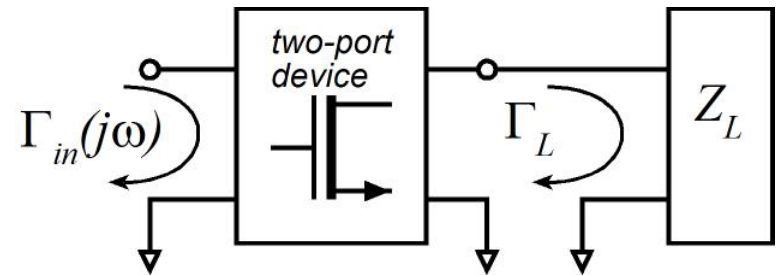
Stability of a Two-Port: Look at Γ_{in}

There are two degrees of freedom :
 Γ_S and Γ_L .



Think of it as a 1 - port :

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$

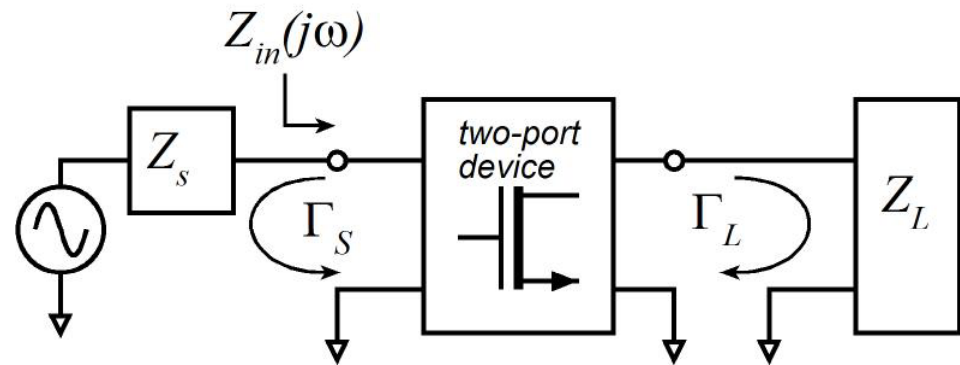


The device is unconditionally stable if $\|\Gamma_{in}(j\omega)\| < 1$
 for all load reflection coefficients Γ_L such that $\|\Gamma_L\| \leq 1$

Why do we not consider $\|\Gamma_L\| > 1$?

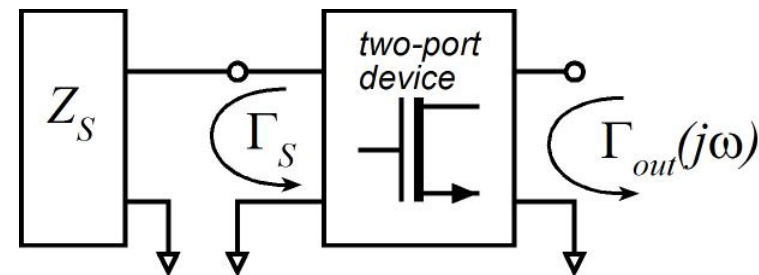
Stability of a Two-Port: Look at Γ_{out} Instead.

There are two degrees of freedom :
 Γ_S and Γ_L .



Think of it as a 1 - port :

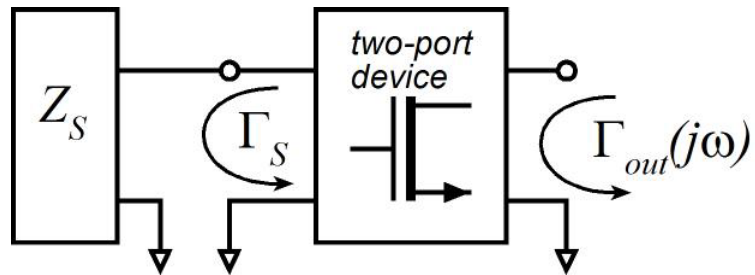
$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$



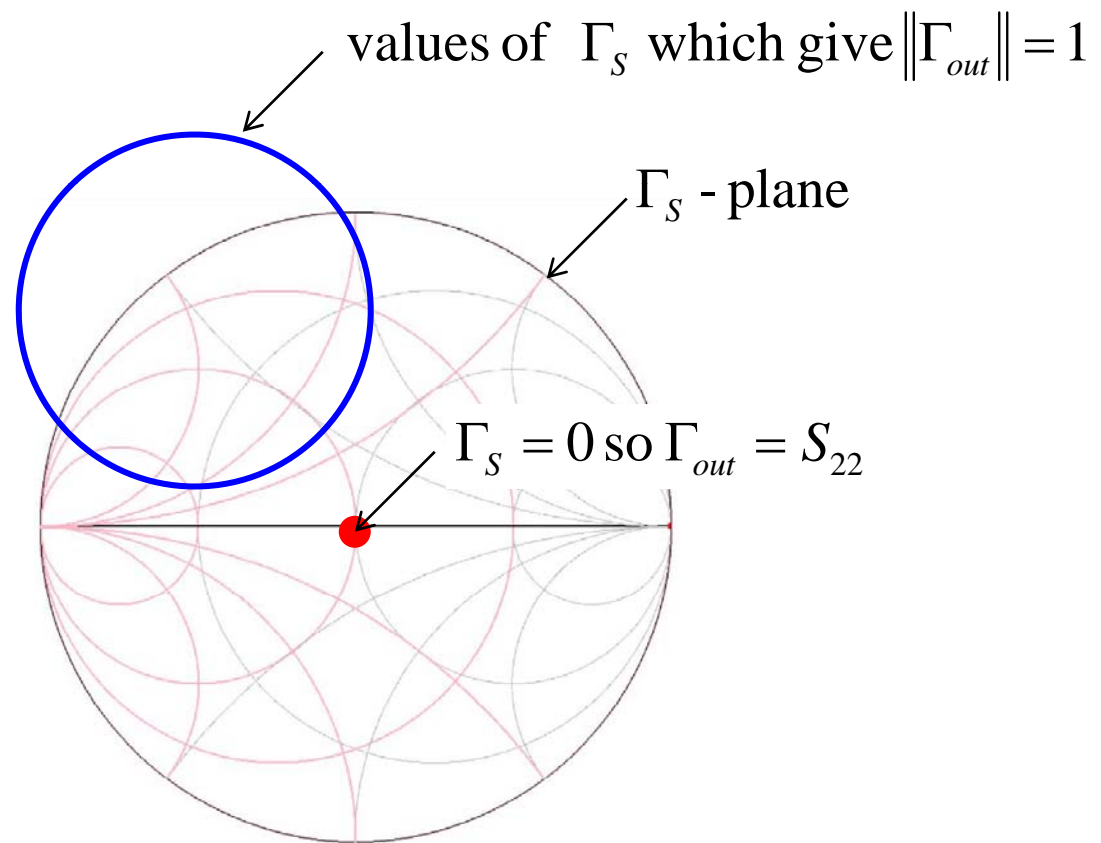
The device is unconditionally stable if $\|\Gamma_{out}(j\omega)\| < 1$
 for all load reflection coefficients Γ_S such that $\|\Gamma_S\| \leq 1$

Why do we not consider $\|\Gamma_S\| > 1$?

Input Stability Circle

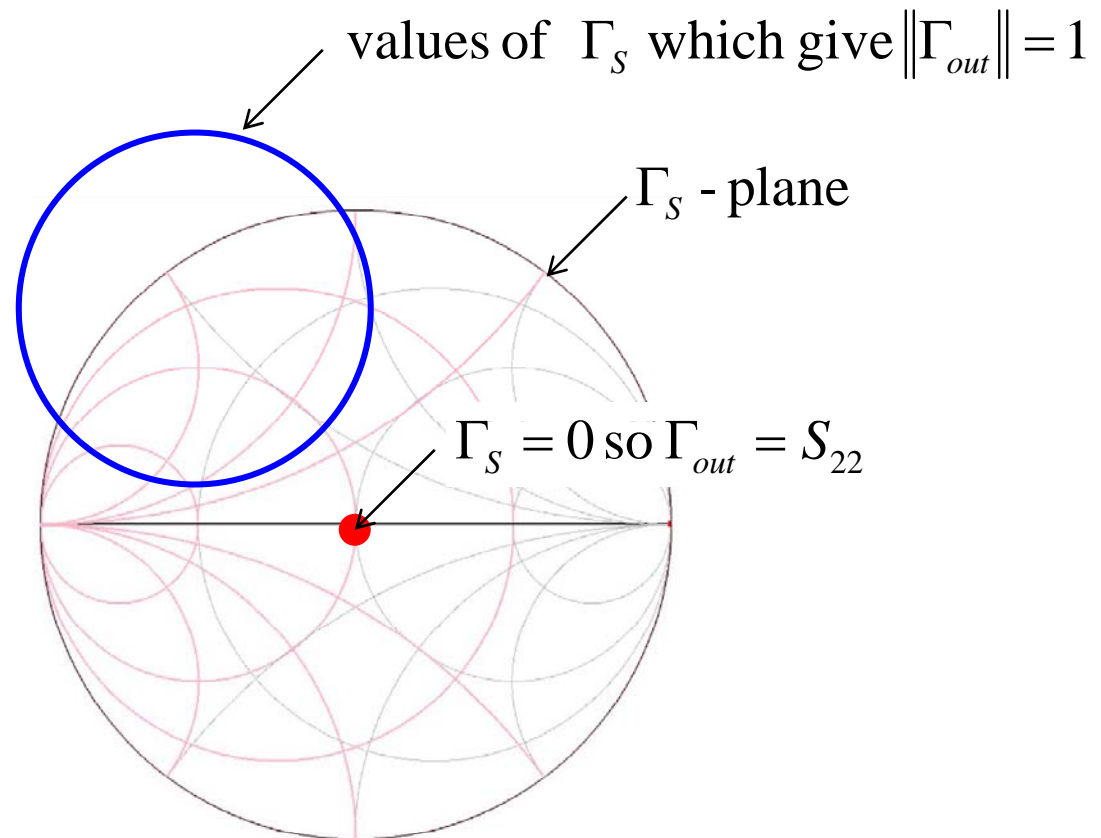


$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$



Is the inside or the outside of the circle stable ?

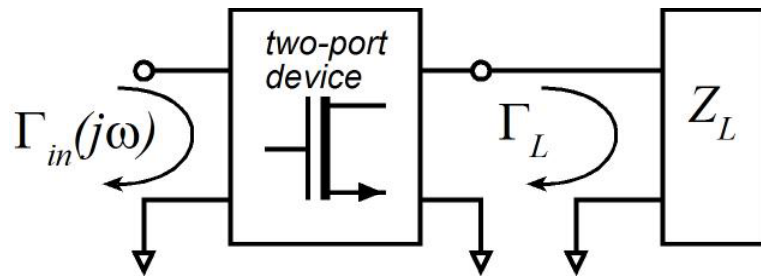
$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$



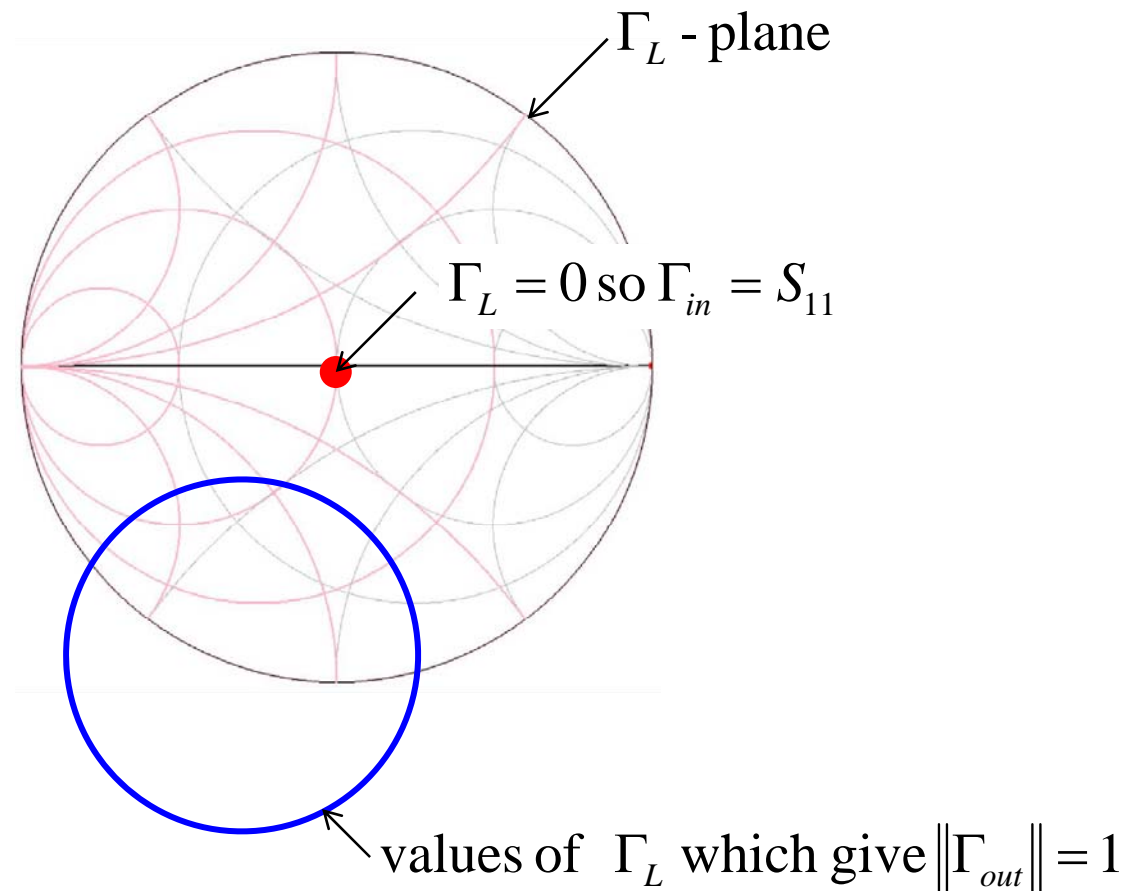
If $\|S_{22}\| < 1$, then the center of the Smith chart is stable.

If $\|S_{22}\| > 1$, then the center of the Smith chart is unstable.

Output Stability Circle

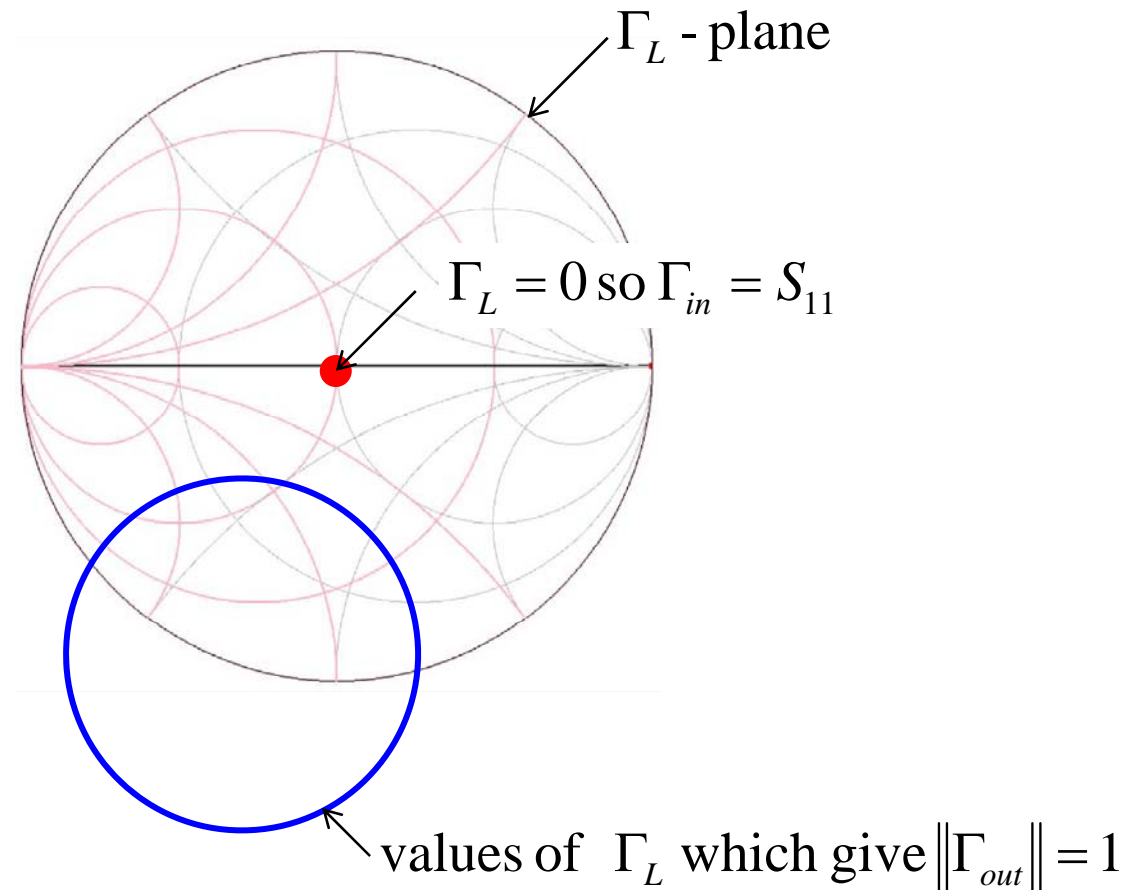


$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



Is the inside or the outside of the circle stable ?

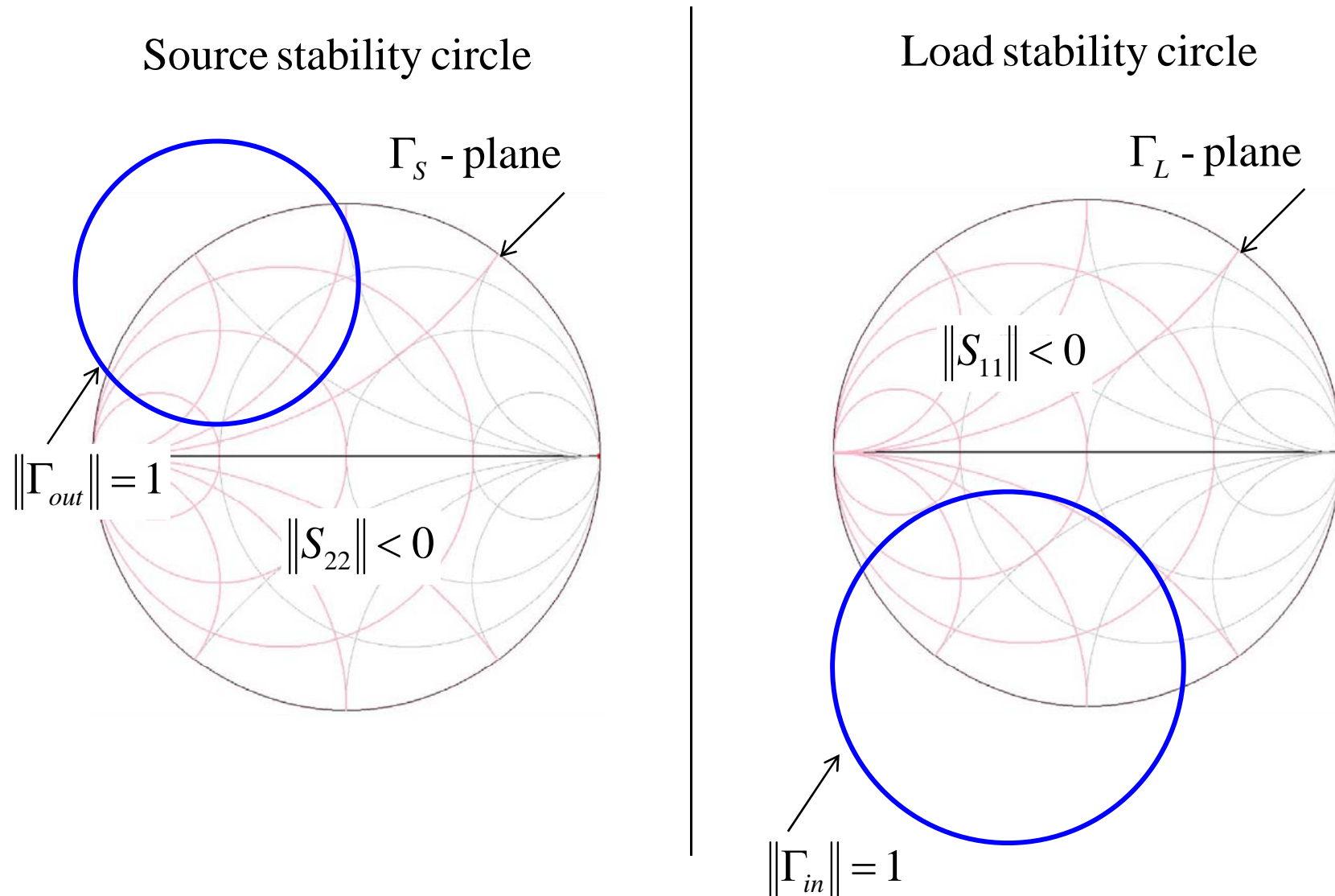
$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



If $\|S_{11}\| < 1$, then the center of the Smith chart is stable.

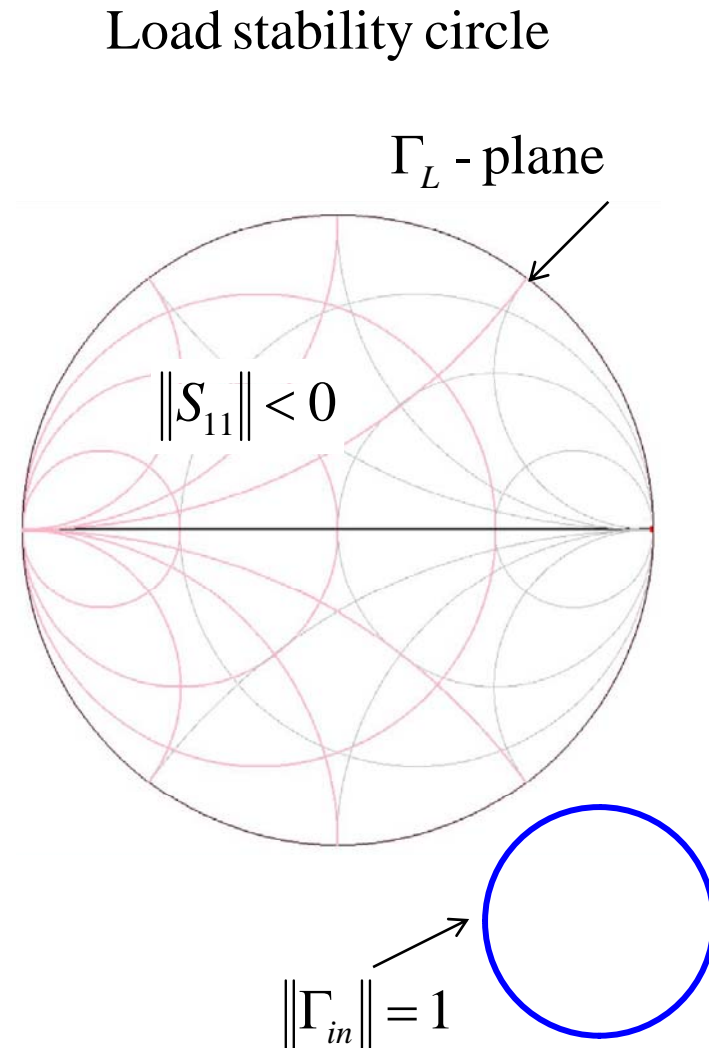
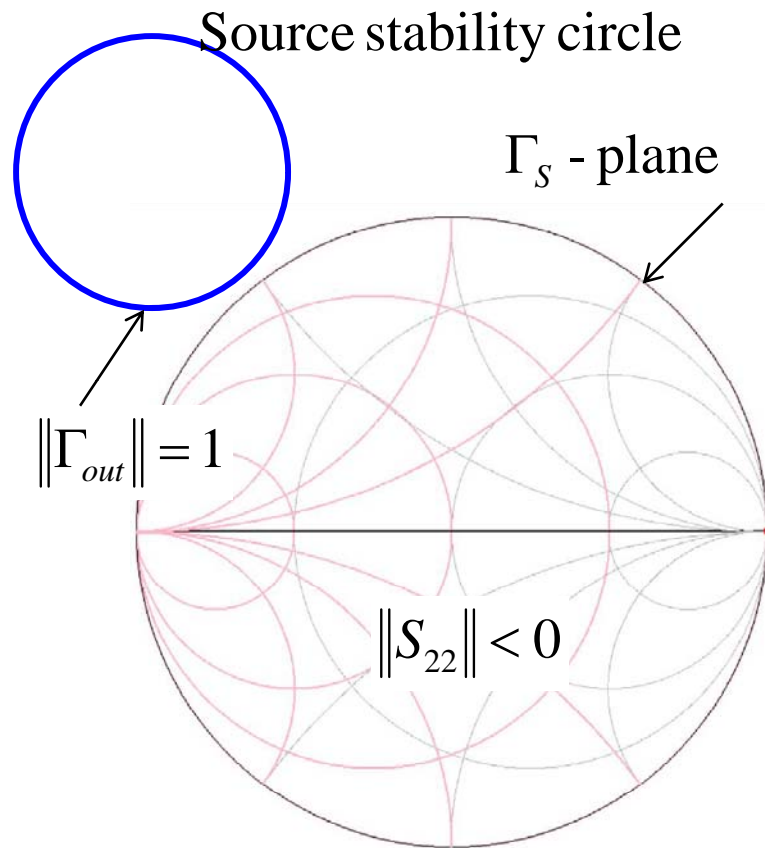
If $\|S_{11}\| > 1$, then the center of the Smith chart is unstable.

Potentially Unstable Amplifier



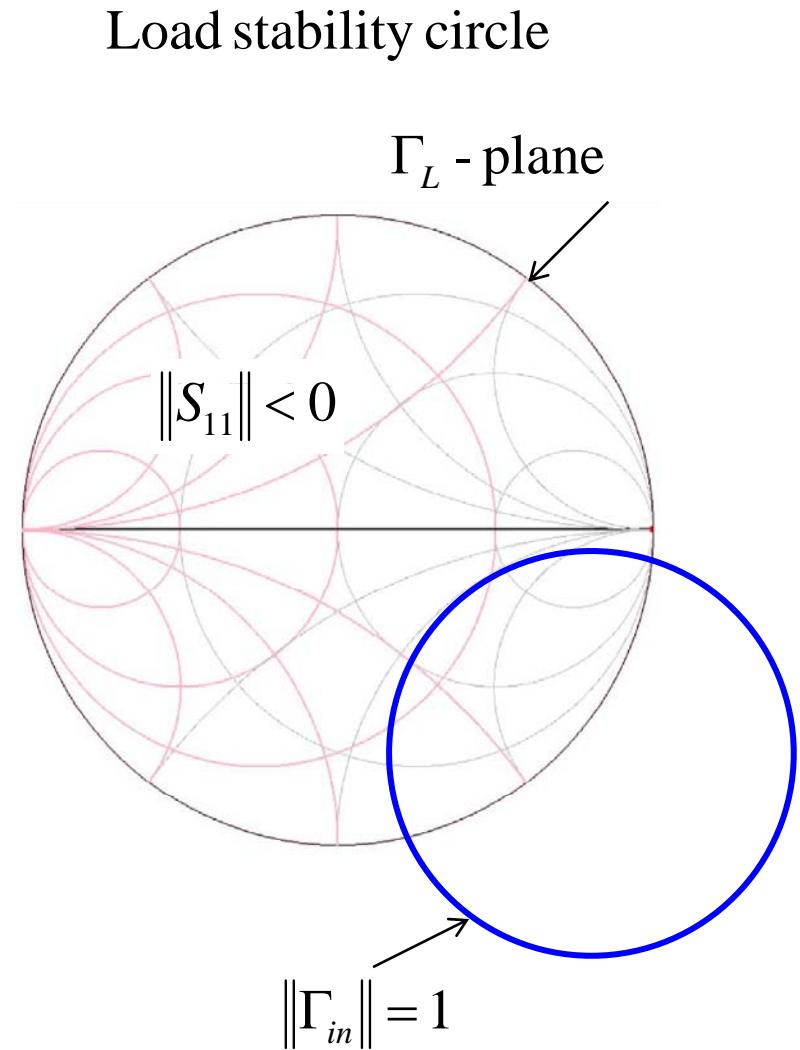
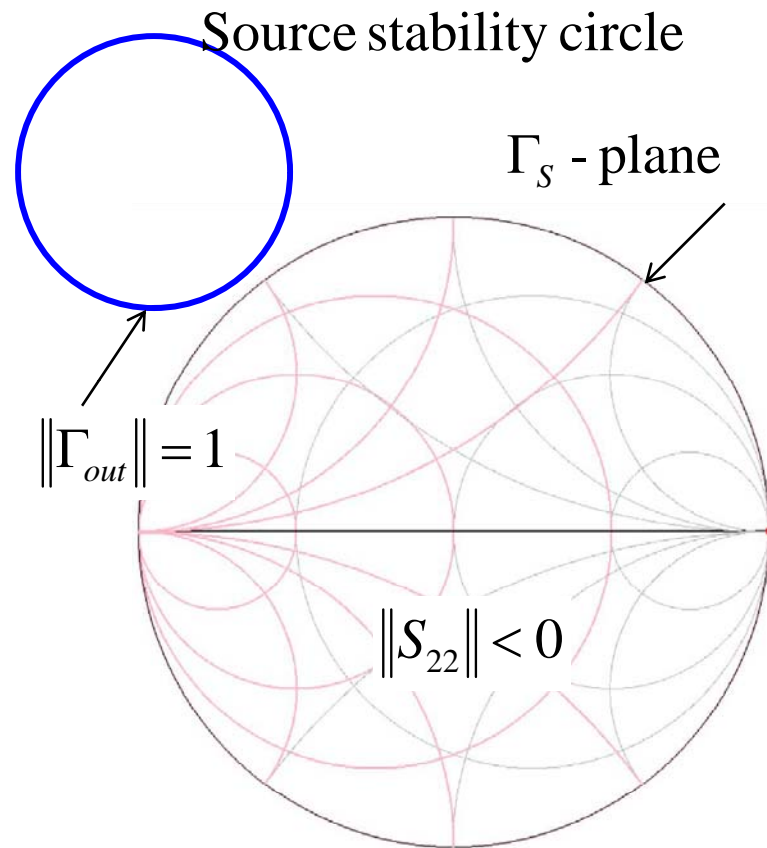
This is a test at one specific frequency; must test at all frequencies.

Unconditionally stable Amplifier



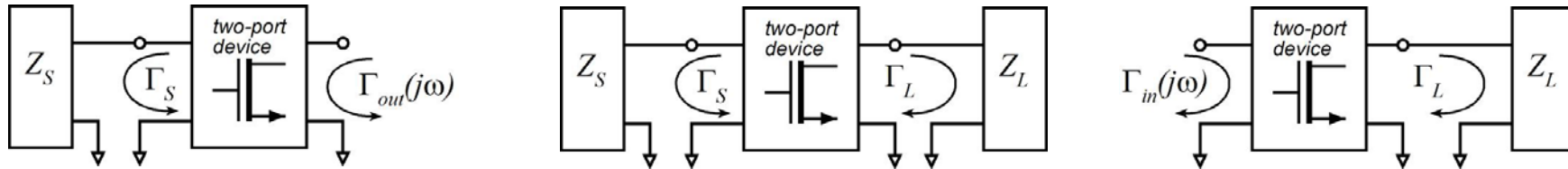
This is a test at one specific frequency; must test at all frequencies.

Is this possible ?????



Stop and think clearly...

We need check only one stability circle

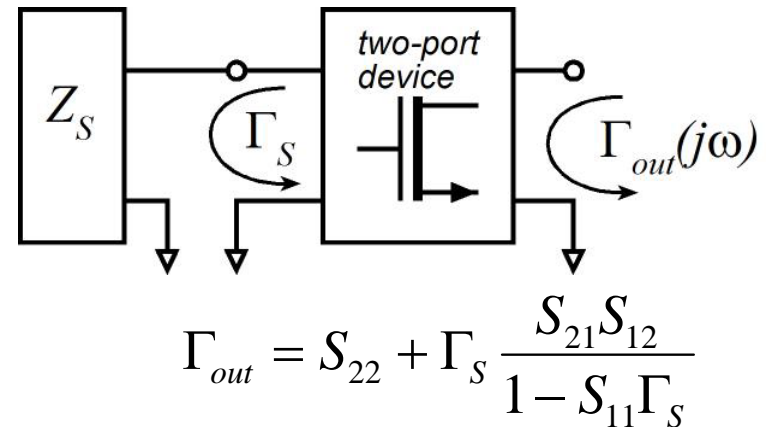
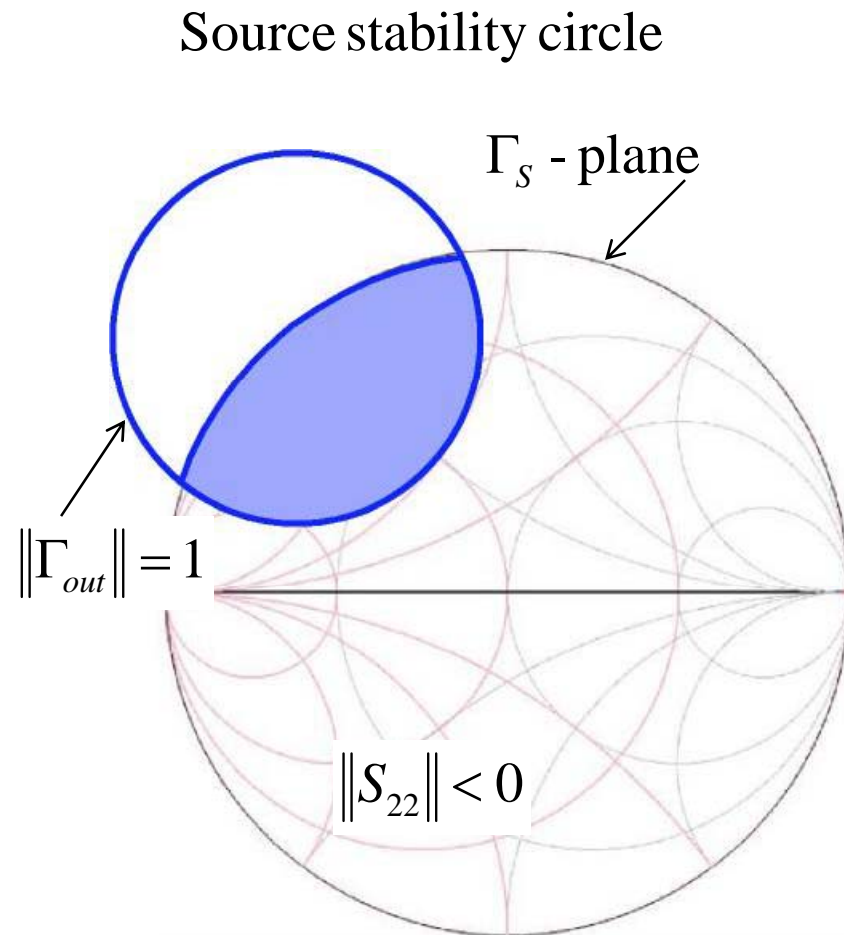


If there is no Γ_L for which $\|\Gamma_{in}\| > 1$,
then no combination of Γ_S and Γ_L can cause oscillation.

If there is no Γ_S for which $\|\Gamma_{out}\| > 1$,
then no combination of Γ_S and Γ_L can cause oscillation.

If one stability circle passes the stability test, then so must the other.

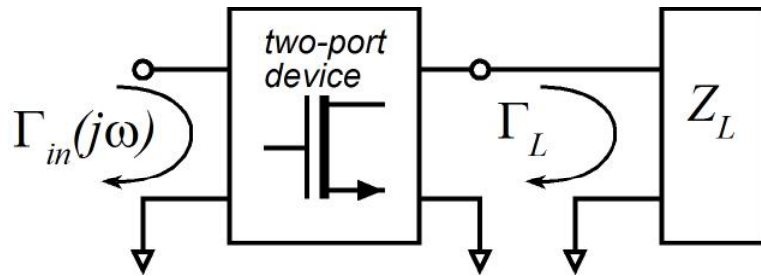
"Safe" and "Unsafe" Impedances



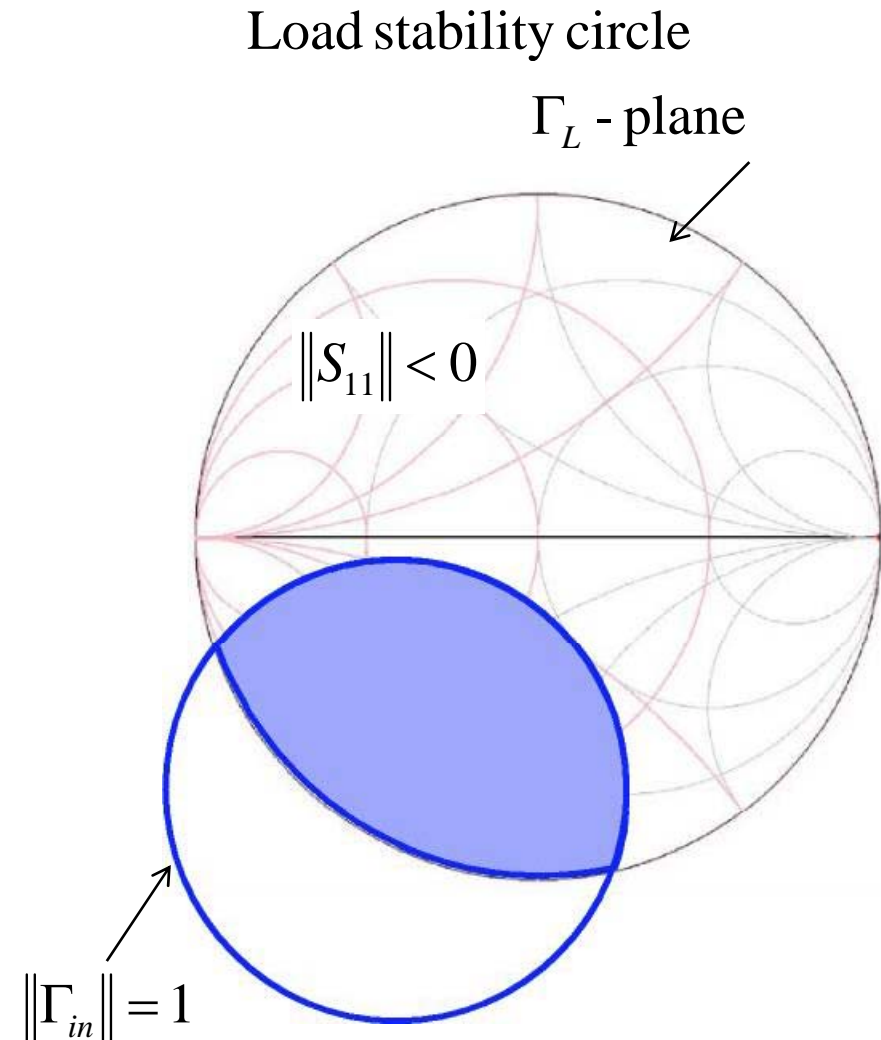
Γ_S outside the shaded region gives $\|\Gamma_{out}\| < 1$, cannot oscillate with any Γ_L .

Γ_S inside in the shaded region gives $\|\Gamma_{out}\| > 1$, might oscillate given with wrong Γ_L .

"Safe" and "Unsafe" Impedances



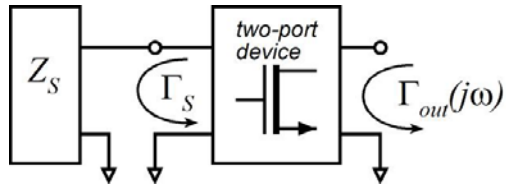
$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



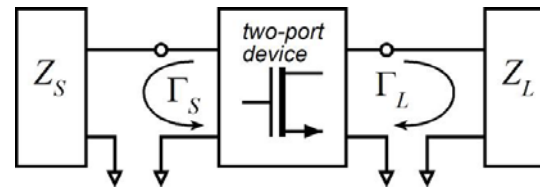
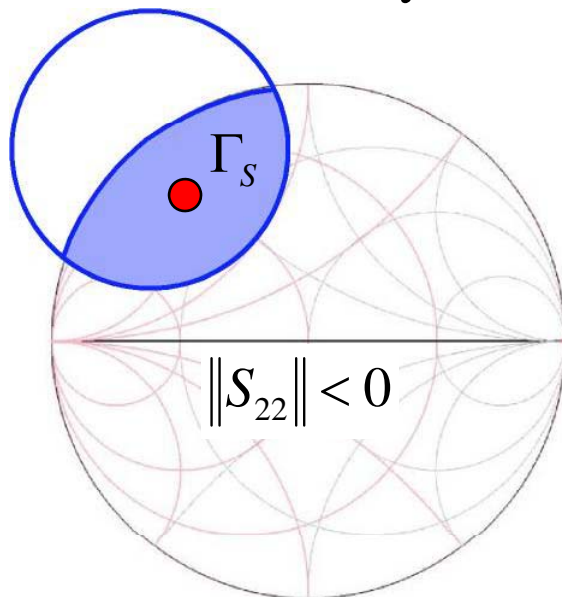
Γ_L outside the shaded region gives $\|\Gamma_{in}\| < 1$, cannot oscillate with any Γ_S .

Γ_L inside in the shaded region gives $\|\Gamma_{in}\| > 1$, might oscillate given with wrong Γ_S .

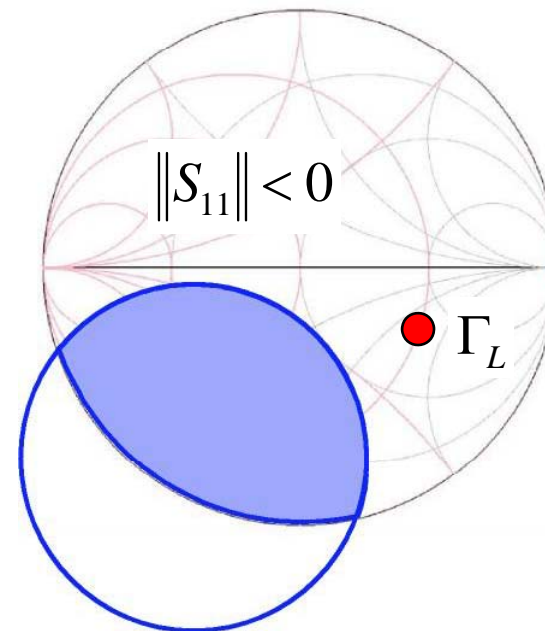
Stable Interfaces to a Potentially Unstable Device



Source stability circle



Load stability circle



If either Γ_S or Γ_L lies outside the danger zones, the circuit will be stable.

Stability Factors

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \qquad \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

If there is no Γ_L for which $\|\Gamma_{in}\| > 1$, the network is unconditionally stable .

If there is no Γ_S for which $\|\Gamma_{out}\| > 1$, the network is unconditionally stable .

A 2 - port is unconditionally stable if the Rollet stability factor $K > 1$, where

$$1) K = \frac{1 - \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2}{2 \cdot \|S_{12}S_{21}\|}$$

and

$$2a) \|S_{11}S_{22} - S_{12}S_{21}\| < 1.$$

An alternative 2nd condition is that the stability measure B_1 be positive, where

$$2b) B_1 = 1 - \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2$$