ECE ECE145B (undergrad) and ECE218B (graduate)

Mid-Term Exam. February 20, 2013

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) *AFTER STATING THEM.*

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Problem 1, 30 points

Circuit Noise calculations
To the left is shown a representation of a bipolar transistor amplifier, and below it the BJT small signal noise model.

Note that the only transistor parasitic element is the finite current gain $\beta$ and hence the presence of the small signal resistance $R_{ne}$.

The base is biased at current $I_{bo}$, producing DC collector current $I_{co} = \beta I_{bo}$. The inductor and capacitor are both very large (infinite inductive reactance, infinite capacitive susceptance).

Part a, 20 points
We will assume that the generator $(V_{gen}, R_{gen})$ has thermal noise at an associated temperature of 300 Kelvin. Device by source transposition the spectral density of the total input-referred noise voltage, including the contributions of the amplifier and of the generator. Please reduce your answer to an algebraic expression involving *only* the following terms: $kT$, $q$, $I_{co}$, $\beta$, $R_{gen}$.

expression for $S_{E_{in}} =$

\[
\frac{4kT R_{gen}}{\beta}\cdot \frac{2qI_{co}}{R_{gen}^2 + 2kT}\cdot \frac{kT}{qI_{co}}\cdot \left[1 + \frac{R_{gen}}{8kT}\right]^{-\frac{3}{2}}
\]

\[\text{thermal} \quad \text{base shot} \quad \text{collector shot noise}\]
\[ E_{\text{in}} = E_{\text{gen}} + I_6 b R_{\text{gen}} + \frac{I_{\text{ic}}}{g_{\text{m}}} \left[ 1 + \frac{R_{\text{em}}}{R_{\text{ic}}} \right] \]

\[ I_{\text{c}} = 4 I_{\text{c}} R_{\text{gen}} \]

\[ V_{\text{gs}} = 2 g I_{\text{c}} \]

\[ R_{\text{se}} = \frac{1}{g_{\text{m}}} = \frac{1}{3 kT} \]

\[ R_{\text{se}} = \frac{1}{g_{\text{m}}} = \frac{1}{3 kT} \]

\[ V_{\text{nt}} = 4 I_{\text{c}} R_{\text{gen}} + 2 g I_{\text{c}} R_{\text{gen}}^2 + \frac{2 g I_{\text{c}}}{(g_{\text{m}} kT)^2} \left[ 1 + \frac{R_{\text{em}}}{R_{\text{ic}}} \right]^2 \]

\[ = 4 I_{\text{c}} R_{\text{gen}} + \frac{2 g I_{\text{c}}}{1/gT} R_{\text{gen}}^2 + 2 kT \frac{4 kT}{g I_{\text{c}}} \left[ 1 + \frac{R_{\text{em}}}{g I_{\text{c}}} \right]^2 \]
Part b, 10 points

Adjusting the collector current (by adjusting the base bias current) will cause the total input-referred noise voltage to vary. What value of collector bias current gives the smallest input-referred noise?

**Hint: please simplify the calculus by assuming that R_{be} is much larger than R_{gm}**

expression for $I_{c,\text{opt}} =$

\[
I_{c,\text{opt}} = \sqrt{\beta} \cdot \frac{KT}{8 R_{gm}} \quad \text{from which}
\]

\[
g_m/ I_{c,\text{opt}} = R_{gm} / \sqrt{\beta}
\]

we simplify by taking $R_e \gg R_{gm}$, which gives:

\[
3 \quad \text{Sent} = \frac{1}{2} K T R_{gm} + 2 \beta I_c \frac{R_{gm}^2}{8 I_0} + 24 T \frac{K T}{8 I_0}
\]

\[
= a_1 + a_2 I_c + a_3 I_c^2
\]

From calculus, minimum is found when $a_2 I_c = a_3 I_c^2$

\[
\rightarrow I_c = \sqrt{a_3/a_2}
\]

\[
= (2 K T \frac{K T}{8})^{\frac{1}{2}} \left(\frac{1}{2 \beta R_{gm}^2} \right)^{\frac{1}{2}} = \left( \frac{\frac{2 K T \cdot K T}{8}}{2 \beta R_{gm}^2} \right)^{\frac{1}{2}} = \left( \frac{K T \cdot K T \cdot \beta}{2 \cdot 8 \cdot \beta \cdot R_{gm}^2} \right)
\]

\[
= \frac{1}{\sqrt{\beta}} \frac{K T}{8 R_{gm}} \quad \text{note this implies}
\]

\[
g_m = \frac{K T}{8 I_c} = \frac{R_{gm}}{\sqrt{\beta}}
\]
Problem 2, 20 points

More circuit noise calculations.
A two-stage FET amplifier is shown at the right.

Ignore DC bias considerations; you don't need these.

The FETs have zero parasitic capacitances, zero parasitic gate, source, and drain resistances.
Both FETs have 10 mS transconductance and a channel noise parameter $\Gamma = 1.5$. $R_{v1} = R_{v2} = 1\, \text{k}\Omega$, $R_{\text{gen}} = 100$ Ohms. Find the spectral density of the total (amplifier plus generator) input-referred noise voltage.

$$S_{\text{in}} = 4.33 \times 10^{-20} \, \nu^2 / \text{Hz}$$
(give units)
\[ S_{\text{neq}} = 4kT R_{\text{gm1}} + \frac{4kT I_1^2}{g_{\text{m1}}} + \frac{4kT}{R_{\text{L1}} g_{\text{m1}}} + \frac{4kT I_2^2}{g_{\text{m2}} g_{\text{m1}} R_{\text{L2}}} + \frac{4kT}{R_{\text{L2}} g_{\text{m2}} g_{\text{m1}}^2 R_{\text{L2}}^2} \]

- **Q1 thermal drain**: \( 2.48 \times 10^{-14} \text{ V}^2/\text{Hz} \)
- **Q2 thermal drain**: \( 2.18 \times 10^{-20} \text{ V}^2/\text{Hz} \)

\[ \text{generator} \quad \begin{cases} 166.10^{-16} \text{ V}^2/\text{Hz} \\ 2.48 \times 10^{-14} \text{ V}^2/\text{Hz} \end{cases} \]

\[ \text{output} \begin{cases} 166.10^{-16} \text{ V}^2/\text{Hz} \\ 2.48 \times 10^{-20} \text{ V}^2/\text{Hz} \end{cases} \]
Problem 3, 30 points

En-In models of circuits, noise figure
A FET is biased with an input resistor $R_{gg}$ as shown.

The FET has
- zero parasitic capacitances,
- zero parasitic gate, source, and drain resistances.
- a channel noise parameter of $\Gamma$

Part a, 10 points
Calculate from the above the spectral density of $E_{na}$ and of $I_{na}$, and their cross-spectral density

expression for $S_{E_{na}} = \frac{4\pi T L I_{g}}{g_{m}}$

expression for $S_{I_{na}} = \frac{4\pi T L I_{g}^2}{g_{m} R_{g}} + \frac{4\pi T L I_{g} T_{1}}{g_{m} R_{g}}$

expression for $S_{E_{na}I_{na}} = \frac{4\pi T L I_{g}^2}{g_{m} R_{g}}$
\[
\begin{align*}
\text{Ind}_{Iq} &= E_{Na} \\
\text{Ind}_{Iq} + \frac{\text{Ind}}{g_{m}R_{q}} &= I_{A} \\
S_{E_{Na}} &= \frac{4kT I_{q}^2}{g_{m}^2} = \frac{4kT I_{q}^2}{g_{m}^2} \\
S_{E_{Na}} &= \frac{4kT I_{q}^2}{R_{q}} + \frac{4kT I_{q}^2}{g_{m}^2 R_{q}} \\
S_{\text{Ind}_{Iq}} &= \frac{1}{g_{m}} S_{\text{Ind}} \frac{1}{g_{m} R_{q}} + \frac{1}{g_{m}^2 R_{q}} S_{\text{Ind}} \\
S_{\text{Ind}_{Iq}} &= \frac{1}{g_{m} R_{q}} S_{\text{Ind}} = \frac{1}{g_{m} R_{q}} 4kT I_{q}^2 \\
S_{\text{Ind}_{Iq}} &= \frac{4kT I_{q}^2}{g_{m} R_{q}}
\end{align*}
\]
Part b. 10 points
We now have a different circuit with

\[ S_{E_{na}} = 10^{-18} V^2 / Hz, \]
\[ S_{I_{na}} = 10^{-22} A^2 / Hz, \]

and \[ S_{E_{na} I_{na}} = 10^{-21} W / Hz. \]

If the generator resistance is 100 Ohms, find the spectral density of the total input-referred noise voltage (including that of the generator).

\[ S_{E_{in,j}} = \frac{3.68 \times 10^{-18} V^2}{Hz}. \] (give units)
Part c, 10 points

Continuing with the same circuit, i.e.
\[ S_{E_{na}} = 10^{-18} V^2 / Hz, \]
\[ S_{I_{na}} = 10^{-22} A^2 / Hz, \]
and \[ S_{F_{in,ref}} = 10^{-21} W / Hz. \]

If the generator resistance is 100 Ohms, find the noise figure.

\[
\text{noise figure} = \frac{2.33.1}{\text{linear units or dB}}
\]

(specify whether the answer is in linear units or dB)

\[ 5 \]

If the amplifier were not present, then

\[ S_{E_{in}} = 4kT R_p = 1.65 \cdot 10^{-18} V^2 / Hz. \]

\[ S_0 = \frac{3.68 \cdot 10^{-18} V^2 / Hz}{1.65 \cdot 10^{-18}} = 3.68 \]

\[ = 2.33.1 \text{ in linear units} \]

\[ = 3.67 \text{ dB} \]
Problem 4, 20 points

Signal/noise calculations

Part a, 10 points

We are now analyzing a generator whose noise voltage is $E_{n, gen}$, connected to an amplifier whose total noise voltage is $E_{n, a}$.

![Diagram showing a generator and amplifier circuit]

We are working in a music recording studio, for which $R_{gen}$ is standardized at 600 Ohms for microphones. The generator noise is thermal at 300K. $E_{n, a}$ has a spectral density whose square root is 5 nV/Hz$^{1/2}$. Working with a standard audio system bandwidth of 20Hz-20kHz, what RMS voltage is required from $V_{gen}$ to obtain a 30 dB signal/noise ratio?

What available signal power does that correspond to?

RMS value of $V_{gen} = \frac{5 \times 6 \mu V}{\sqrt{2}}$ (give units)

Available signal power from the generator = $2.9 \text{ pW}$ (give units)
\( \text{Gain} \) \[ R_{\text{in}} \quad \text{Gain} \quad \text{Gain} \quad E_{\text{in}} \]

1. \( S_{\text{eq}} = 4 \pi T R_{\text{in}} = 9.94 \times 10^{-19} \text{ V}^2/\text{Hz} \)

2. \( S_{\text{inc}} = (5 \sqrt{\text{V}/\text{Hz}})^2 = 25 \times 10^{-18} \text{ V}^2/\text{Hz} \)

3. \( S_{\text{tot}} = S_{\text{eq}} + S_{\text{inc}} = 5.5 \times 10^{-17} \text{ V}^2/\text{Hz} \)

Bandwidth \( = 20 \text{kHz} - 20 \text{kHz} \approx \text{approximately} \ 20 \text{kHz} \)

\[ f(\text{V}^2) = 3.5 \times 10^{-17} \text{ V}^2/\text{Hz} \cdot 2 \times 10^4 \text{ Hz} = 7.0 \times 10^{-12} \text{ V}^2 \]

\[ V_{\text{in}} = \sqrt{f(\text{V}^2)} = 2.64 \mu\text{V} \]

For 30dB SNR, we need \( V_{\text{in}} \approx 100 \mu\text{V} \) bigger than this.

\[ V_{\text{in}} = 83.6 \mu\text{V} \]

Power \( = \frac{(83.6 \mu\text{V})^2}{4 \times 600 \Omega} = 2.9 \times 10^{-12} \text{ W} = 2.9 \text{ pW} \)

Note \( \frac{2.9 \text{ pW}}{2 \times 10^12 \text{ W}} = 1.46 \times 10^{-16} \text{ W/Hz} \)
Part b. 10 points
We are now analyzing a radio receiver.

The antenna radiation resistance is at 300K. The antenna has negligible conductor resistance. The amplifier has 3 dB noise figure.

The receiver is receiving QPSK data at 100 megabits/second data rate.

If we use ideal raised-cosine filters with zero excess bandwidth ($\beta=0$), what receiver bandwidth do we need?
receiver bandwidth:\[ 100 \text{ Mbit/s} \] Hz

If no error-correcting code is used, we need a signal/noise ratio of 36:1 to obtain $10^{-9}$ bit error rate. What is the corresponding received signal power?
Signal power = \[ -75.3 \text{ dBm} \] (give units)
or 30pW] Either answer fine.

\[ P_s = Q^2 \cdot KT \cdot FB \]
\[ = 36 \cdot KT \cdot (2) \cdot 10^{-6} + 3 \cdot 10^{-11} \text{ W} = -75.3 \text{ dBm} \]

\[ P_s = 10 \log (8) - 173.8 \text{ dBm} (1/2) + 3 \text{ dB} + 10 \log \left( \frac{1 \text{ MHz}}{1 \text{ Hz}} \right) \]
\[ = 15.56 \text{ dBm} - 173.8 \text{ dBm} + 3 \text{ dB} + 8 \text{ dB} \]
\[ = -75.2 \text{ dBm} \]