ECE 145B / 218B, notes set 11: PLLs and Synthesizers

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**Elementary Phase Lock Loop**

Loop consists of:
- Phase detector
- Loop filter
- Voltage controlled oscillator

Loop operation:
- Detector measures phase difference of VCO and reference
- Phase error signal amplified and filtered
- Error signal adjusts VCO phase and frequency
- With good design / luck, VCO phase tracks that of reference
Mixer as Phase Detector

\[ V_{RF}(t) = V_{RF} \cos(\omega_{RF} t + \theta_{RF}) \]
\[ V_{VCO}(t) = V_{VCO} \sin(\omega_{VCO} t + \theta_{VCO}) \]
\[ \Delta \omega = \omega_{VCO} - \omega_{RF}; \, \Delta \theta = \theta_{VCO} - \theta_{RF} \]

Idealized mixer:
\[ V_{\theta}(t) = V_{RF}(t)V_{VCO}(t) / V_x = (V_{RF} V_{VCO} / V_x) (\sin(\Delta \omega \cdot t + \Delta \theta) + \text{term at } (\omega_{RF} + \omega_{VCO}) \]

Real mixer:
\[ V_{\theta}(t) = A_{mixer} \cdot V_{RF}(t) \cdot \frac{V_{VCO}(t)}{\| V_{VCO}(t) \|} = A_{mixer} \cdot V_{RF} (\sin(\Delta \omega \cdot t + \Delta \theta) + \text{terms at } (\omega_{RF} + N\omega_{VCO}) \]

If \( \Delta \omega = 0 \rightarrow \text{Phase detector} \rightarrow \quad V_{\theta}(t) = A_{mixer} \cdot V_{RF} \sin(\Delta \theta) = K_{pd} \sin(\Delta \theta) \]

For small \( \Delta \theta, \quad V_{\theta}(t) = K_{pd}(\Delta \theta) \]
XOR Gate as Phase Detector

\[ V_{RF}(t) = +1, -1, +1, -1, \ldots \text{squarewave (binary sequence), frequency } \omega_{RF}, \text{Phase } \theta_{RF} \]

\[ V_{VCO}(t) = +1, -1, +1, -1, \ldots \text{squarewave, frequency } \omega_{LO}, \text{Phase } \theta_{LO} \]

XOR Gate:

\[ V_\theta(t) = V_{RF}(t) \otimes V_{VCO}(t) \]
XOR Gate as Phase Detector

Linear voltage - phase characteristics only if signal period is long compared to switching times.
VCO Characteristics

VCO Frequency: \( \omega_{VCO}(t) = \omega_0 + K_{VCO} V_C(t) \)

\( K_{VCO} \) : rad/sec/volt

Taking the deviation relative to that of \( \omega_0 \), the VCO phase is

\[ \theta_{VCO}(t) = K_{VCO} \int V_C(t) \, dt \]

In the LaPlace domain:

\[ \theta_{VCO}(s) = K_{VCO} V_C(s) / s \]
Loop Transmission, loop transfer function

Phase detector: \( V_\theta = K_{pd} (\theta_R - \theta_{VCO}) \)

Loop Filter: \( H_F(s) \)

VCO: \( \theta_{VCO}(s) = K_{VCO} V_C(s) / s \)

Feedback loop transmission: 
\[
T(s) = K_{pd} H_F(s) K_{VCO} / s
\]

Phase Transfer function
\[
\frac{\theta_{VCO}(s)}{\theta_{REF}(s)} = \frac{T(s)}{1+T(s)}
\]

Design of \( T(s) \) follows standard feedback loop theory.
Loop Transmission, loop transfer function

Assume Loop Filter is integrator with compensating zero:

\[ H_F(s) = \frac{(1 + s\tau_z)}{s\tau_i} \]

Loop Transmission:

\[
T(s) = \frac{K_{pd}H_F(s)K_{VCO}}{s} = \frac{K_{pd}K_{VCO}(1 + s\tau_z)}{s^2\tau_i} = \left(\frac{\omega_x}{s}\right)^2 \cdot \left(1 + s/\omega_{zero}\right)
\]

\(T(s)\) decreases at -40 dB/decade, *would* cross through 0dB @ \(\omega_x\).

The compensating zero (typically \(\sim 50\%\) of \(\omega_{loop}\)) increases phase margin, ...and increases the loop bandwidth \(\omega_{loop}\) to somewhat above \(\omega_x\).

\[
\omega_x^2 = \frac{K_{pd}K_{VCO}}{\tau_i} \quad \text{and} \quad \omega_{zero} = 1/\tau_i; \text{it is helpful to check units.}
\]
Why Use Loop Integrator? → Static Phase Error!

Suppose we did not use a loop integrator

\[ H_F(s) = H_0 \]

and \( T(s) = \frac{K_{pd} H_0 K_{VCO}}{s} \)

VCO frequency: \( \omega_{VCO} = \omega_0 + K_{VCO} V_c \)

\[ \rightarrow V_c = (\omega_{VCO} - \omega_0) / K_{VCO} \]
and \( V_\theta = (\omega_{VCO} - \omega_0) / K_{pd} H_0 \)

\[ \rightarrow \theta_{\text{Ref}} - \theta_{VCO} = (\omega_{VCO} - \omega_0) / K_{pd} H_0 K_{VCO} \]

As we tune the VCO frequency, the phase of the VCO deviates from that of the reference → phase error.

Static phase error is avoided by making \( H_f(s) \) infinite at DC.
**Phase Error With Frequency Ramp**

Assume a loop integrator

\[ H_F(s) = \frac{(1 + s \tau_z)}{s \tau_i} \quad \text{at low frequencies} \rightarrow \frac{1}{s \tau_i} \]

\[ T(s) = \frac{K_{pd} K_{VCO} (1 + s \tau_z)}{s^2 \tau_i} = \left( \frac{\omega_x}{s} \right)^2 \cdot \left(1 + s / \omega_{zero}\right) \]

VCO frequency: \( \omega_{VCO} = \omega_0 + K_{VCO} V_C \)

PLLs are often used to scan signal frequencies: \( d \omega_{VCO} / dt = \text{constant} \).

\( \rightarrow (dV_C / dt) = (d \omega_{VCO} / dt) / K_{VCO} \)

The loop filter is an integrator: \( V_\theta \equiv \tau_i (dV_C / dt) = (d \omega_{VCO} / dt)(\tau_i / K_{VCO}) \)

\( \rightarrow \theta_{\text{Ref}} - \theta_{VCO} = (d \omega_{VCO} / dt)(\tau_i / K_{VCO}K_{pd}) \)

A fixed frequency scan rate causes a fixed phase deviation of the VCO from that of the reference. \( \rightarrow \) *ramp* phase error.

Ramp phase error can be avoided by adding another integrator and compensating zero to \( H_f(s) \).
PLL Slew Rate.

Given a finite frequency scan rate \( d\omega_{VCO} / dt = \text{constant} \).

\[
(dV_c / dt) = (d\omega_{VCO} / dt) / K_{VCO}
\]

The loop filter is an integrator: \( V_\theta \approx \tau_i (dV_c / dt) = (d\omega_{VCO} / dt)(\tau_i / K_{VCO}) \)

In the last page, we had assumed \( V_\theta (t) = K_{pd} (\Delta \theta) \).

This gave \( \theta_{\text{Ref}} - \theta_{VCO} = (d\omega_{VCO} / dt)(\tau_i / K_{VCO} K_{pd}) \).

If the phase detector has sinusoidal characteristics: \( V_\theta (t) = K_{pd} \sin(\Delta \theta) \)

Then the maximum range of \( V_\theta \) is \( \pm K_{pd} \)

Maximum Frequency scan rate.

\[
\| d\omega_{VCO} / dt \|_{\text{max}} = K_{VCO} K_{pd} / \tau_i = \omega_x^2
\]

Faster frequency slew rates can be obtained by replacing the mixer-based phase detector with an up/down counter: maximum measurable phase range is then the maximum numerical count times 360 degrees.
Maximum Loop Acquisition Bandwidth

If $V_{\text{REF}}$ is at frequency $\omega_{\text{REF}}$, and $V_{\text{VCO}}$ is at frequency $\omega_{\text{VCO}}$, then $V_\theta$, the output of the phase detector, is at frequency $\pm(\omega_{\text{REF}} - \omega_{\text{VCO}})$.

If this frequency is much greater than the loop bandwidth $\omega_{\text{loop}}$, then the loop will not respond; no significant error signal is generated to bring the VCO into lock with the reference.

PLLs will not acquire lock if $(\omega_{\text{REF}} - \omega_{\text{VCO}})$ is more than a few times $\omega_{\text{loop}}$.

Even approximate analysis of loop capture dynamics is complex, and beyond our scope.
Frequency Difference Detector

\[ V_{RF} \sin(\omega_{RF} t + \theta_{RF}) \]

\[ V_{VCO} \cos(\omega_{VCO} t) \]

\[ V_{VCO} \sin(\omega_{VCO} t) \]

\[ I \]

\[ Q \]

\[ \tau \]

Analysis to be posted online later...
Data Pattern Driven Detector: BPSK

\[ V_{RF}\sin(\omega_{RF}t + \theta_{RF}) \]
\[ V_{VCO}\cos(\omega_{VCO}t) \]
\[ V_{VCO}\sin(\omega_{VCO}t) \]

IQ<0
IQ>0

"Stable" states
Data Pattern Driven Detector: QPSK

\[ V_{\text{VCO}} \cos(\omega_{\text{VCO}} t) \]

\[ V_{\text{VCO}} \sin(\omega_{\text{VCO}} t) \]

\[ I \]

\[ I+Q \]

\[ I-Q \]

\[ Q \]

IQ<0

IQ>0

“Stable” states

\[ V_{\text{OUT}} \]

\[ \theta \ [\text{deg}] \]
Frequency Synthesis

Frequency divider:
a digital counter which counts
clock pulses, reaching count \( N \),
and then resetting.

→ Output frequency is \( 1/N \) times the input frequency.
(Also: output phase is \( 1/N \) times the input phase)

Divider in the feedback path
→ VCO frequency \( \omega_{VCO} \) forced to \( N\omega_{REF} \)
Frequency Synthesis

To tune the VCO frequency, the divider ratio is tuned.

This is done dynamically, by dithering the divider ratio between $N$ and $(N+1)$.

The dithering is done at a rate much larger than the loop bandwidth.

Dithering techniques (delta sigma vs. fractional frequency synthesis) are in Steve Long's notes.
**Loop Phase Transfer Function.**

![Loop Phase Transfer Function Diagram](image)

**Loop Transmission:**

\[
T(s) = \frac{K_{pd} K_{VCO} (1 + s \tau_z)}{Ns^2 \tau_i}
\]

If the loop filter is \( H_F(s) = (1 + s \tau_z) / s \tau_i \), then

\[
T(s) = \frac{K_{pd} K_{VCO} (1 + s \tau_z)}{Ns^2 \tau_i} = \frac{\omega_x^2 (1 + s / \omega_z)}{s^2}; \text{ where } \omega_x^2 = \frac{K_{pd} K_{VCO}}{N \tau_i}
\]

**Phase transfer function:**

\[
\frac{\theta_{VCO}(s)}{\theta_{REF}(s)} = N \frac{T(s)}{1 + T(s)}
\]

Note the \( N : 1 \) increase in phase deviation.
Phase Noise Transfer Functions

\[ \theta_{VCO} = \theta_{N,VCO} + \frac{K_{pd} K_{VCO} H_f(s)}{s} \left( \theta_{REF} - \theta_{VCO} / N \right) \]

\[ = \theta_{N,VCO} + \frac{K_{pd} K_{VCO} H_f(s)}{N s} \left( N \theta_{REF} - \theta_{VCO} \right) \]

\[ \theta_{VCO} = \theta_{N,VCO} + T(s) \left( N \theta_{REF} - \theta_{VCO} \right) \]

\[ \theta_{VCO} = \frac{1}{1 + T(s)} \theta_{N,VCO} + \frac{T(s)}{1 + T(s)} \left( N \theta_{REF} \right) \]

The closed-loop phase deviations \( \theta_{VCO} \) include the open-loop VCO phase deviations \( \theta_{N,VCO} \) multiplied by \( 1/(1+T) \) and the reference phase deviations multiplied by \( T/(1+T) \).